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Discrete location planning

By

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ABSTRACT: Two new models for discrete location planning under static competition are introduced. The empirical context is an enterprise that is planning to set up a number of stores in various locations. The probability that a customer chooses a specific store is obtained from a multinomial logit (MNL) model. In the first model we apply the basic MNL model where the choice set contains all potential locations. To obtain the choice probabilities of a reduced choice set, we take advantage of the property of constant substitution patterns which can be modelled by linear constraints. In the second model we consider the case where flexible substitution patterns are accounted for. The main idea is to simulate, for a given number of individuals, their alternative specific utility values. Thus for each individual, we can identify which locations have to be opened to attract an individual as a customer. We consider two parcel service providers in the City of Dresden. Both approaches can be solved very fast within few minutes with a small solution gap by a state-of-the-art solver.

KEY WORDS: ***Discrete location, competition, multinomial logit model, constant substitution patterns, flexible substitution patterns, simulation, parcel service provider***

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1 Introduction

In this paper our focus is on an enterprise which is going to establish a specified number of stores for supplying services or products in an urban area where other competitors are active in the market. The available locations are already known, so we are faced with a discrete location problem. As an application, we have in mind a parcel service provider who is planning to set up a new chain of shops or to restructure an existing in order to increase their market share.

A potential customer has firstly to decide which shop they want to visit for posting a parcel. This means, that they have to solve a discrete choice problem in which the shops represent the alternatives from which exactly one has to be chosen. For shops far away from the home place and in a district which is a long distance from other activities, like working and shopping, the choice probability will be close to zero. On the other hand, we expect high choice probabilities for shops which are located in the neighbourhood or in a nearby shopping centre. However, there may exist several non-locational factors which influence the decisions of the customers.

For analysing such discrete choice problems the multinomial logit-analysis is well established (Hensher et al. 2005). Several models and specific estimation methods have been developed. In the base MNL model a linear utility function with constant substitution patterns of the alternatives, and independent extreme value distributed utility components are assumed.

In particular, the assumption of constant substitution patterns might be very restrictive or not fulfilled in real applications. To take into account flexible types of substitution patterns the nested and the mixed logit model have been proposed in the literature, and for its estimation, tailored methods have been developed based on maximum likelihood and simulation techniques.

From a logit analysis we obtain, for each individual, the probability of choosing a certain alternative. These choice probabilities can then be aggregated across the sampled population to obtain the expected sales of a shop or the expected market share of the whole system. At a first glance, this seems to be straightforward. But in location planning, the set of alternatives are not given in advance, as its determination is the subject of a location model. Moreover, considering a utility based objective function we cannot assume known objective function coefficients which represent expected market shares or similar economic measurements. In such a case the market shares have to be modelled by variables as the choice probabilities depend on the set of locations, i.e. they depend on the realised choice set. For this case, but with another utility functions, in the literature, non-linear models and, for their solution, tailored enumerative and heuristic methods have been proposed (e.g. Serra et al. (1999), Drezner and Drezner (2007), Marianov et al. (2008)). The subject of this paper is to develop models which take advantage of the results of an MNL analysis and can be solved with standard linear programming methods.

The paper is organised as follows. In the next section we give a brief overview of the literature on location and discrete choice analysis. In Section 3 we then describe some relevant well-known basics of the multinomial logit model. In particular, we discuss the independence of irrelevant alternative property of the basic multinomial logit model. This property can be used to formulate linear constraints which guarantee that the choice probabilities of the location will be calculated without bias from selected locations as shown in Section 4. The new location model is closely related to the maximum cover problem. In the case that we consider a more general multinomial logit model, a more general discrete location model is proposed in Section 5. The formulation is based on simulated decisions of a large number of individuals. In Section 6 we present some computational results which are derived from a more or less real application in which a parcel service provider in the City of Dresden, the former government monopolist, is considered who has to close some shops due to the new competition situation. We show that both location models can be solved with a state-of-the-art solver in reasonable time and with a small solution gap.

2 Literature overview

A classification of location problems can be found in Klose and Drexl (2005) and in ReVelle and Eiselt (2005). Drezner and Drezner (2004) proposes a model to maximise the market share in problems associated with competitive (facility) locations. An overview, with a further classification of such problems, is given by Plastria (2001). A recent overview of discrete location planning problems is given in ReVelle et al. (2008).

Utility based approaches are not new in location planning. Drezner (1994a) considers a deterministic utility function based on the distance between the customer's residence and the store location, as well as on some store characteristics. This has been extended to a stochastic utility function in Drezner and Drezner (1996). Moreover, Drezner et al. (1998) show that the stochastic utility function can be approximated by a Logit-S-function.

Gravity based approaches for continuous competitive location for a single location and multiple locations have been presented in Drezner (1994b) and Drezner and Drezner (2002), respectively. Both proposed models integrate a so-called revealed preference approach by including the gravity model of Huff (1994) and the market share model of Nakanishi and Cooper (1974). Recently, Dasci and Laporte (2005) have introduced a model with changing market conditions. Plastria (2005) explores the effect of quality standards of competitors and the cannibalising in a chain of offices. McGarvey and Cavlier (2005) present a formulation with elastic and gravity-theoretical demand, forbidden building sites, capacities, and a budget.

In Ghosh and Graig (1984) and in Dobson and Karmarkar (1987) a profit maximising objective function is considered, where the authors assume that customers choose the nearest distance location. The so-called maximum capture problem (MAXCAP), introduced by ReVelle (1986), has the objective of maximising the supply for the population. Modifications are extensively discussed in ReVelle (1996) as well as in Serra et al. (1999). In these approaches, the mapping of the competition is based on the assumption that a customer goes to the nearest location.

In Achabal et al. (1982) an extension of the so called multiplicative competitive interaction model (MCI) is presented. Eiselt and Laporte (1989) extend MAXCAP in such a way that the gravity model and Voronoi diagrams, also called as Thissen-Polygone, can be included. Serra et al. (1999) present two new MAXCAP models with different customer choice rules, in such a way that a direct assignment of customers to a location is not necessary.

Drezner and Drezner (2007) introduce a formulation for the p-median problem in which the choice probabilities of customers are taken into account. In the contribution of Abooliana et al. (2007) the location choice problem is connected with the design question for which a MCI-type formulation is considered.

For an introduction on discrete choice analysis we refer to Koppelman and Sethi (2000), Ben-Akiva and Lerman (1985), Train (2003) and Hensher et al. (2005). The brief explanations of the following section are based on Train (2003).

3 Multinomial Logit Models

For analysing the decision process of an utility maximising individual i who has to choose one alternative j from a finite set of alternatives J, the multinomial logit approach is selected. The individual's utility, U_{ij}, from the alternative j comprises a deterministic, V_{ij}, and a stochastic component, ϵ_{ij}, as follows:

$$U_{ij} = V_{ij} + \epsilon_{ij}. \tag{1}$$

with

$$V_{ij} = \sum_k \beta_{jk} x_{ijk} \tag{2}$$

where

β_{jk} is the utility of alternative j $(j = 1, \ldots, J)$ per unit of attribute k $(k = 1, \ldots, K)$,

x_{ijk} the value of attribute k according to the alternative j and the individual i $(i = 1, \ldots, I)$, and

ϵ_{ij} the stochastic utility of alternative j for individual i.

Assuming that the stochastic components are independently and identical extreme value distributed (IID, EVD) , then it can be shown that

$$P_{ij} = \frac{e^{V_{ij}}}{\sum_{\tilde{j}} e^{V_{i\tilde{j}}}} \tag{3}$$

which gives the probability that individual i chooses alternative j (Train, 2003). Moreover, the ratio of the choice probabilities of the two alternatives j and j' is independent from other alternatives, i.e.

$$\frac{P_{ij}}{P_{ij'}} = \frac{e^{V_{ij}}}{\sum_{\tilde{j}} e^{V_{i\tilde{j}}}} / \frac{e^{V_{ij'}}}{\sum_{\tilde{j}} e^{V_{i\tilde{j}}}} = \frac{e^{V_{ij}}}{e^{V_{ij'}}} \tag{4}$$

which is also called as independence of irrelevant alternatives (IIA) property. This means, that this fraction remains unchanged when adding or removing one alternative to or from the choice set, respectively, i.e. the substitution patterns are constant.

For our location problem, this property might not be valid. For example, consider a block in which 50% of the residents prefer a shop of the considered enterprise and the remaining residents prefer a shop of a competitor. The choice set may consist of the three alternatives j_1, j_2, and j_3 from which j_1 and j_2 belong to the considered enterprise, and j_3 to a competitor. Let the choice probabilities be $P_{i,j_1} = 0.25$, $P_{i,j_2} = 0.25$, and $P_{i,j_3} = 0.5$. Then, for the alternatives j_1 and j_3 we obtain the ratio

$$\frac{P_{i,j_1}}{P_{i,j_3}} = \frac{0.25}{0.50} = \frac{1}{2}.$$

Therefore, if the shop j_2 has been closed, the new choice probabilities must be $P_{i,j_1} = 1/3$ and $P_{i,j_3} = 2/3$ which is not reasonable as 50% of the block's residents still prefer a shop of the considered enterprise.

In such a case an approach which allows flexible substitution patterns is preferred. Examples are the nested and the mixed logit models (Hensher et al. 2005). In particular, we have to consider correlations between the stochastic components. Now, let us extend our numerical example by enforcing for the stochastic component the dependency structure

$$\epsilon_{i,j_1} := -\epsilon_{i,j_3}, \text{ and } \epsilon_{i,j_2}, := -\epsilon_{i,j_3}$$

where ϵ_{i,j_3} is discrete uniformly distributed over $\{-1, 1\}$. If we remove shop j_2 from the choice set we will obtain the expected choice probabilities $P_{i,j_1} = 0.5$ and $P_{i,j_3} = 0.5$.

4 Discrete Location Planning with Constant Substitution Patterns

In this section we use the IIA-property of the basic multinomial logit model to obtain a mixed-integer formulation for a discrete location problem which is closely related to the MAXCAP. For the mathematical formulation, we define the following sets:

- R The set of small administrative quarters (blocks).
- J The set of shops of all competitors, including also the shops of the considered enterprise.
- $\tilde{J}$ The subset of shops which belong to the considered enterprise where $\tilde{J} \subset J$.

Secondly, the data are given by the following parameters:

- E_r The number of (homogeneous) residents of block r.
- p_{rj} The probability that a customer from block r chooses shop j under the assumption that the networks of the competitors remain unchanged and all potential shops of the considered enterprise are setup, i.e. we assume that all $j \in J$ exist.

S The number of shops of a considered company which have to be established, where $S <\mid \tilde{J} \mid$.

We also introduce some variables:

y_j =1, if the considered company establishes shop $j \in \tilde{J}$ (=0, otherwise).

x_{rj} the expected market share of shop $j \in J$ in block r.

Note, if shop j is not set up, then its market shares is zero. In the case that shop j is set up, the market shares x_{rj} will be (equal to or) greater than the associated p_{rj} as the choice set in the solution is smaller then the choice set used for the calculation of the choice probabilities. Moreover, the ratio of the market shares in block r of two set up shops, say j and j', have to be identical, with the ratio of the corresponding choice proabilities, i.e. if $y_j = 1$ and $y_{j'} = 1$ then $x_{rj}/x_{rj'} = p_{rj}/p_{rj'}$.

Let us define the mathematical model:

The objective function

$$\max F = \sum_{r \in R} \sum_{j \in J} E_r x_{rj} \tag{5}$$

maximises the expected total market share. By

$$\sum_{j \in J} x_{rj} = 1 \qquad r \in R \tag{6}$$

the demand of each block will be satisfied. Note, that the decision that no shops will be chosen can be an alternative. We assume, however, that the decision maker has already determined that S shops have to be opened:

$$\sum_{j \in \tilde{J}} y_j = S \tag{7}$$

The set of constraints

$$x_{rj} \leq y_j \qquad r \in R, j \in \tilde{J} \tag{8}$$

satisfy that a customer can only visit a set up shop. To satisfy the IIA-property, we define

$$x_{rj} p_{rj'} \leq x_{rj'} p_{rj} + (1 - y_j) I(j \in \tilde{J}) + (1 - y_{j'}) I(j' \in \tilde{J}) \qquad j, j' \in J, r \in R \tag{9}$$

where $I(\cdot)$ denotes the indicator function. Thus it is not necessary to calculate the fractional values by the non-linear constraints

$$x_{rj} = \frac{e^{V_{rj}} \cdot y_j}{\sum_{j'} e^{V_{rj'}} \cdot y_{j'}}$$

and to solve a non-convex quadratic model.

Eventually, we have to define the domains of the variables as follows:

$$x_{rj} \geq 0 \qquad r \in R, j \in \tilde{J} \tag{10}$$
$$y_j \in \{0, 1\} \qquad j \in \tilde{J} \tag{11}$$

Note, the sets of constraints in (9) includes redundant constraints because of transitivity. Moreover, very small choice probability values might cause numerical instabilities. Therefore, at first we obtain, for each block, r, the competitor $j'_r \in J - \tilde{J}$ with the largest choice probability, i.e. $p_{rj'_r} \geq p_{rj}$ for all $j \in J - \tilde{J}$. Secondly we define a small value γ, e.g., $\gamma = 0.001$ and then we replace (9) by the following constraints:

$$x_{rj} = 0 \qquad j \in J, r \in R \mid p_{rj} \leq \gamma \tag{12}$$

$$\begin{aligned} x_{rj}p_{rj'_r} \leq x_{rj'_r}p_{rj} + (1 - y_j)I(j \in \tilde{J}) + (1 - y_{j'})I(j' \in \tilde{J}) \\ j \in \tilde{J}, r \in R \mid p_{jr} > \gamma. \end{aligned} \tag{13}$$

This may have an impact on the optimal solution and we also lose the possibility of deriving valid upper bounds from the LP-relaxation. But on one hand, if the locations are determined, then we can enhance the values of x_{rj} by applying (3). On the other hand, very small choice probabilities may indicate no-real alternatives, which have to be removed from the choice set. Following this argument we have to define, for each block r, the choice sets J_r and $\tilde{J}_r$ and then we will obtain optimal solutions or at least upper bounds based on the LP-relaxation. However, we expect only marginal differences in the solutions.

5 Discrete Location Planning with Flexible Substitution Patterns

In this section we formulate a new model in which the IIA-property need not be satisfied. In particular, the stochastic utility components can be correlated and must not be maximum value distributed. In the following mathematical formulation we consider n individuals. They are taken from the blocks, proportional to the number of residents in a block. We obtain the utility of an individual for an alternative by generating the stochastic component at random. Then we determine for each individual i, the largest utility u_i^* associated with the shops of the competitors. Individual i will select a shop of the considered company, if the considered company runs at least one shop $j \in \tilde{J}$ for which

$$u_{i,j} > u_i^* \tag{14}$$

is satisfied.

Defining additionally

- J_i as the set of shops for which the simulated utility of customer i is larger than the largest simulated utility of a competitor and
- $\tilde{x}_i$ equal to 1, if individual i chooses a shop of the considered enterprise (0, otherwise);

then the objective function

$$\max = \frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \tag{15}$$

estimates the expected market share. By the constraints

$$\tilde{x}_i \leq \sum_{j \in J_i} y_j \qquad \text{for all } i \mid J_i \neq \emptyset \tag{16}$$

we ensure that customer i chooses the shop which maximise utility. Again, by

$$\sum_{j \in \tilde{J}} y_j = S \tag{17}$$

we constitute that S shops will be set up. Eventually, the domains of the (relevant) variables have to be defined:

$$y_j \in \{0,1\} \qquad \text{for all } j \in \tilde{J}, \tag{18}$$

$$x_i \in [0,1] \qquad \text{for all } i = 1, \ldots, n \mid J_i \neq \emptyset. \tag{19}$$

Note, the flexible substitution patterns have to be taken into account when generating the stochastic components. For the determination of correlated random variables from independent generated random variables the Cholesky decomposition can be applied. Moreover, the number of variables is not influenced by the chosen administrative units and the number of competitors.

6 Computational Investigation for Model Comparison

The empirical illustration is based on an application which often arises in a real market. The following example has been motivated by the total liberalisation of the postal market in Germany. We consider the two main parcel service providers of the City of Dresden, the Deutsche Post AG and the Hermes Logistik Group. We assume that the shop network of the Hermes Logistik Group remains unchanged, and the former state monopolist has to reduce their network to cut cost. For all locations of the two competitors, we have identified their geographic positions. Then for each location we have computed the distance in metre to the centre of each block. The German Post AG operates 49 shops and the competitor Hermes Logistik Group operates 69 shops, i.e., in total we consider 118 shops. These shops are representing the initial choice set of the individuals. The City of Dresden is divided into 6,406 blocks. Figure 1 illustrates the current situation.

In addition to the calculated data, we have chosen the results of a survey undertaken in the City of Dresden with a sample of 637 individuals which have been interviewed face to face in some parcel shops (Hoppe 2009). The results of the basic multinomial logit analysis are summarised in Table 1.

Thus a shop j of the Hermes Logistik Group, which is 500 m away from the home of the customer i, located in a shopping centre and open 40 hours per week, has a (relative) utility of

$$u_{ij} = \ln(500) \cdot (-1.86) + 1.02 \cdot 1 + 1.48 \cdot 0 + 40 \cdot 0.028.$$

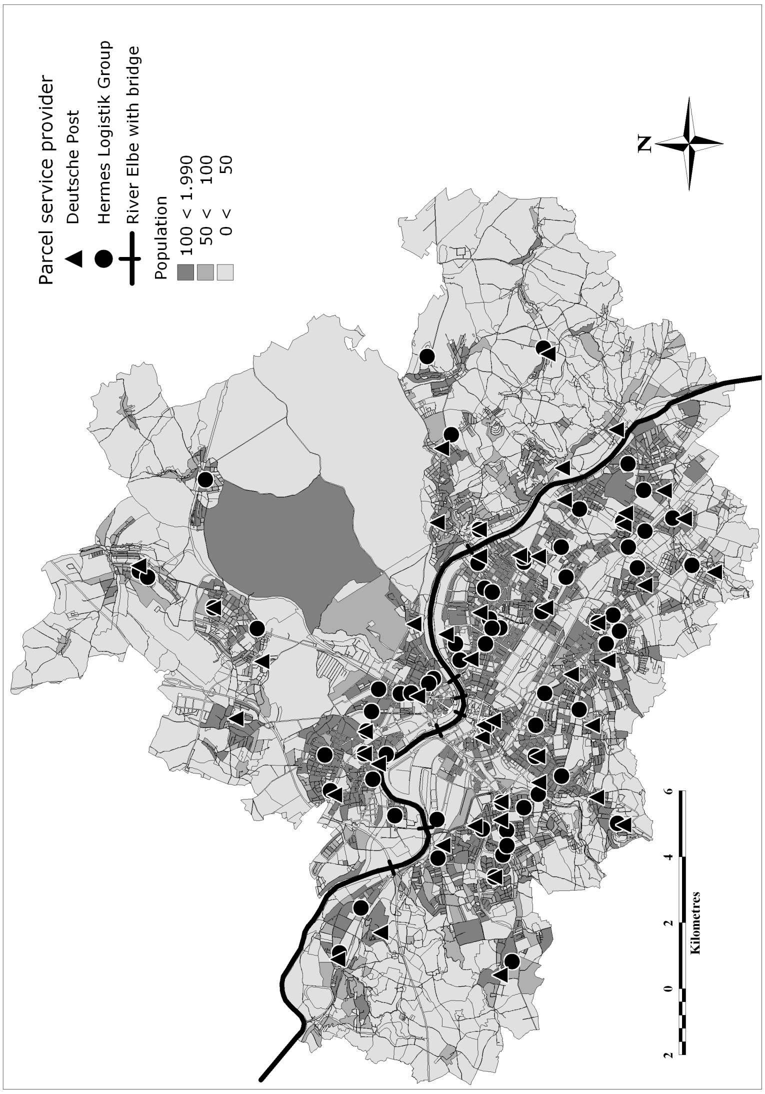

Figure 1: Locations of the main competitors in the City of Dresden

Table 1: Survey results (revealed preferences, Hoppe (2009))

Variable	unit	$\hat{\beta}$	t-ratio
Logarithmised distance to parcel shop	metres	-1.860	-7.30
Distance to parcel shop < 1800 metres?	1,0	1.020	2.95
Parcel shop of the German Post AG?	1,0	1.480	4.09
Parcel shop is located in a shopping centre	1,0	1.600	3.70
Opening time per week	hours	0.028	3.47
Number of observations : 637			
$\bar{\rho}^2$: 0.689			

For the sake of comparison, we assume for both models that the assumptions of the base MNL are fulfilled. Thus to simulate, for the second model, maximum value distributed random variables ϵ_{ij} we have to generate a uniformly distributed variable g and then calculate

$$\epsilon_{ij} = -\ln(-\ln(g)) \tag{20}$$

where the right hand side corresponds to the inverse cumulated distribution function of the maximum value probability function (double exponential).

In the following, we use the abbreviation LOC-IIA to refer to the first model. LOC-SIM refers to the second model, based on simulation. To analyse the solution quality of the LOC-SIM, we considered different sample sizes. Moreover, for a given sample size n we solved the problem m times. By $\bar{F}$ we denote the average objective function value of the m iterations. For a given set of locations, we can apply (3) to obtain a lower bound, denoted by LB. For the LOC-SIM we keep only the maximum lower bound of the m iterations.

All problems have been implemented in GAMS 22.9 and solved with Cplex 11.2.0 on a MacBook with a 2 GHz Intel Core 2 Duo Processor and 2 GB 1067 MHz DDR3 memory under the operating system Mac OS X 10.5.5. We optimise the locations of the German Post AG, i.e., the set of shops of the Hermes Logistik Group remains unchanged. We assume that 8 shops of the German Post AG have to be closed. Therefore, we choose $S = 41$. Table 2 shows the computational results. We see, for our test instances and parameter settings, LOC-SIM provides slightly better results than LOC-IIA in shorter time where for LOC-IIA it was necessary to allow a solution gap of 1% to obtain a solution in reasonable time. The best found solution is illustrated in Figure 2.

7 Summary and Future Work

In this paper we have shown how discrete choice analyses can be integrated in a mathematical optimisation approach in which the determination of the choice set is the subject of the optimisation problem. We have illustrated the methods with a discrete location

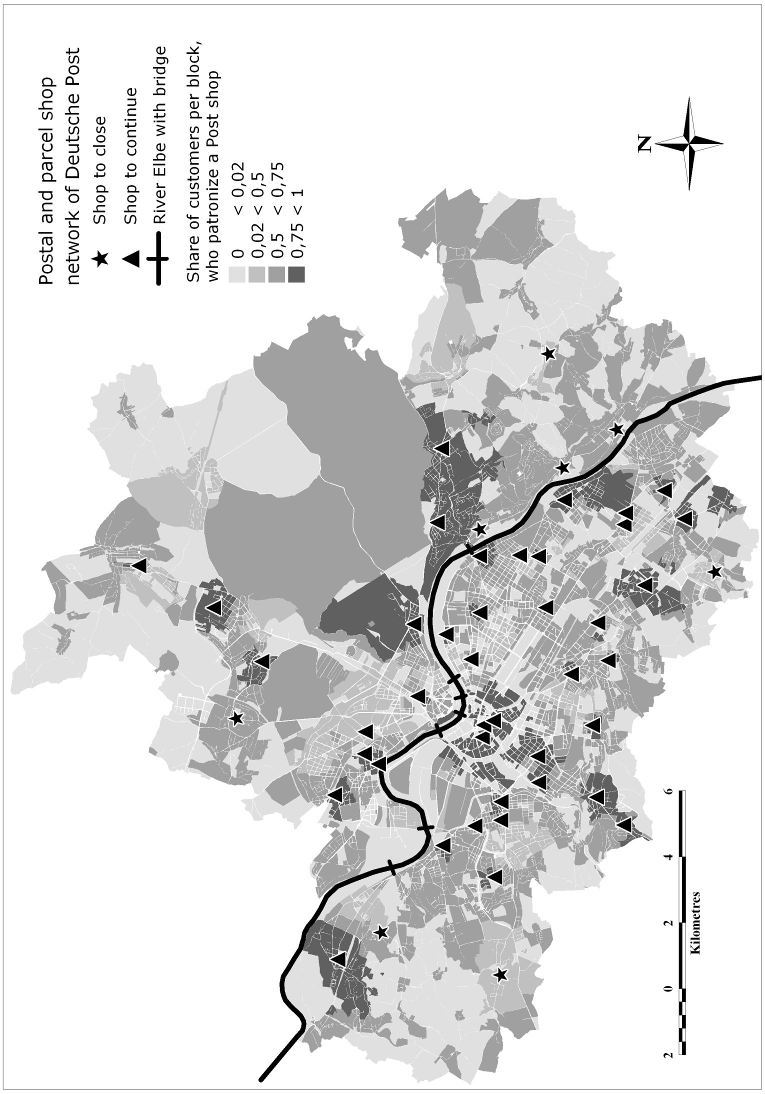

Figure 2: Illustration of best solution with $S = 41$

Table 2: Numerical results ($S = 41$)

Model	γ	$m \times n$	$\bar{F}$	LB	gap	cpu time
LOC-IIA	0.001	-	0.6195	0.6135	0.91%	342 sec.
LOC-SIM	-	50×1.000	0.6153	0.6135	0.00%	110 sec.
LOC-SIM	-	10×5.000	0.6199	0.6135	0.00%	53 sec.
LOC-SIM	-	5×10.000	0.6111	0.6135	0.00%	71 sec.
LOC-SIM	-	2×25.000	0.6125	0.6138	0.00%	142 sec.
LOC-SIM	-	1×50.000	0.6144	0.6137	0.00%	278 sec.
LOC-SIM	-	10×10.000	0.6124	0.6138	0.00%	142 sec.
LOC-SIM	-	25×10.000	0.6135	0.6138	0.00%	351 sec.

planning problem where the locations (of the optimal solution) constitute the choice set. In the first model we take advantage of the IIA-property of the basic multinomial logit model, i.e. we assume constant substitution patterns between the alternatives. For cases with flexible substitution patterns we have introduced a model which is based on simulated utilities of a large number of individuals.

Both models have been implemented in GAMS solved with Cplex. For a computational study we have considered a parcel service provider in a competitive market. For our set of instances, small solution gaps have been obtained within reasonable time.

Ongoing research will apply these approaches to other optimisation problems. In particular, we are working on the school location problem where we have take into account the free school choice and the transport mode choice of the students. We are also investigating a line planning approach in which demand depends on the provided service quality. Another important application is revenue management for the airline industry.

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