The Multiple Discrete-Continuous Extreme Value (MDCEV) model: Role of utility function parameters, identification considerations and model extensions

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Overview

- Introduction
- Functional form of utility function
- Stochastic form of utility function
- Specific model structures
- Empirical illustration
- Conclusions

Introduction

Several consumer demand choices are characterized by multiple discreteness

- Vehicle type holdings and usage
- Activity type choice and duration of participation
- Airline fleet mix and usage
- Carrier choice and transaction level
- Brand choice and purchase quantity
- Stock choice and investment amount

- Multiple discreteness
  - Choice of multiple alternatives simultaneously
- Modeling methodologies of multiple discrete situations
  - Traditional random utility-based (RUM) single discrete choice models
    - Number of composite alternatives explodes with the number of elemental alternatives
  - Multivariate probit (logit) methods
    - Not based on a rigorous underlying utility-maximizing framework of multiple discreteness
  - Other issues with these methods
    - Cannot accommodate the diminishing marginal returns (*i.e.*, satiation) in the consumption of an alternative
    - Cumbersome to include a continuous dimension of choice

- Modeling methodologies of multiple discrete situations
  - Two alternative methods proposed by Wales and Woodland (1983)
    - Amemiya-Tobin approach
    - Kuhn-Tucker approach
  - Both approaches assume a direct utility function *U(x)* that is assumed to be quasi-concave, increasing, and continuously differentiable with respect to the consumption quantity vector *x*
  - Approaches differ in how stochasticity, non-negativity of consumption, and corner solutions (*i.e.*, zero consumption of some goods) are accommodated

- Methods proposed by Wales and Woodland
  - Amemiya-Tobin approach
    - Extension of the classic microeconomic approach of adding normally distributed stochastic terms to the budget-constrained utility-maximizing share equations
    - Direct utility function *U(x)* assumed to be deterministic by the analyst, and stochasticity is introduced post-utility maximization
  - Kuhn-Tucker (KT) approach
    - Based on the Kuhn Tucker or KT (1951) first-order conditions for constrained random utility maximization
    - Employs a direct stochastic specification by assuming the utility function *U(x)* to be random (from the analyst's perspective) over the population
    - Derives the consumption vector for the random utility specification subject to the linear budget constraint by using the KT conditions for constrained optimization
    - Stochastic nature of the consumption vector in the KT approach is based fundamentally on the stochastic nature of the utility function

- Advantages of KT approach
  - Constitutes a more theoretically unified and consistent framework for dealing with multiple discreteness consumption patterns
  - Satisfies all the restrictions of utility theory
  - Stochastic KT first-order conditions provide the basis for deriving the probabilities for each possible combination of corner solutions (zero consumption) for some goods and interior solutions (strictly positive consumption) for other goods
  - Accommodates for the singularity imposed by the “adding-up” constraint
- Problems with KT approach used by Wade and Woodland
  - Random utility distribution assumptions lead to a complicated likelihood function that entails multi-dimensional integration

- Studies that used the KT approach for multiple discreteness
  - Kim *et al.* (2002)
    - Used the GHK simulator to evaluate the multivariate normal integral appearing in the likelihood function in the KT approach
    - Used a generalized variant of the well-known translated constant elasticity of substitution (CES) direct utility function
    - Not realistic for practical applications and is unnecessarily complicated
  - Bhat (2005)
    - Introduced a simple and parsimonious econometric approach to handle multiple discreteness
    - Based on the generalized variant of the translated CES utility function but with a multiplicative log-extreme value error term
    - Labeled as the multiple discrete-continuous extreme value (MDCEV) model
    - MDCEV model represents the multinomial logit (MNL) form-equivalent for multiple discrete-continuous choice analysis and collapses exactly to the MNL in the case that each (and every) decision-maker chooses only one alternative
  - Several studies in the environmental economics field
    - Phaneuf *et al.*, 2000; von Haefen *et al.*, 2004; von Haefen, 2003a; von Haefen, 2004; von Haefen and Phaneuf, 2005; Phaneuf and Smith, 2005
    - Used variants of the linear expenditure system (LES) and the translated CES for the utility functions, and used multiplicative log-extreme value errors

Functional form of utility function

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left\{ \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\}$$

- $U(\boldsymbol{x})$ is a quasi-concave, increasing, and continuously differentiable function with respect to the consumption quantity vector *x*
- ψ_k , γ_k and α_k are parameters associated with good *k*

Assumptions

- Additive separability
  - All the goods are strictly Hicksian substitutes
  - Marginal utility with respect to any good is independent of the level of consumption of other goods
- Weak complementarity

- Role of ψ_k

$$\frac{\partial U(\boldsymbol{x})}{\partial x_k} = \psi_k \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1}$$

  - ψ_k : baseline (at zero consumption) marginal utility
  - $\frac{\psi_k}{\psi_l}$: marginal rate of substitution at zero consumption
  - Higher baseline ψ_k implies less likelihood of a corner solution for good *k*

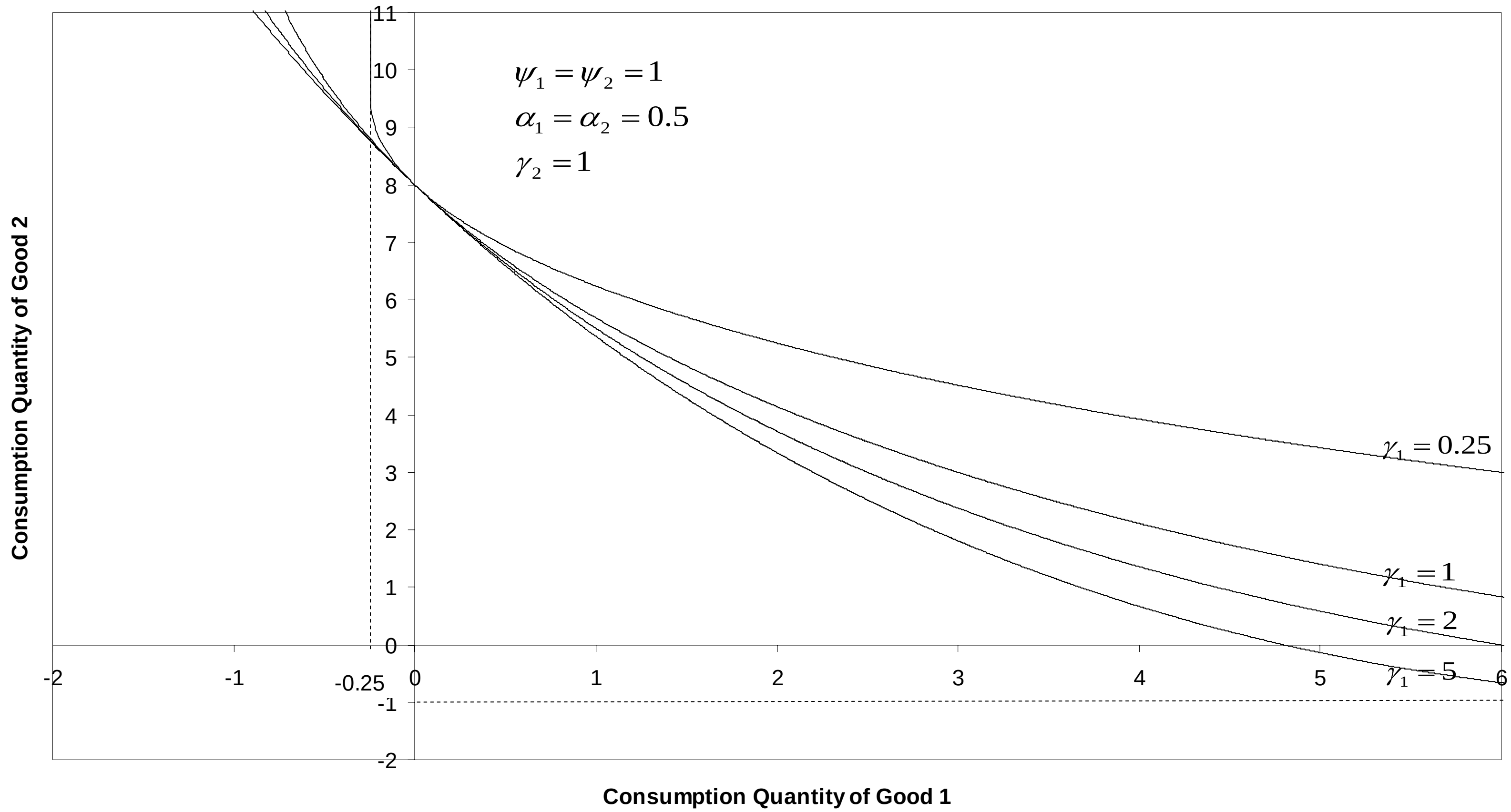
- Role of $\gamma_k (\gamma_k > 0)$

$$\text{Slope}(x_1, x_2) = \frac{\partial U(\boldsymbol{x}) / \partial x_1}{\partial U(\boldsymbol{x}) / \partial x_2} = \frac{\left(\dfrac{x_2}{\gamma_2} + 1\right)^{1-\alpha_2}}{\left(\dfrac{x_1}{\gamma_1} + 1\right)^{1-\alpha_1}} \times \frac{\psi(x_1)}{\psi(x_2)}$$

$$\text{At } x_1 = -\gamma_1, \text{ slope} = \infty$$

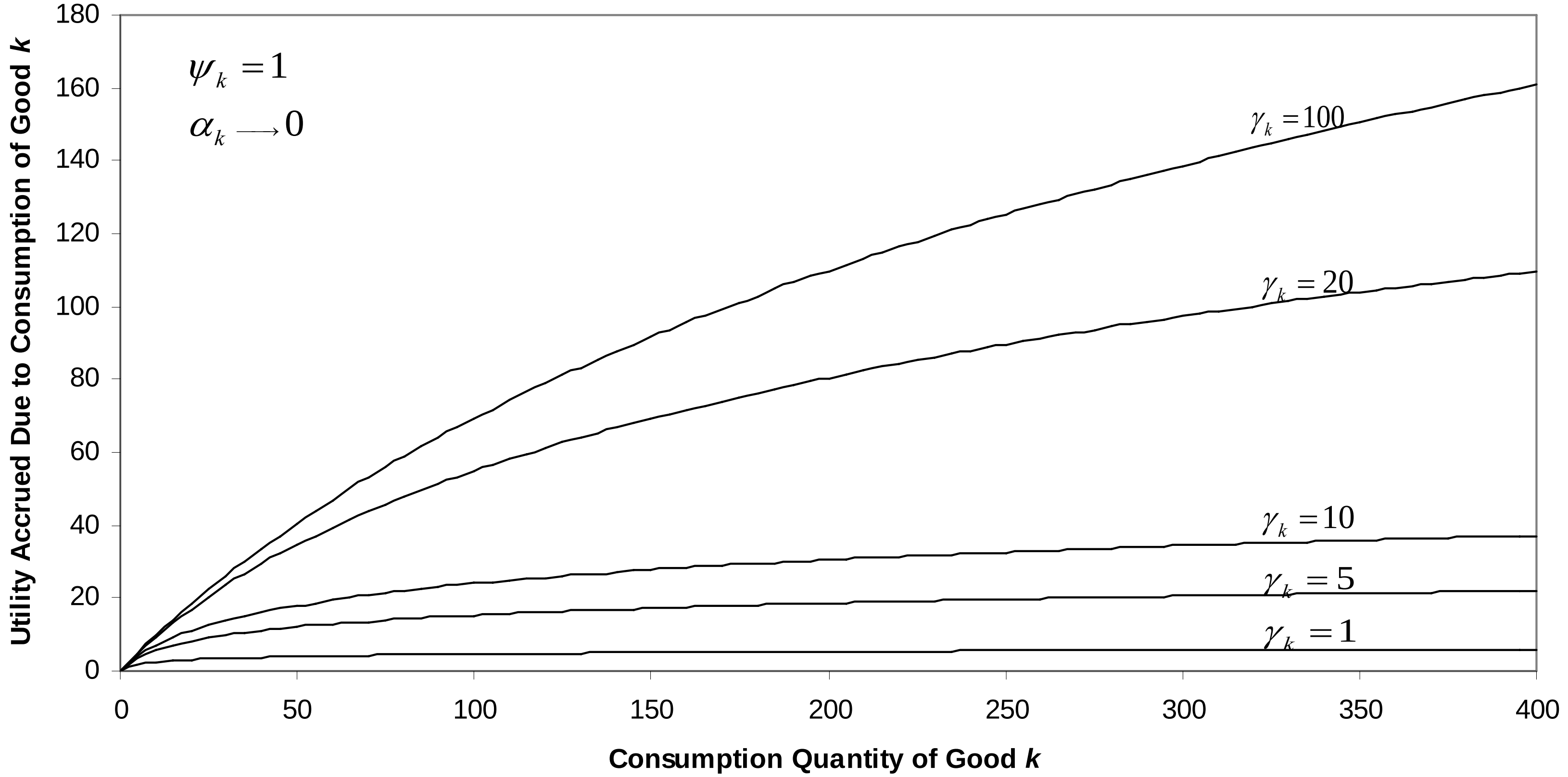
$$\text{At } x_2 = -\gamma_2, \text{ slope} = 0$$

Indifference Curves



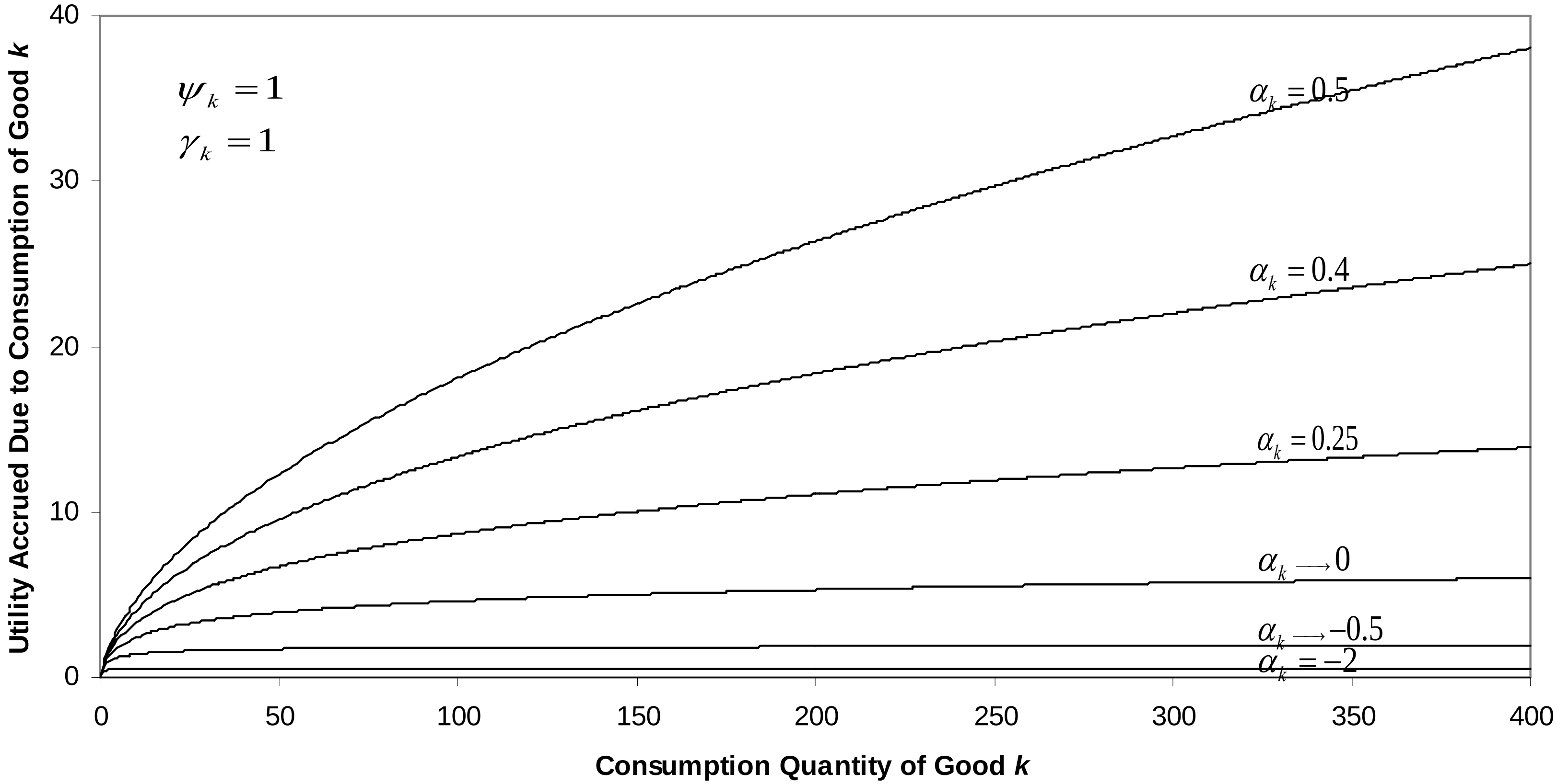
Indifference Curves Corresponding to Different Values of γ_1

Role of γ_k



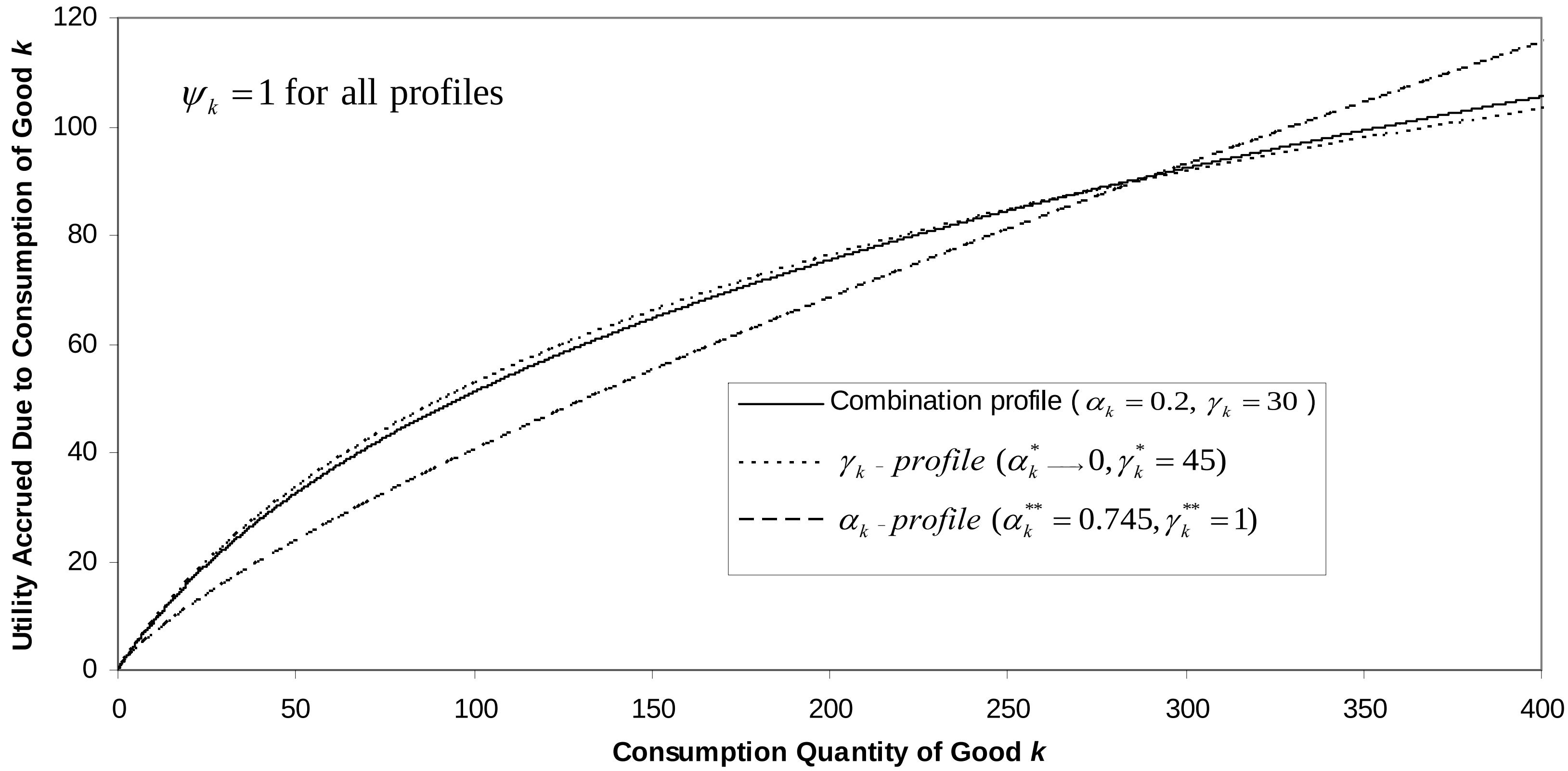
Effect of γ_k Value on Good *k*'s Subutility Function Profile

- Role of α_k



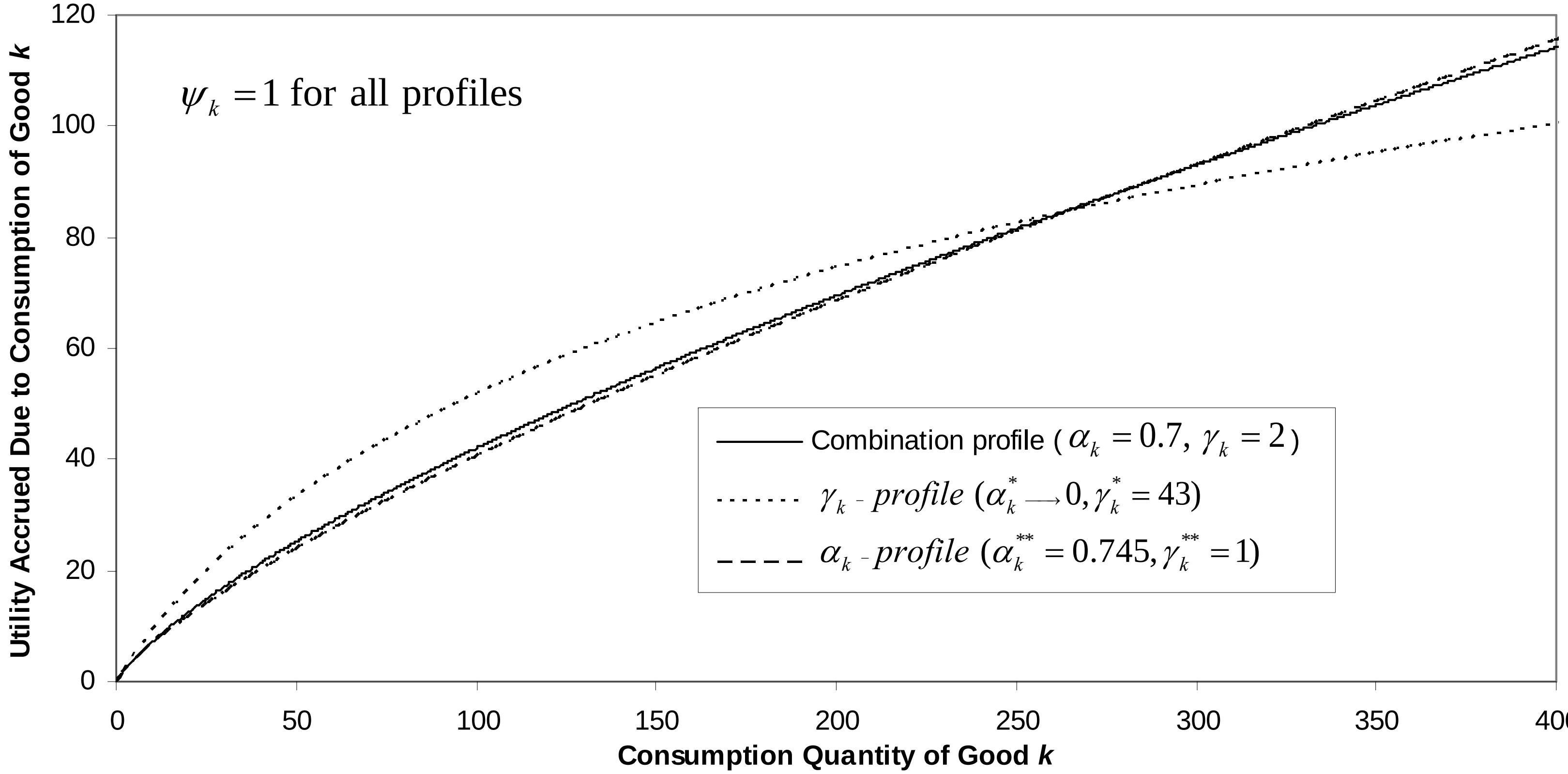
Effect of α_k Value on Good *k*'s Subutility Function Profile

- Empirical identification issues associated with utility form



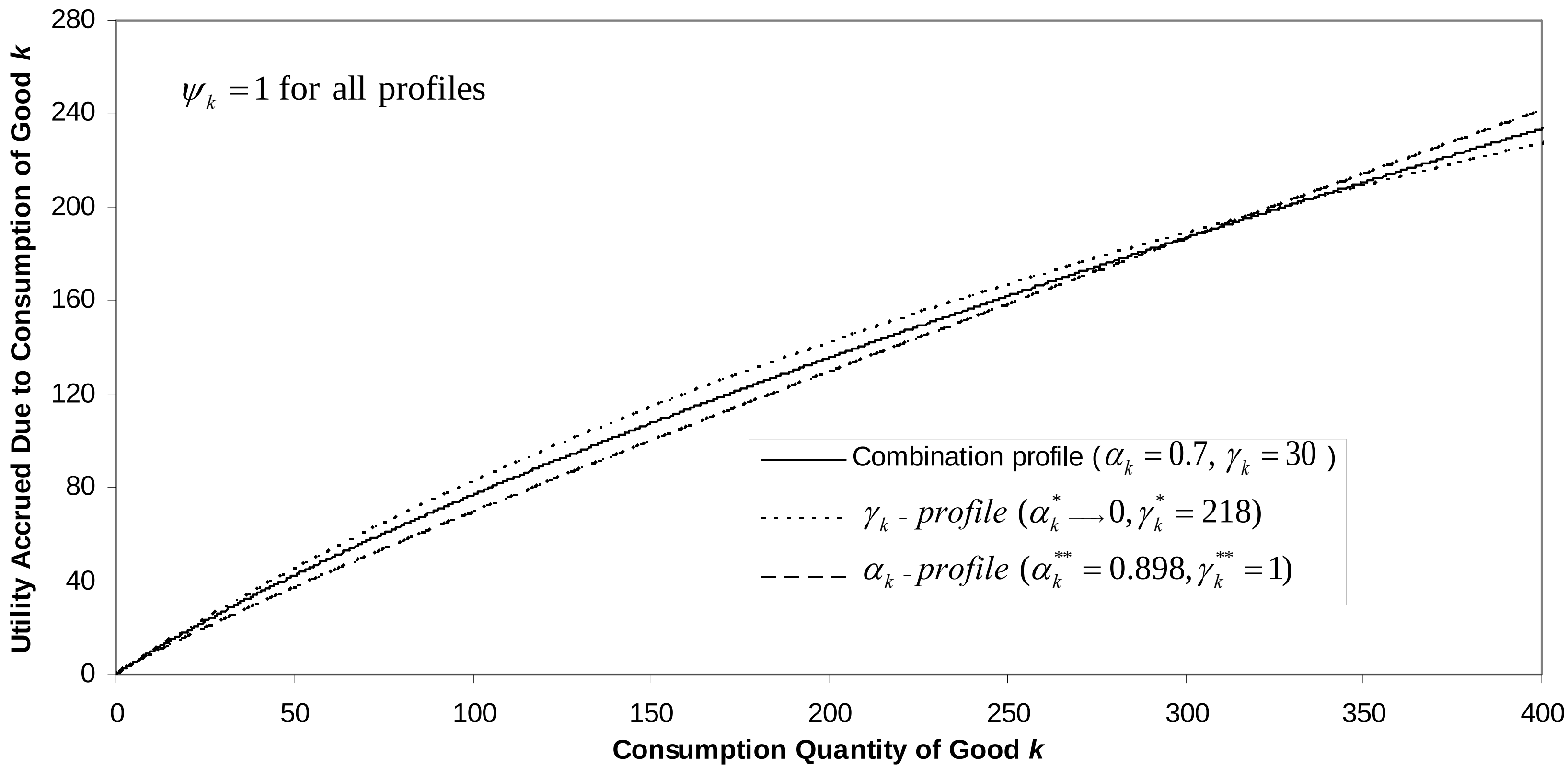
Alternative Profiles for Moderate Satiation Effects with Low α_k Value and High γ_k Value

- Empirical identification issues associated with utility form



Alternative Profiles for Moderate Satiation Effects with High α_k Value and Low γ_k Value

❖ Empirical identification issues associated with utility form



Alternative Profiles for Low Satiation Effects with High α_k Value and High γ_k Value

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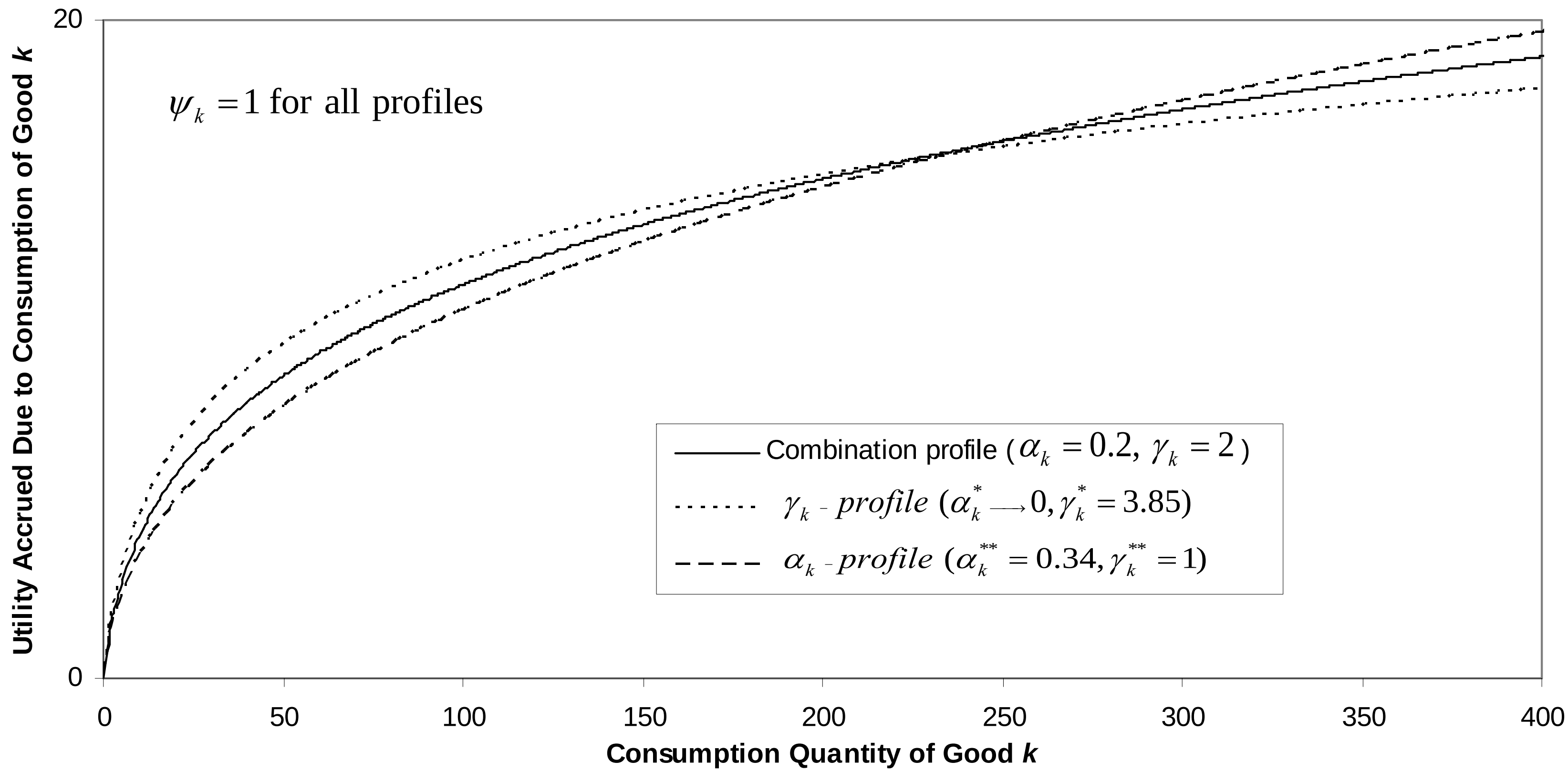


Alternative Profiles for High Satiation Effects with Low α_k Value and Low γ_k Value

Stochastic form of utility function

- Overall random utility function

$$\mathrm{U}(\boldsymbol{x}) = \sum_k \frac{\gamma_k}{\alpha_k}\left[\exp(\beta' z_k + \varepsilon_k)\right] \cdot \left\{\left(\frac{x_k}{\gamma_k} + 1\right)^{\alpha_k} - 1\right\}$$

- Random utility function for optimal expenditure allocations

$$\mathrm{U}(\boldsymbol{x}) = \sum_k \frac{\gamma_k}{\alpha_k}\left[\exp(\beta' z_k + \varepsilon_k)\right] \cdot \left\{\left(\frac{e_k}{\gamma_k p_k} + 1\right)^{\alpha_k} - 1\right\}$$

Lagrangian and KT Conditions

$$\mathscr{L} = \sum_k \frac{\gamma_k}{\alpha_k}[\exp(\beta' z_k + \varepsilon_k)]\left\{\left(\frac{e_k}{\gamma_k p_k}+1\right)^{\alpha_k} - 1\right\} - \lambda\left[\sum_{k=1}^{K} e_k - E\right]$$

$$\left[\frac{\exp(\beta' z_k + \varepsilon_k)}{p_k}\right]\left(\frac{e_k^*}{\gamma_k p_k}+1\right)^{\alpha_k - 1} - \lambda = 0, \text{ if } e_k^* > 0, k = 1, 2, \ldots, K$$

$$\left[\frac{\exp(\beta' z_k + \varepsilon_k)}{p_k}\right]\left(\frac{e_k^*}{\gamma_k p_k}+1\right)^{\alpha_k - 1} - \lambda < 0, \text{ if } e_k^* = 0, k = 1, 2, \ldots, K$$

$$\lambda = \frac{\exp(\beta' z_1 + \varepsilon_1)}{p_1}\left(\frac{e_1^*}{\gamma_1 p_1} + 1\right)^{\alpha_1 - 1}$$

- KT conditions

$$V_k + \varepsilon_k = V_1 + \varepsilon_1 \quad \text{if} \quad e_k^* > 0 \qquad (k = 2, 3, \ldots, K)$$

$$V_k + \varepsilon_k < V_1 + \varepsilon_1 \quad \text{if} \quad e_k^* = 0 \qquad (k = 2, 3, \ldots, K), \quad \text{where}$$

$$V_k = \beta' z_k + (\alpha_k - 1)\ln\left(\frac{e_k^*}{\gamma_k p_k} + 1\right) - \ln p_k \qquad (k = 1, 2, 3, \ldots, K)$$

❖ General econometric model structure and identification

$$P(e_1^*, e_2^*, e_3^*, ..., e_M^*, 0, 0, ..., 0) = |J| \int_{\varepsilon_1=-\infty}^{+\infty} \int_{\varepsilon_{M+1}=-\infty}^{V_1-V_{M+1}+\varepsilon_1} \int_{\varepsilon_{M+2}=-\infty}^{V_1-V_{M+2}+\varepsilon_1} \cdots \int_{\varepsilon_{K-1}=-\infty}^{V_1-V_{K-1}+\varepsilon_1} \int_{\varepsilon_K=-\infty}^{V_1-V_K+\varepsilon_1}$$

$$f(\varepsilon_1, V_1 - V_2 + \varepsilon_1, V_1 - V_3 + \varepsilon_1, ..., V_1 - V_M + \varepsilon_1, \varepsilon_{M+1}, \varepsilon_{M+2}, ..., \varepsilon_{K-1}, \varepsilon_K)$$

$$d\varepsilon_K d\varepsilon_{K-1} ... d\varepsilon_{M+2} d\varepsilon_{M+1} d\varepsilon_1 ,$$

where J is the Jacobian whose elements are given by:

$$J_{ih} = \frac{\partial[V_1 - V_{i+1} + \varepsilon_1]}{\partial e_{h+1}^*} \quad ; i, h = 1, 2, \ldots, M-1$$

$$P(e_1^*, e_2^*, e_3^*, ..., e_M^*, 0, 0, ..., 0) = |J| \int_{\tilde{\varepsilon}_{M+1,1}=-\infty}^{V_1-V_{M+1}} \int_{\tilde{\varepsilon}_{M+2,1}=-\infty}^{V_1-V_{M+2}} \cdots \int_{\tilde{\varepsilon}_{K-1,1}=-\infty}^{V_1-V_{K-1}} \int_{\tilde{\varepsilon}_{K,1}=-\infty}^{V_1-V_K}$$

$$g(V_1 - V_2, V_1 - V_3, ..., V_1 - V_M, \tilde{\varepsilon}_{M+1,1}, \tilde{\varepsilon}_{M+2,1}, ..., \tilde{\varepsilon}_{K,1}) d\tilde{\varepsilon}_{K,1} d\tilde{\varepsilon}_{K-1,1} ... d\tilde{\varepsilon}_{M+1,1}$$

Specific Model Structures

- The MDCEV model structure

$$P(e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0)$$

$$= |J| \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \left(\prod_{i=2}^{M} \frac{1}{\sigma} \lambda \left[\frac{V_1 - V_i + \varepsilon_1}{\sigma} \right] \right) \right\} \times \left\{ \prod_{s=M+1}^{K} \Lambda \left[\frac{V_1 - V_s + \varepsilon_1}{\sigma} \right] \right\} \frac{1}{\sigma} \lambda \left(\frac{\varepsilon_1}{\sigma} \right) d\varepsilon_1$$

$$|J| = \left(\prod_{i=1}^{M} c_i \right) \left(\sum_{i=1}^{M} \frac{1}{c_i} \right), \text{ where } c_i = \left(\frac{1-\alpha_i}{e_i^* + \gamma_i p_i} \right)$$

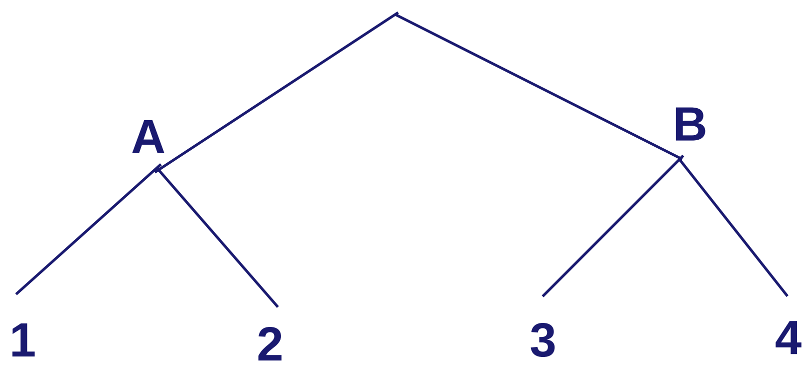
$$P(e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0) = \frac{1}{\sigma^{M-1}} \left[\prod_{i=1}^{M} c_i \right] \left[\sum_{i=1}^{M} \frac{1}{c_i} \right] \left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma} \right)^M} \right] (M-1)!$$

- Can be derived from the differenced form also

- The MDCGEV model
  - Generalized Extreme Value error structure

  - A nested logit example with four alternatives
  - Start with the following expression for density function

$$\Lambda(\varepsilon_1 < s_1, \varepsilon_2 < s_2, \varepsilon_3 < s_3, \varepsilon_4 < s_4) = \exp\left[-\left(e^{-s_1/\sigma\theta_A} + e^{-s_2/\sigma\theta_A}\right)^{\theta_A} - \left(e^{-s_3/\sigma\theta_B} + e^{-s_4/\sigma\theta_B}\right)^{\theta_B}\right]$$



- The MDCGEV model (nested logit example)

$$P(x_1^*,0,0,0) = \frac{A_{12}^{\theta_A - 1} \cdot e^{V_1/\sigma\theta_A}}{H}$$

$$P(x_1^*,x_2^*,0,0) = \frac{1}{\sigma} |J| \frac{e^{V_1/\sigma\theta_A} \cdot e^{V_2/\sigma\theta_A} \cdot A_{12}^{\theta_A - 2}}{H} \left(\frac{A_{12}^{\theta_A}}{H} + \frac{1-\theta_A}{\theta_A} \right)$$

$$P(x_1^*,0,x_3^*,0) = \frac{1}{\sigma} |J| \frac{e^{V_1/\sigma\theta_A} \cdot e^{V_3/\sigma\theta_B} \cdot A_{12}^{\theta_A - 1} B_{34}^{\theta_B - 1}}{H^2}$$

$$P(x_1^*,x_2^*,x_3^*,0) = \frac{1}{\sigma^2} |J| \frac{e^{V_1/\sigma\theta_A} \cdot e^{V_2/\sigma\theta_A} \cdot e^{V_3/\sigma\theta_B} \cdot A_{12}^{\theta_A - 2} B_{34}^{\theta_B - 1}}{H^2} \left(\frac{2A_{12}^{\theta_A}}{H} + \frac{1-\theta_A}{\theta_A} \right)$$

$$P(x_1^*,x_2^*,x_3^*,x_4^*) = \frac{1}{\sigma^3} |J| \frac{e^{V_1/\sigma\theta_A} \cdot e^{V_2/\sigma\theta_A} \cdot e^{V_3/\sigma\theta_B} \cdot e^{V_4/\sigma\theta_B} \cdot A_{12}^{\theta_A - 2} B_{34}^{\theta_B - 1}}{H^2}$$

$$\left\{ \frac{6A_{12}^{\theta_A} B_{34}^{\theta_B}}{H^2} + \frac{2(1-\theta_A)}{\theta_A} \cdot \frac{B_{34}^{\theta_B}}{H} + \frac{2(1-\theta_B)}{\theta_B} \cdot \frac{A_{12}^{\theta_A}}{H} + \left(\frac{(1-\theta_A)}{\theta_A} \right) \left(\frac{(1-\theta_B)}{\theta_B} \right) \right\}$$

$$A_{12} = \left(e^{V_1/\sigma\theta_A} + e^{V_2/\sigma\theta_A} \right)$$

$$B_{34} = \left(e^{V_3/\sigma\theta_B} + e^{V_4/\sigma\theta_B} \right)$$

$$H = A_{12}^{\theta_A} + B_{34}^{\theta_B}$$

- The Mixed MDCEV model

$$P\left(e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0\right)$$

$$= \int_{\eta} \frac{1}{\sigma^{M-1}} \left[\prod_{i=1}^{M} c_i\right] \left[\sum_{i=1}^{M} \frac{1}{c_i}\right] \left[\frac{\prod_{i=1}^{M} e^{(V_i+\eta_i)/\sigma}}{\left(\sum_{k=1}^{K} e^{(V_k+\eta_k)/\sigma}\right)^M} \right] (M-1)!\, dF(\eta).$$

Empirical Analysis

- Increasing dependence on automobiles
- Wide-ranging impacts of automobile dependency
  - Household level
  - Community level
  - Regional level

- A widely used indicator of automobile dependency is vehicle holdings and use
  - 92% of US households owned at least one motorized vehicle in 2003 (compared to 80% in the early 1970s)
  - Household VMT has increased 300% between 1997-2001 relative to a population increase of 30% during same period
- Important to examine vehicle holdings and usage
  - Travel demand forecasting
  - Transportation policy analysis

- Examine several dimensions of household vehicle holdings and usage decisions
  - Number of vehicles owned
  - Vehicle body type
  - Vehicle age (i.e., vintage)
  - Vehicle make and model
  - Vehicle usage

- Incorporate a comprehensive set of determinants of vehicle holdings and usage decisions
  - Household demographics
  - Individual characteristics
  - Vehicle characteristics
  - Built environment characteristics
- Develop a comprehensive econometric model to analyze the many dimensions of vehicle holdings and use that accommodates for
  - Multiple discreteness
  - Satiation effects

Data

- 2000 San Francisco Bay Area travel survey (BATS)
  - Designed and administered by MORPACE International Inc.
  - 2-day survey of 15000 households
  - Information on vehicle fleet mix of households, individual and household socio-demographics, individual characteristics and activity episodes
- Data on vehicle make/model attributes from secondary data sources
  - Consumer Guides
  - EPA Fuel Economy Guide
- Land use/Demographic coverage data from MTC of San Francisco Bay area
- GIS layer of bicycle facilities from MTC of San Francisco Bay area
- Census 2000 Tiger files

Sample Characteristics

- Final sample: 8107 households
- 10 motorized vehicle types
  - Coupe
  - Mini/Subcompact Sedan
  - Compact Sedan
  - Mid-size Sedan
  - Large Sedan
  - Hatchback/Station Wagon
  - Sports Utility Vehicle (SUV)
  - Pickup Truck
  - Minivan
  - Van
- 2 vintages considered for each motorized vehicle type
  - New vehicles (age of the vehicle less than or equal than 5 years)
  - Old Vehicles (age of the vehicle is more than 5 years)
- Twenty-one vehicle types/vintages studied including
  - 20 motorized vehicle type/vintages
  - Non-motorized form of transportation

Classification of Vehicle type/vintage

Vehicle Type/ Vintage

Vehicle Type/Vintage	Makes/models
Coupe Old	33 makes/models
Coupe New	23 makes/models
Sedan Mini/Subcompact Old	10 makes/models
Sedan Mini/Subcompact New	7 makes/models
Sedan Compact Old	25 makes/models
Sedan Compact New	19 makes/models
Sedan Mid-size Old	24 makes/models
Sedan Mid-size New	21 makes/models
Sedan Large Old	16 makes/models
Sedan Large New	12 makes/models
Hatchback/Station Wagon Old	23 makes/models
Hatchback/Station Wagon New	12 makes/models
SUV Old	15 makes/models
SUV New	23 makes/models
Pickup Truck Old	12 makes/models
Pickup Truck New	13 makes/models
Minivan Old	13 makes/models
Minivan New	15 makes/models
Van Old	6 makes/models
Van New	5 makes/models
Non-motorized vehicles	

Distribution of Vehicles

Number of vehicles owned by the household	Total No. of households	% of households
1	4459	55%
2	2918	36%
3	644	8%
4 or more	86	1%

Utility Structure

$$\mathrm{U} = \sum_{k=1}^{K} \left[\exp\left(\sum_{l \in N_k} \delta_{lk} w_{lk} \right) \right] (m_k + 1)^{\alpha_k}, \quad s.t$$

$$\sum_{l} \delta_{lk} = 1, \ \forall k$$

$$\sum_{k} m_k = M$$

Simplified version

$$\text{U} = \sum_{k=1}^{K} \left[\exp\left(\max_{l \in N_k} \{ w_{lk} \} \right) \right] (m_k + 1)^{\alpha_k}, \quad s.t$$

$$\sum_k m_k = M$$

Random utility structure

$$W_{lk} = \beta' x_k + \gamma' z_{lk} + \eta_{kl}$$

$\beta' x_k$ is the overall observed component utility of vehicle type k

z_{lk} is an exogenous variable vector influencing the utility of vehicle make/model l of vehicle type k

γ is a corresponding coefficient vector to be estimated

η_{kl} is an unobserved error component specific to make/model l of vehicle type k

N_k is the set of makes/models l within vehicle type k

Econometric Model

$$\eta_{kl} = \lambda_k + \lambda_{kl}$$

λ_k is a common unobserved utility component shared by all vehicle make/model alternatives of vehicle type *k*

λ_{kl} is an extreme value term distributed identically with scale parameter θ_k

$$H_k = \beta' x_k + \lambda_k + \underset{l \in N_k}{Max}\{\gamma' z_{lk} + \lambda_{kl}\} + \ln \alpha_k + (\alpha_k - 1)\ln(m_k^* + 1)$$

$$= \beta' x_k + \theta_k \ln \sum_{l \in N_k} \exp\left(\frac{\gamma' z_{lk}}{\theta_k}\right) + \ln \alpha_k + (\alpha_k - 1)\ln(m_k^* + 1) + \varepsilon_k$$

Marginal probability that the household uses first Q of the *K* vehicle types (Q ≥ 1) for annual mileages $m_1^*, m_2^*, \ldots m_Q^*$, using Multiple Discrete-Continuous Extreme Value model (MDCEV derived by Bhat, 2005):

$$P(m_1^*, m_2^*, \ldots m_Q^*, 0, 0, 0, \ldots, 0) = \left[\prod_{k=1}^{Q} r_k\right]\left[\sum_{k=1}^{Q} \frac{1}{r_k}\right]\left[\frac{\prod_{k=1}^{Q} e^{V_k}}{\left(\sum_{k=1}^{K} e^{V_k}\right)^Q}\right](Q-1)! \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where,

$$r_k = \left(\frac{1-\alpha_k}{m_k^* + 1}\right)$$

$$V_k = \beta' x_k + \ln \alpha_k + (\alpha_k - 1)\ln(m_k^* + 1), \qquad \text{if } k \notin B, k \geq 1$$

$$= \beta' x_k + \theta_k \ln \sum_{l \in N_k} \exp\left(\frac{\gamma' z_{lk}}{\theta_k}\right) + \ln \alpha_k + (\alpha_k - 1)\ln(m_k^* + 1), \text{ if } k \in B, k \geq 1$$

Conditional probability that vehicle make/model *l* will be used for an annual mileage $m_k^* (l \in N_k, k \in B)$,

given that $m_k^* > 0$

$$P(l \mid m_k^* > 0;\ l \in N_k) = \frac{\exp\left(\dfrac{\gamma' z_{lk}}{\theta_k}\right)}{\sum_{g \in N_k} \exp\left(\dfrac{\gamma' z_{gk}}{\theta_k}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Unconditional probability that household uses vehicle make/model *a* of vehicle type 1 for annual mileage, m_{1a}^*, make/model *b* of vehicle type 2 for m_{2b}^* ……. Make/model *s* for vehicle type *S* for m_{sS}^* is

$$P(m_{1a}^*, m_{2b}^*, m_{3c}^*, \ldots m_{Ss}^*, m_{S+1}^*, m_{S+2}^* \ldots m_Q^*, 0, 0 \ldots 0)$$
$$= P(m_1^*, m_2^*, \ldots m_Q^*, 0, 0, \ldots 0) \times P(a \mid m_1^* > 0) \times P(b \mid m_2^* > 0) \ldots P(s \mid m_S^* > 0)$$

❖ Econometric Model – Mixed MDCEV-MNL model

Unconditional probability of vehicle holdings and usage:

$$P(m_{1a}^*, m_{2b}^*, m_{3c}^*, \ldots m_{Qq}^*, 0, 0, 0, \ldots 0)$$

$$= \int_{\beta}\int_{\gamma} \left\{ P(m_1^*, m_2^*, \ldots m_Q^*, 0, 0, \ldots 0) \times P(a \mid m_1^* > 0) \times P(b \mid m_2^* > 0) \right.$$

$$\left. \ldots \times P(q \mid m_Q^* > 0) \mid (\beta, \gamma) \right\} \phi(\beta)\phi(\gamma)\, \mathrm{d}\beta\, \mathrm{d}\gamma$$

Descriptive Statistics of Vehicle Type/Vintage Holdings

Vehicle type/vintage	Total number (%) of households owning	Annual Mileage	No. of households who own (%): Only Vehicle type/vintage	No. of households who own (%): Vehicle type/vintage and other Vehicle type/vintages
New Coupe	389 (5%)	7763	132 (34%)	257 (66%)
Old Coupe	1024 (13%)	7766	374 (37%)	650 (63%)
New Subcompact Sedan	292 (4%)	7838	127 (43%)	165 (57%)
Old Subcompact Sedan	513 (6%)	9570	238 (46%)	275 (54%)
New Compact Sedan	767 (9%)	8321	342 (45%)	425 (55%)
Old Compact Sedan	1175 (14%)	9614	495 (42%)	680 (58%)
New Midsize Sedan	987 (12%)	7688	361 (37%)	626 (63%)
Old Midsize Sedan	1543 (19%)	9342	636 (41%)	907 (59%)
New Large Sedan	250 (3%)	7418	71 (28%)	179 (72%)
Old Large Sedan	377 (5%)	8339	151 (40%)	226 (60%)
New Station Wagon	242 (3%)	7869	80 (33%)	162 (67%)
Old Station Wagon	728 (9%)	8248	254 (35%)	474 (65%)
New SUV	707 (9%)	8920	245 (35%)	462 (65%)
Old SUV	711 (9%)	9813	213 (30%)	498 (70%)
New Pickup Truck	578 (7%)	8887	153 (26%)	425 (74%)
Old Pickup Truck	1198 (15%)	8679	301 (25%)	897 (75%)
New Minivan	459 (6%)	9156	115 (25%)	344 (75%)
Old Minivan	480 (6%)	9890	130 (27%)	350 (73%)
New Van	39 (1%)	10640	8 (21%)	31 (79%)
Old Van	122 (2%)	8203	33 (27%)	89 (73%)
Non-Motorized form of transportation	201 (3%)	2695	-	201 (100%)

Empirical Results

- Variables considered
  - Household socio-demographics
    - Household income, presence of children in the household, presence of a senior adult in the household, household size and number of employed people in the household
  - Household location attributes
    - Area type variables (central business district, urban zone, suburban zone and rural zone), residential density and employment density variables
  - Built environment characteristics of the residential neighborhood
    - Percentages and absolute values of acreage in residential, commercial/industrial, and other land-use categories; fractions and number of single family and multi-family dwelling units, and fractions and number of households living in single family and multi-family dwelling units, bikeway density, street block density, highway density
  - Characteristics of the household head
    - Age (classified into less than 30 years of age, 31 to 45 years of age and greater than 45 years of age), gender and ethnicity (primarily, Caucasian, African-American, Hispanic, Asian and Other)
  - Vehicle Characteristics
    - Purchase price, fuel cost, seating capacity, luggage volume, engine size, number of cylinders, front headroom space, front legroom space, rear headroom space, rear legroom space, standard payload capacity (for pickup trucks only), wheelbase, length, height, width, horse power, vehicle weight, type of fuel used, amount of greenhouse gas emissions (tons/year), types of drive wheels, type of vehicle make

- MDCEV model – Effects of Household Demographics
  - Medium income (35-90K) and high income (>90K) households have a high baseline preference for new SUVs as compared to low-income households and a low preference for old vans
  - High income households have a lower baseline preference for old vehicles compared to low/middle income households
  - High income households less likely to undertake activities using non-motorized forms of transportation
  - Households with very small children (less than 4 years of age) are more likely to use compact sedans, mid-size sedans, and SUVs than other households
  - Households with kids between 5 and 15 years of age have a high baseline preference for minivans than other households
  - Households with senior adults (greater than 65 years) are more likely to use compact, mid-size, and large sedans relative to coupes and subcompact sedans
  - As the size of the household increases, the household is more likely to use mid-size sedans, large sedans, station wagons, SUVs, pickup trucks, minivans and vans
  - Household with more number of employed members have a high baseline preference for new vehicle types such as subcompact sedans and compact sedans, and a low baseline preference for large sedans and minivans

- MDCEV model – Effects of Household Location Characteristics
  - Households residing in suburban zones are less likely to own and use old vehicles relative to households in urban zones
  - Households residing in suburban and rural zones are more likely to own and use pickup trucks relative to urban households

- MDCEV model – Effects of Built Environment Characteristics of the Residential Neighborhood
  - Households located in highly residential/commercial areas are less likely to prefer large vehicle types such as pickup trucks and vans, irrespective of the age of the vehicle
  - Households located in a neighborhood with high bike lane density have a high baseline preference for non-motorized modes of transportation
  - Households located in a neighborhood with high street block density are more likely to prefer smaller vehicle types (such as subcompact and compact sedans) and older vehicles.

- MDCEV model – Characteristics of the Household Head
  - Older households (*i.e.*, households whose heads are greater than 30 years) are generally more likely to own vehicles of an older vintage compared to younger households (*i.e.*, households whose heads are less than or equal to 30 years of age)
  - Older households are more likely to own minivans and old vans, and travel by non-motorized forms of transportation
  - Households have higher baseline preference for older and larger vehicles if the male is the oldest member (or only adult) in the household relative to households with the female being the oldest member (or only adult)
  - Asians more likely to own sedans and new minivans, and less likely to own pickup trucks, than other races.

- MDCEV model – Random Error Components/Coefficients
  - Households preferring old coupes due to unobserved factors also prefer new coupes
  - Intangible unobserved factors that affect utilities of all old vehicles

MNL model for Vehicle Make/Model Choice

Variable	Parameter	t-stat
Purchase Price (in $)/Income (in $/yr) [x 10]		
Mean Effect	- 0.173	- 5.71
Standard Deviation	- 0.064	- 4.44
Fuel Cost (in $/yr) /Income (in $/yr) [x 10]	- 0.003	- 1.61
Seat Capacity * Household Size less than equal to 2 dummy variable	- 0.075	- 5.11
Luggage Volume (in 10s of cubic feet)	0.023	3.54
Standard Payload Capacity (for Pickup Trucks only) (in 1000 lbs)	0.196	5.13
Horsepower (in HP) /Vehicle Weight (in lbs) [in 10s]	1.102	4.89
Engine Size (in liters)	- 0.045	- 2.42
Dummy variable for All-Wheel-Drive (base: rear-wheel-drive)	- 0.214	- 3.81
Dummy Variable for Vehicle Make - Chevy	- 0.149	- 1.25
Dummy Variable for Vehicle Make - Ford	0.716	5.37
Dummy Variable for Vehicle Make - Honda	1.444	5.37
Dummy Variable for Vehicle Make - Toyota	0.752	5.29
Dummy Variable for Vehicle Make - Cadillac	0.880	4.36
Dummy Variable for Vehicle Make - Volkswagen	0.374	2.55
Dummy Variable for Vehicle Make - Dodge	0.699	4.96
Amount of Greenhouse Gas Emissions (in 10s of tons/yr)	- 0.429	- 2.71
Dummy variable for Premium Fuel (base: regular fuel)	- 0.552	- 5.01

- Satiation Effects
  - All the satiation parameters are very significantly different from 1
  - Middle and High income households are more likely to get satiated with the increasing use of any vehicle type/vintage compared to low income households
  - Low income households are least likely to get satiated with the increasing use of old subcompact sedans, new and old compact sedans, and old midsize sedans
  - Satiation effect is highest for non-motorized mode of transportation compared to all vehicle type/vintage categories
- Logsum Parameters
  - Indicate the presence of common unobserved attributes that affect the utilities of all makes/models corresponding to old SUV, old minivan, new minivan, old van, and new van vehicle type/vintage categories

Application of the Model

Vehicle Type	Impact of a 25% increase in bike lane density		Impact of a 25% increase in street block density		Impact of a 25% increase in fuel cost	
	% change in holdings of vehicle type	% change in overall use of vehicle type	% change in holdings of vehicle type	% change in overall use of vehicle type	% change in holdings of vehicle type	% change in overall use of vehicle type
Compact Car	-	-2.2%	8.5%	3.4%	1.3%	-0.9%
Midsize and Large Sedan	-2.2%	-2.1%	-	-0.8%	-	-0.6%
SUV	-0.6%	-0.4%	-	-	-	-
Pickup Truck	-1.4%	-0.4%	-2.1%	-1.7%	-5.7%	-2.3%
Minivan and Van	-	-0.7%	-	-0.6%	-2.6%	-
Non-motorized modes of transportation	7.4%	13.9%	-4.0%	-3.3%	1.5%	0.8%

Applications of the MDCEV model

- Time-use analysis
  - Discretionary time-use on weekends (Bhat, 2005; and Bhat *et al.*, 2006)
  - Discretionary time-use by activity purpose and accompaniment arrangement (Kapur and Bhat, 2007)
  - Children's physical activity participation (Copperman and Bhat, 2006)
  - Social context of children's discretionary activity participation (Sener and Bhat, 2007)
  - Structure of children's discretionary activity participation on weekdays and weekends (Copperman, Sener, and Bhat, 2007...in progress)
  - Weekly discretionary time-use (Spissu *et al.*, 2007...in progress)
  - Residential sorting effects in the interactions between built environment and discretionary time-use decisions on weekdays (Pinjari, Bhat, and Hensher, 2007...in progress)
- Household vehicle holdings and usage analysis
  - Household vehicle type choice and usage decisions (Bhat and Sen, 2006)
  - Household vehicle type/vintage, make/model, and usage decisions (Sen and Bhat, 2007)

- Directions for further research
  - Accommodating more than one constraint in the utility maximization problem (for example, recognizing both time and money constraints in activity type choice and duration models)
  - Incorporating latent consideration sets in a theoretically appropriate way within the MDCEV structure
  - Using more flexible utility structures that can handle both complementarity as well as substitution among goods, and that do not impose the constraints of additive separability

Model with an Outside Good

$$U(\boldsymbol{x}) = \frac{1}{\alpha_1}\exp(\varepsilon_1)x_1^{\alpha_1} + \sum_{k=2}^{K} \frac{1}{\alpha_k}\exp(\beta' z_k + \varepsilon_k)\left\{\left(x_k + 1\right)^{\alpha_k} - 1\right\}$$

$$U(\boldsymbol{x}) = \frac{1}{\alpha_1}\exp(\varepsilon_1)x_1^{\alpha_1} + \sum_{k=2}^{K} \gamma_k \exp(\beta' z_k + \varepsilon_k)\ln\left(\frac{x_k}{\gamma_k} + 1\right)$$

$$U(\boldsymbol{x}) = \frac{1}{\alpha}\exp(\varepsilon_1)x_1^{\alpha} + \sum_{k=2}^{K} \frac{\gamma_k}{\alpha}\exp(\beta' z_k + \varepsilon_k)\left\{\left(\frac{x_k}{\gamma_k} + 1\right)^{\alpha} - 1\right\}$$

- The MDCEV model structure
  - Probability of the consumption pattern of the goods (rather than the expenditure pattern) is

$$P\left(x_1^*, x_2^*, x_3^*, \ldots, x_M^*, 0, 0, \ldots, 0\right)$$

$$= \frac{1}{p_1} \cdot \frac{1}{\sigma^{M-1}} \left[\prod_{i=1}^{M} f_i\right] \left[\sum_{i=1}^{M} \frac{p_i}{f_i}\right] \left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma}\right)^M}\right] (M-1)!,$$

where

$$f_i = \left(\frac{1-\alpha_1}{x_i^* + \gamma_i}\right)$$

Comparison With Earlier Multiple Discrete-Continuous Models

- Kim *et al.*'s model

$$U = \sum_{k=1}^{K} \psi_k^1 (x_k + \gamma_k^1)^{\alpha_k^1}$$

$$U = \sum_{k=1}^{K} \psi_k^1 \left\{ (x_k + \gamma_k^1)^{\alpha_k^1} - (\gamma_k^1)^{\alpha^1} \right\}$$

$$\tau_k = \tau_k^1 + \ln\left(\frac{\alpha_k^1}{\alpha_1^1}\right) \; \forall \; k$$

$$\alpha_k = \alpha_k^1 \; \forall \; k$$

$$\beta = \beta^1$$

- Models in environmental economics

$$U = \sum_{k=1}^{K} \psi_k^2 \ln(x_k + \gamma_k^2)$$

$$U = \sum_{k=1}^{K} \psi_k^2 \ln(\frac{x_k}{\gamma_k^2} + 1)$$

$$\tau_k = \tau_k^2 + \ln\left(\frac{\gamma_1^2}{\gamma_k^2}\right) \quad \forall\ k$$

$$\gamma_k = \gamma_k^2 \quad \forall\ k$$

$$\beta = \beta^2$$

$$P\left(x_1^*, x_2^*, x_3^*, \ldots, x_M^*, 0, 0, \ldots, 0\right)$$

$$= |J|_x \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{\left(\prod_{i=2}^{M} \frac{1}{\sigma}\lambda\left[\frac{V_1 - V_i + \varepsilon_1}{\sigma}\right]\right)\right\} \times \left\{\prod_{s=M+1}^{K} \Lambda\left[\frac{V_1 - V_s + \varepsilon_1}{\sigma}\right]\right\} \frac{1}{\sigma}\lambda\left(\frac{\varepsilon_1}{\sigma}\right) d\varepsilon_1$$

$$= |J|_x \left\{\left(\prod_{i=2}^{M} \frac{1}{\sigma}\lambda\left[\frac{V_1 - V_i}{\sigma}\right]\right)\right\} \times \left\{\prod_{s=M+1}^{K} \Lambda\left[\frac{V_1 - V_s}{\sigma}\right]\right\}, \qquad for\ \varepsilon_1 = 0$$

$$= |J|_x \left\{\prod_{i=2}^{M} \frac{\exp(-g_i/\sigma)}{\sigma}\right\} \left\{\exp\left(-\sum_{k=2}^{K} \exp(-g_k/\sigma)\right)\right\}, \quad for\ \varepsilon_1 = 0$$

$$where,\ |J|_x = \left(\prod_{i=1}^{M} f_i\right)\left(\sum_{i=1}^{M} \frac{p_i}{f_i}\right),\ f_i = \left(\frac{1-\alpha_i}{x_i^* + \gamma_i}\right),\ and\ \ g_k = V_1 - V_k$$

$$P\left(x_1^*, x_2^*, x_3^*, \ldots, x_M^*, 0, 0, \ldots, 0\right)$$

$$= |J|_x \left\{ \prod_{i=2}^{M} \frac{\exp(-g_i/\sigma)}{\sigma} \right\} \left\{ \exp\left(-\sum_{k=2}^{K} \exp(-g_k/\sigma) \right) \right\}, \; for\ \varepsilon_l = 0; l = 1$$

$$= \frac{1}{p_l} |J|_x \{\exp(+g_l/\sigma)\}^{M-1} \left\{ \prod_{\substack{i=1 \\ i \neq l}}^{M} \frac{\exp(-g_i/\sigma)}{\sigma} \right\} \left\{ \exp\left(-\sum_{\substack{k=1 \\ k \neq l}}^{K} \exp(-g_k/\sigma) \right) \right\}^{\exp(+g_l/\sigma)} , \; for\ \varepsilon_l = 0;\ l \le M$$

$$= |J|_x \left\{ \prod_{i=2}^{M} \frac{\exp(-g_i/\sigma)}{\sigma} \right\} \left\{ \sum_{\substack{k=1 \\ k \neq l}}^{K} \exp(-g_k/\sigma) \right\}^{-M}$$

$$\left\{ (M-1)! - \exp(d_l)(-1)^{M+1} \left(d_l^{M-1} - (M-1)d_l^{M-2} + (M-1)(M-2)d_l^{M-3} + \ldots (-1)^{M-1}(M-1)! \right) \right\}, \; for\ \varepsilon_l = 0;\ l > M$$

$$where,\ |J|_x = \left(\prod_{i=1}^{M} f_i \right) \left(\sum_{i=1}^{M} \frac{p_i}{f_i} \right),\ f_i = \left(\frac{1-\alpha_i}{x_i^* + \gamma_i} \right),\ g_k = V_1 - V_k,\ and\ d_l = -\left\{ \sum_{\substack{k=1 \\ k \neq l}}^{K} \exp(-g_k/\sigma) \right\} \exp(+g_l/\sigma)$$

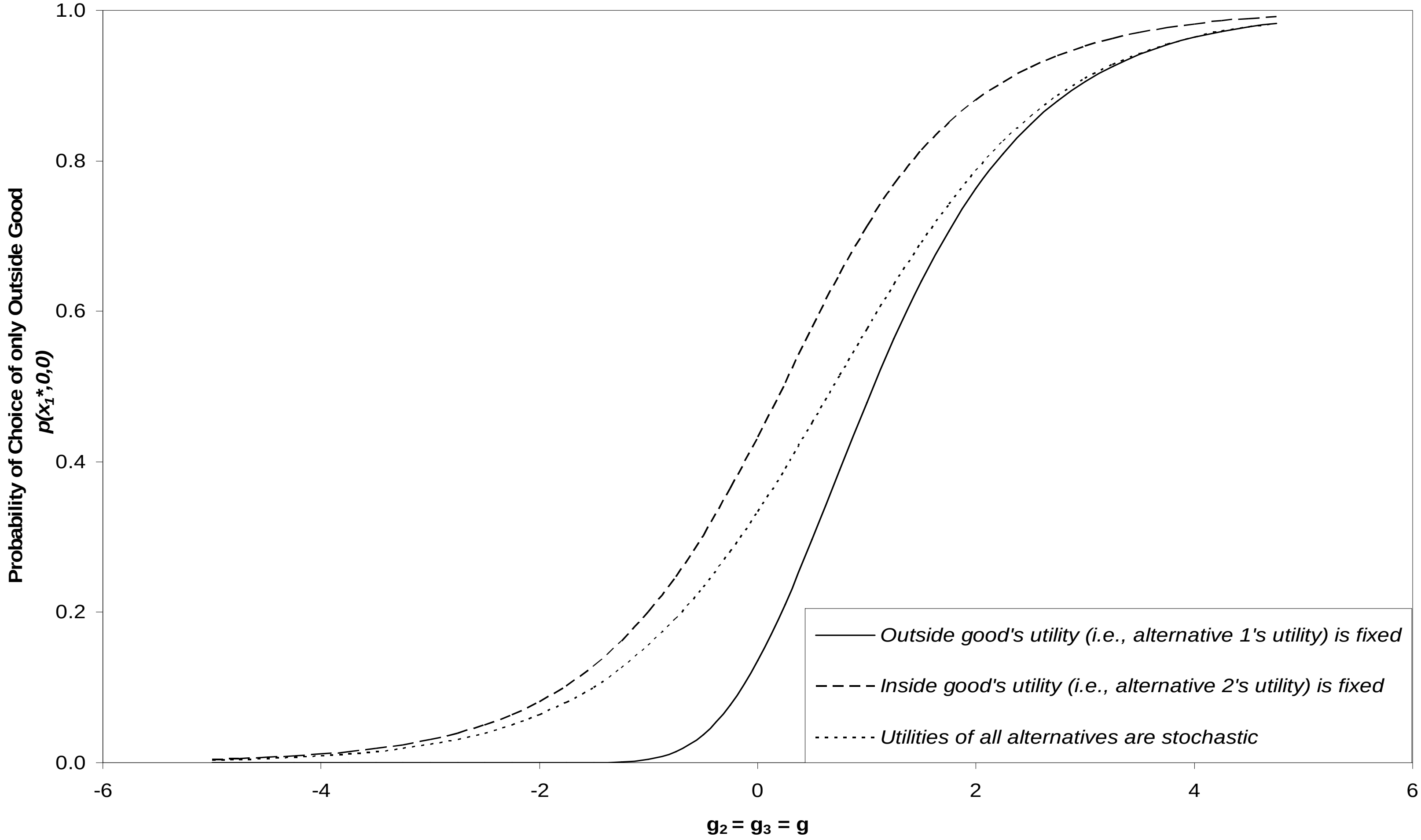
Three Good Case: Probability of Choice of Only Outside Good

$$P^1(x_1^*,0,0) = \exp\left[-\sum_{k=2}^{3} \exp(-g_k/\sigma)\right]$$

$$P^2(x_1^*,0,0) = \{1+\exp(-g_3/\sigma)\}^{-1}\left\{1-\exp\left[-\{1+\exp(-g_3/\sigma)\}\{\exp(+g_2/\sigma)\}\right]\right\}$$

$$P^3(x_1^*,0,0) = \frac{\exp(V_1/\sigma)}{\sum_{k=1}^{3} \exp(V_k/\sigma)} = \frac{1}{1+\exp(-g_2/\sigma)+\exp(-g_3/\sigma)}$$

- Utility of outside good is assumed to be deterministic (i.e., $\varepsilon_1 = 0$)
- A three-good example



Empirical Illustrations

- Specifications for the "No Outside Good" case with no price variables

Parameters	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Baseline marginal utility (β_k)												
Passenger car	-	-	-	-	-	-	-	-	-	-	-	-
SUV	-2.204	-38.81	-4.409	-38.81	-5.800	-29.74	-10.817	-12.05	-2.003	-37.96	-4.006	-37.96
Pickup-Truck	-1.956	-35.82	-3.912	-35.82	-5.146	-30.62	-9.597	-11.43	-1.766	-35.12	-3.533	-35.12
Minivan	-2.748	-42.08	-5.497	-42.08	-7.230	-27.57	-13.484	-12.20	-2.549	-41.17	-5.098	-41.17
Van	-4.587	-34.28	-9.175	-34.28	-12.069	-22.11	-22.508	-12.06	-4.391	-33.19	-8.782	-33.19
Satiation Parameters												
$\alpha_{Passenger\ Car}$	0.619	38.28	0.239	7.40	0.000	(fixed)	-0.865	-4.91	0.000	(fixed)	-1.000	(fixed)
α_{SUV}	0.886	60.16	0.773	26.24	0.701	16.19	0.443	5.10	0.000	(fixed)	-1.000	(fixed)
$\alpha_{Pick-up}$	0.796	51.36	0.592	19.10	0.463	9.16	0.000	(fixed)	0.000	(fixed)	-1.000	(fixed)
$\alpha_{Minivan}$	0.881	47.18	0.762	20.40	0.687	12.93	0.416	4.00	0.000	(fixed)	-1.000	(fixed)
α_{Van}	0.810	16.85	0.620	6.45	0.500	3.88	0.069	0.28	0.000	(fixed)	-1.000	(fixed)
$\gamma_{Passenger\ Car}$	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	11.470	14.79	11.469	14.79
γ_{SUV}	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	31.107	7.75	31.127	7.75
$\gamma_{Pick-up}$	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	17.880	10.56	17.877	10.56
$\gamma_{Minivan}$	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	29.155	6.60	29.159	6.60
γ_{Van}	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	19.672	3.19	19.591	3.21
Scale parameter	1.000	(fixed)	2.000	(fixed)	2.630	23.47	4.906	13.14	1.000	(fixed)	2.000	(fixed)
Log-Likelihood value at convergence	-9648.48		-9648.48		-9648.48		-9648.48		-9218.89		-9218.89	

Specifications for the "No Outside Good" case with price variables

Parameters	Model 1 (Consumption-based)		Model 2 (Consumption-based)		Model 3 (Expenditure-based)		Model 4 (Expenditure-based)		Model 5 (Expenditure-based)		Model 6 (Expenditure-based)	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Baseline marginal utility (β_k)												
Passenger car	-	-	-	-	-	-	-	-	-	-	-	-
SUV	-5.088	-15.44	-5.087	-15.35	-5.086	-15.40	-1.814	-32.02	-2.925	-19.95	-1.584	-29.89
Pickup-Truck	-4.465	-15.08	-4.464	-14.99	-4.464	-15.05	-1.564	-28.66	-2.522	-19.05	-1.344	-26.57
Minivan	-6.124	-14.89	-6.123	-14.81	-6.122	-14.86	-1.996	-30.52	-3.488	-18.90	1.764	-28.30
Van	-10.502	-14.50	-10.501	-14.43	-10.499	-14.47	-3.529	-26.33	-6.333	-17.99	-3.302	-24.91
Satiation Parameters												
$\alpha_{Passenger\ Car}$	0.000	0.00	0.000	0.00	0.000	0.00	0.543	35.20	0.000	(fixed)	0.000	(fixed)
α_{SUV}	0.746	20.48	0.746	20.48	0.746	20.49	0.906	71.55	0.000	(fixed)	0.000	(fixed)
$\alpha_{Pick-up}$	0.560	12.56	0.560	12.55	0.560	12.56	0.837	59.74	0.000	(fixed)	0.000	(fixed)
$\alpha_{Minivan}$	0.868	22.47	0.868	22.47	0.868	22.47	0.952	66.51	0.000	(fixed)	0.000	(fixed)
α_{Van}	0.774	7.87	0.776	7.94	0.776	7.95	0.913	24.49	0.000	(fixed)	0.000	(fixed)
$\gamma_{Passenger\ Car}$	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	4.197	11.79	8.513	17.84
γ_{SUV}	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	20.971	6.69	40.921	7.09
$\gamma_{Pick-up}$	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	11.628	8.12	24.360	9.35
$\gamma_{Minivan}$	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	32.795	4.23	68.228	3.93
γ_{Van}	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	28.589	1.68	361.995	1.56
Scale parameter	2.565	17.93	2.565	17.85	2.564	17.90	1.000	(fixed)	1.718	26.76	1.000	(fixed)
Log-Likelihood value at convergence	-9947.07		-8986.11		-9047.08		-9348.95		-8803.51		-8939.42	

Specifications for case with outside good and with price variables

Parameters	Model 1 (Expenditure-based)		Model 2 (Consumption-based)		Model 3 (Consumption-based)		Model 4 (Consumption-based)		Model 5 (Consumption-based)	
	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
Baseline marginal utility (β_k)										
Maintenance activity	-	-	-	-	-	-	-	-	-	-
In-home relaxation	-8.562	-31.53	-8.562	-32.21	-6.795	-52.54	-8.369	-36.75	-7.664	-29.44
In-home recreation	-8.053	-29.11	-8.053	-29.73	-6.216	-47.95	-7.785	-33.75	-7.074	-26.92
Non-work internet use	-10.546	-26.06	-10.546	-26.47	-7.514	-43.93	-9.875	-29.38	-9.025	-25.66
Social	-8.307	-29.16	-8.307	-29.77	-6.371	-48.65	-8.038	-33.74	-7.317	-27.23
Out-of-home meals	-6.768	-26.68	-6.768	-27.27	-5.238	-41.12	-6.698	-31.32	-6.019	-24.15
Out-of-home maintenance	-6.464	-25.12	-6.464	-25.67	-4.876	-38.31	-6.350	-29.33	-5.667	-22.60
Out-of-home volunteer	-9.995	-31.41	-9.995	-32.03	-7.692	-55.68	-9.590	-36.17	-8.826	-30.31
Out-of-home recreation	-7.446	-27.40	-7.446	-27.99	-5.674	-43.93	-7.214	-31.79	-6.511	-25.06
Pure recreation	-10.943	-30.66	-10.943	-31.21	-8.278	-55.02	-10.406	-35.08	-9.600	-30.21
Satiation Parameters										
α_1	0.000	0.00	0.000	0.00	0.186	9.05	0.000	0.00	0.108	2.83
α_2	0.739	42.43	0.739	42.53	0.841	90.43	0.000	(fixed)	0.108	2.83
α_3	0.846	56.79	0.846	56.83	0.911	107.05	0.000	(fixed)	0.108	2.83
α_4	0.773	18.92	0.773	18.93	0.862	34.83	0.000	(fixed)	0.108	2.83
α_5	0.749	38.57	0.749	38.63	0.847	78.22	0.000	(fixed)	0.108	2.83
α_6	0.622	29.21	0.622	29.30	0.768	71.92	0.000	(fixed)	0.108	2.83
α_7	0.618	27.02	0.618	27.09	0.767	65.53	0.000	(fixed)	0.108	2.83
α_8	0.708	27.88	0.708	27.92	0.823	56.65	0.000	(fixed)	0.108	2.83
α_9	0.809	48.83	0.809	48.88	0.889	95.66	0.000	(fixed)	0.108	2.83
α_{10}	0.619	15.80	0.619	15.82	0.765	33.25	0.000	(fixed)	0.108	2.83

Specifications for case with outside good and with price variables (continued)

Parameters	Model 1 (Expenditure-based)		Model 2 (Consumption-based)		Model 3 (Consumption-based)		Model 4 (Consumption-based)		Model 5 (Consumption-based)	
	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
Satiation Parameters										
γ_1	0.000	(fixed)	0.000	(fixed)	0.000	(fixed)	0.000	(fixed)	0.000	(fixed)
γ_2	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	118.015	10.77	125.194	10.77
γ_3	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	209.869	9.40	219.289	9.55
γ_4	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	92.063	3.99	97.161	4.05
γ_5	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	97.962	9.23	104.049	9.27
γ_6	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	39.642	12.01	42.158	11.95
γ_7	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	29.663	11.26	31.679	11.20
γ_8	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	86.222	8.15	91.102	8.21
γ_9	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	131.518	9.97	137.766	10.11
γ_{10}	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)	35.945	6.17	38.278	6.22
Scale parameter	1.608	30.18	1.608	30.50	1.000	(fixed)	1.402	35.02	1.329	37.22
Log-Likelihood value at convergence	-30054.40		-30013.80		-30229.00		-28220.00		-28216.40	

General econometric model structure and identification

$$\tilde{U} = \sum_k \frac{\gamma_k}{\alpha_k^*}\left[\exp\{\sigma \times (\beta' z_k + \varepsilon_k)\}\right] \cdot \left\{\left(\frac{x_k}{\gamma_k} + 1\right)^{\alpha_k^*} - 1\right\}$$

KT conditions for optimal expenditure for this modified utility function can be shown to be:

$$V_k^* + \sigma\varepsilon_k = V_1^* + \sigma\varepsilon_1 \quad \text{if} \quad e_k^* > 0 \qquad (k = 2, 3, \ldots, K)$$

$$V_k^* + \sigma\varepsilon_k < V_1^* + \sigma\varepsilon_1 \quad \text{if} \quad e_k^* = 0 \qquad (k = 2, 3, \ldots, K), \quad \text{where}$$

$$V_k^* = \sigma\beta' z_k + (\alpha_k^* - 1)\ln\left(\frac{e_k^*}{\gamma_k p_k} + 1\right) - \ln p_k \qquad (k = 1, 2, 3, \ldots, K)$$

$$= \sigma\beta' z_k + \sigma(\alpha_k - 1)\ln\left(\frac{e_k^*}{\gamma_k p_k} + 1\right) - \ln p_k \quad (k = 1, 2, 3, \ldots, K)$$

- Research objectives
  - Reformulate the utility specification used in earlier studies
  - Present identification considerations related to both the functional form as well as the stochastic nature of the utility specification
  - Derive the MDCEV model expression for the case when there is price variation across goods and extend the MDCEV model to accommodate generalized extreme value (GEV)-based and other correlation structures
  - Discuss the relationship between the models of Kim *et al.* (2002), the KT formulations used in Environmental Economics, and the MDCEV formulation
  - Illustrate the technical issues related to the properties and identification of the MDCEV model through empirical illustrations

- The Mixed MDCEV model - Heteroscedastic structure
  - The heteroscedastic structure may be specified in the form of the following covariance matrix for $\varepsilon = (\varepsilon_{k1}, \varepsilon_{k2}, \varepsilon_{k3}, \varepsilon_{k4})$:

$$Cov(\varepsilon) = \frac{\pi^2 \sigma^2}{6} \begin{bmatrix} 1 & 0 & 0 & 0 \\ & 1 & 0 & 0 \\ & & 1 & 0 \\ & & & 1 \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 & 0 & 0 \\ & \omega_2^2 & 0 & 0 \\ & & \omega_3^2 & 0 \\ & & & \omega_4^2 \end{bmatrix}$$

  - Two ways to proceed with a normalization
    - Normalize σ and estimate the heteroscedastic covariance matrix of $\eta\,(i.e.,\ \omega_1, \omega_2, \omega_3 \text{ and } \omega_4)$
    - Normalize one of the ω_k terms instead of the σ term

- The Mixed MDCEV model - General error covariance structure
  - Allows correlation in unobserved factors influencing the baseline utility of alternatives
  - Requires appropriate identification normalizations to be placed on σ and the covariance matrix of η
  - One way to achieve identification in the most general error covariance structure, and when there is price variation
    - Normalize the scale parameter σ to be a small value such that the variance of the minimum variance alternative exceeds $\pi^2\sigma^2/6$
    - Normalize ω_k for the minimum variance alternative *k* to zero
    - Normalize all correlations of this minimum variance alternative with other alternatives to zero
    - These normalizations leave only $K(K-1)/2$ parameters to be estimated, and are adequate for identification