

# 1 Introduction

An entrepreneur might have a great idea but not have the money to execute it. Consider, for instance, a cash-constrained entrepreneur who has access to a profitable investment project that can produce a certain return  $R = sK$  in one period, where  $s$  is a constant rate of return greater than one and  $K$  is the cost of physical capital required to complete the project. With no wealth the entrepreneur has no choice but to find an outside investor to finance the project, who must receive a large enough part of the return so as to at least cover her costs. A problem can arise, however, if the initial agreement between the entrepreneur and the investor is not enforceable. In this case the entrepreneur may repudiate the initial contract by threatening to withdraw his human capital from the project. Anticipating this renegotiation (or hold-up problem), the investor will not finance some profitable projects (see Hart and Moore 1994).

To illustrate this point, consider the following example. The rate of return  $s$  is equal to 1.8 and total cost of capital investment  $K$  is equal to \$1. The entrepreneur borrows \$1 from the investor and promises to return \$1 to her in one period. After the investor lends the money the entrepreneur repudiates the initial agreement, causing the parties to renegotiate. Renegotiation occurs after the up-front investment (of \$1) is sunk but prior to the realization of the return of the project. For simplicity, assume that both agents have equal bargaining powers. As the investment is sunk the investor will receive half of the final return, that is  $sK/2 = \$0.9$ , which is less than the total capital invested. Consequently, the investor will not provide up-front finance for an otherwise profitable project.

In an important contribution Neher (1999) showed that financing a project in stages can help mitigate this commitment problem by allowing the accumulation of alienable physical assets to be used as collateral by the investor in the event of renegotiation. To illustrate this point, assume the entrepreneur stages the capital investment such that  $\gamma K$  dollars, where  $\gamma \in (0, 1)$ , are invested immediately and the remaining  $(1 - \gamma)K$  dollars are invested in one period's time. Such a project returns  $R = sK$  dollars in two periods. The output produced in the first period can be used as collateral in the second period. This collateral reassures the investor that the entrepreneur will not repudiate the initial agreement in the second period. First-period repudiation will also not occur because the entrepreneur's net return with repudiation is smaller than his net return specified in the initial contract. Hence, the investor may be able to finance the project with the investment staged in two rounds, which is something she would not be willing to do if all the investment had to be provided up-front.

Let us go back to our example and show that it is possible to finance the project in 2 stages. Let the up-front investment be  $I_1 = \$0.6$  and the investment in one period time be  $I_2 = \$0.4$ . From the completed project the entrepreneur receives  $R = \$1.8$  over the two periods and promises to pay back the investor \$1; the entrepreneur's net return is \$0.8. What if the entrepreneur repudiates the initial contract straight after  $I_1$  has been invested?  $I_1$  is sunk and the net return of the project is  $R - I_2 = \$1.4$ .

As both parties have equal bargaining powers the entrepreneur receives  $\frac{1}{2}(R - I_2) = \$0.7$  and the investor receives the remainder, that is  $I_2 + \frac{1}{2}(R - I_2) = \$1.1$ . The entrepreneur's return in the case of repudiation is smaller than his return specified in the initial contract. Therefore, he has no incentive to repudiate the initial contract in the first period. What if the entrepreneur repudiates the initial contract after  $I_2$  has been invested? In this case the investor has an option to liquidate the project and receive the output from the first period. The return of the investor is the collateral accumulated after the first period of  $sI_1 = \$1.08$ , which is greater than her costs. Again, the entrepreneur has no incentive to repudiate the initial contract. In this manner the investor can finance the project in two stages, which was not possible using a single investment period.

In essence, staged investment allows the entrepreneur's inalienable human capital to be converted into tangible physical assets, increasing the investor's collateral in event of repudiation. If the accumulation of collateral is sufficiently large, the entrepreneur will not want to trigger renegotiation because his return is greater if the project continues on the basis of the original contract.<sup>1</sup> As in the example above, Neher (1999) assumed that: assets that could be used as collateral in the event of repudiation accumulate at a constant rate; and that this rate was the same as the rate of return on the final project. More realistically, collateral accumulation could differ significantly over different stages of the project. A non-constant rate of accumulation allows for an examination of a wider scope of projects including, for example, projects with fixed costs or increasing and decreasing rates of return.<sup>2</sup> In this paper we relax both of these assumptions made in the previous literature (and as outlined in the example above); that is, we allow for both a variable rate of return over the life of the project and we decouple the one-to-one relationship between the returns of the project and collateral.

As shown below, relaxing these assumptions significantly alter the predictions concerning the optimal investment path - this is the main contribution of this paper. For instance, Neher (1999) predicts that positive wealth of the entrepreneur and a lack of discounting (very short time duration of one period) ensures that all profitable projects go forward. On the contrary, the model presented here shows that this is not the case. Unlike in Neher, if the project does not produce enough collateral in the early investment periods the project will not be financed, even if it is profitable. This arises because if the conversion of the entrepreneur's human capital in early periods is relatively low, the investor will not be afforded sufficient collateral protection, and the entrepreneur will have an incentive to repudiate the original contract.

Another difference arises with respect to the optimal investment path itself when

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<sup>1</sup>In the example above, if the collateral generated after the first-period investment (of  $I_1 = 0.6$ ) was insufficient, the investor would not have adequate protection from renegotiation and the project would not proceed.

<sup>2</sup>For example, the human capital of a researcher working on a new innovation may only become tangible when it can be converted into an alienable asset, like a patent. Research will not necessarily be converted into patents at a constant rate - it may be that only after certain stages of the project the researcher's (or entrepreneur's) efforts manifest themselves into patentable research output.

we allow for a more general relationship between investment and collateral. Neher (1999) finds that, starting from the second investment, the optimal investment path is always monotonically increasing. The result presented here is that the optimal investment path in general is not monotonically increasing with respect to the size of the investment. In both models the values of investments are determined by the way collateral accumulates. However, if collateral accumulates at a decreasing rate, the sequence of investments will be decreasing as well.

We derive necessary and sufficient conditions for the project to be financed when there are no costs of delay. We also identify the different roles of the final return of the project and collateral that accrues to the investor. The investment in the first period is supported by the final return. Once the project has commenced, however, further investments are supported by the collateral.

The issue of commitment has been examined in a number of other papers. Hart and Moore (1994) constructed a model in which the capital borrowed from the investor plus entrepreneur's wealth are invested by the entrepreneur at the beginning of the first period. This investment generates a stream of nonnegative returns and a stream of nonnegative liquidation values. The returns promised to the investor may not be credible as the entrepreneur cannot commit not to remove his valuable human capital from the project and renegotiate the investor's return. If in some period the outcome of renegotiation leaves the investor with a combined return less than his investment minus returns he has already received, he would not be willing to finance the project *ex ante*. Solving for all possible repayment paths such that the entrepreneur has no incentive to repudiate, the authors show how repayment paths change with the maturity structure of the project return stream, and with durability and specificity of the project assets.

Admati and Perry (1991) explored the role of staged financing in overcoming a commitment problem. In their model, two players invest consecutively one after the other until the total investment exceeds some value, in which case both players receive the benefits of the completed project. The crucial feature of their model is the tradeoff between free-riding and costs of delay; free-riding gives incentives for the players to invest less (in the hope that the other party will incur the cost instead), while the costs of delay push them to invest more (so as to receive the benefits of the project sooner than later). Although there is no collateral in their model, the results of their model accord well with the basic intuition of our model: that is, some of the projects that cannot be financed up-front due to the commitment problem can be financed in stages.

The focus in this paper is how staged financing can alleviate commitment problems when contracts are not enforceable. Alternatively, staging can arise as a result of uncertainty or asymmetric information, as has been also explored in a number of papers; for example Gompers (1995), Sahlman (1988 and 1990), Admati and Pfleiderer (1994), Bolton and Scharfstein (1990), and Roberts and Weitzman (1981).

## 2 The Model

Consider an entrepreneur who does not have any wealth and therefore must get an outside investor to finance a profitable project. Both physical assets and human capital need to be invested in the project. Physical capital is supplied solely from the investor in the form of an investment path  $I_1, I_2, \dots, I_T$ , where the structure of the investment path - that is size of each investment  $I_t$  and the number of periods of investment  $T$  - is determined endogenously. This structure is chosen before the investment begins and, once chosen, it cannot be altered. Further, the total amount of the physical capital investment required to complete the project is equal to  $K$ , so that

$$\sum_{t=1}^{t=T} I_t = K. \quad (1)$$

Working with the physical assets, the entrepreneur invests his human capital. There are no effort costs on the part of the entrepreneur. It is also assumed his outside option is zero. Both the entrepreneur and the investor discount the future with a some common per period discount factor  $\beta$ , where  $\beta \in (0, 1]$ .

If all investments are made for the  $T$  periods, the entrepreneur combines the physical investments with their human capital to produce a return of  $R$  after  $T$  periods, where  $R > K/\beta$  to ensure profitability. There is no other returns from the project except for those accruing at its completion. The project can be terminated, however. In this case the tangible return is the outside value - or the ‘liquidation’ value - of the assets established at the time of termination. If the project is terminated in period  $i$  the liquidation value is denoted as  $L_i$ .

The value of the liquidated project depends on the combination of the entrepreneur’s human capital with the physical investments. To capture this we assume that any physical investment is sunk in the period in which is made. After an investment  $I_i$  has been worked with by the entrepreneur for a whole period (and combined with his human capital), it then becomes tangible (alienable); only at this point does the investment contribute to the liquidation value of the project. As a consequence, the liquidation value in period  $t$  depends only on the investments made in periods 1 to  $t - 1$ ; the liquidation value does not depend on the investment made in period  $t$  as this investment remains sunk during the period in which it was made.

The way the liquidation value is determined is the most important departure from Neher (1999). We assume

$$L_t = f(\Lambda_{t-1}) \quad (2)$$

where  $\Lambda_t = \sum_{i=1}^{i=t} I_i$  and  $f(\cdot)$  is a monotonically increasing function. Further, if it is efficient to complete the project the liquidation value of the project at any point along the investment path will be less than the value of the completed project - this requires that  $L_t < R$  for any  $t \leq T$ . The advantage of this framework is that it does not restrict the way the liquidation value of the project accumulates along the investment path. Note, however, Neher’s (1999) assumption that  $R = s\Lambda_T$  and that

$L_t = s\Lambda_{t-1}$  where  $s$  is constant is a special case of the model presented here.

The timing of the model is presented in Figure 1. At the beginning of every period  $t$  the entrepreneur receives a piece of investment  $I_t$ . During all this period the liquidation value is  $L_t$ . At the end of period  $T$  the final return  $R$  is realized.

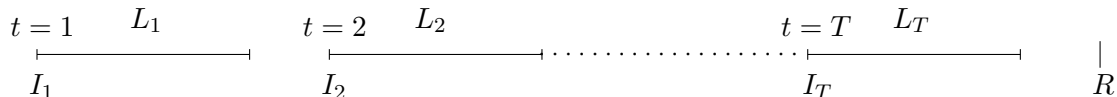


Figure 1: Time line for the project

In every period  $t$  after the investor puts  $I_t$  in the project, the entrepreneur has the option to initiate renegotiation of the initial contract by refusing to work.<sup>3</sup> Given the entrepreneur's unique human capital contribution, the project cannot be started again with some other entrepreneur. The two parties can renegotiate and agree to continue the project. If the project is continued it recommences from the point at which renegotiation began, with all the previous physical assets remaining in place.

We follow Hart and Moore (1994) in assuming that in every period the entrepreneur has an option to repudiate the contract and make the conditions of the investor worse than in the initial contract. The worst penalty that the investor can impose on the entrepreneur is expropriating all available physical assets from the project (and leaving the entrepreneur with the outside option of 0). Thus, the maximum the investor can receive if renegotiation fails is the liquidation value.

The entrepreneur, if he works on the project for all  $T$  periods as specified in the original contract, will receive a payoff of  $R - P$ , where  $P$  the payment to the investor detailed in the initial contract. If the contract is repudiated by the entrepreneur in period  $i$ , the investor can liquidate the physical assets and receive  $L_i$ . If the parties successfully renegotiate, the entrepreneur again takes control of the project, and the investor recommences providing investments in return for her new promised payment of  $P^{New}$ . For simplicity, we assume that the investor receives an expected return of 0 in the initial contract.<sup>4</sup>

An important aspect of this model is that there is an efficiency cost to financing the project via multiple rounds versus a single round. The final return  $R$  is the same regardless of the number of periods there is, but because of discounting there is a costly delay in realizing this return. On the other hand, the single round financing might not be feasible, because the entrepreneur might not be able to commit that he will not repudiate. As a result, the objective of the entrepreneur is to construct such an investment path that gives him the maximal net present value of the project. In order to ensure the project proceeds this path has to allow the investor to at least break even.

<sup>3</sup>The investor, on the other hand, is assumed to have no option to trigger renegotiation.

<sup>4</sup>Neher (1999) makes the assumption that there is a competitive market for potential investors.

To summarize the discussion above, the model has the following timing in each period: (i) investment made by the investor; (ii) the entrepreneur chooses whether or not to repudiate the existing contract; (iii) if repudiation occurred, the parties renegotiate according to their bargaining powers, given investor's option to liquidate the project; (iv) the entrepreneur makes their human capital input; and (v) the physical and human capital inputs combine into an increase in the collateral. We now discuss the renegotiation process in the event of repudiation.

## 2.1 Renegotiation

Now, let us describe the renegotiation process. We denote the present value to the investor of the outcome of renegotiation in period  $t$  by  $U_t$  and the surplus being bargained over in period  $t$  by  $S_t(T)$ , where<sup>5</sup>

$$S_t(T) := \beta^{T+1-t}R - \sum_{i=t+1}^T \beta^{i-t}I_i. \quad (3)$$

Further,  $\beta^{T+1-t}R$  is the value in period  $t$  of the project's final return, while  $\sum_{i=t+1}^T \beta^{i-t}I_i$  is the value in period  $t$  of the investments to be made after period  $t$ .

We model the renegotiation process using a reduced-form bargaining outcome arising from the alternating-offers model with outside options of Shaked and Sutton (1984).<sup>6</sup> The key element of this bargaining outcome is that an outside option only affects the distribution of surplus if it is binding (its adoption is credible in the relevant subgames).

For simplicity, we assume that the entrepreneur and the investor have equal bargaining powers; that is, if they start bargaining over some surplus under equal conditions they will share the surplus equally. At renegotiation, the outside option for the investor is to terminate the project and receive the liquidation value. The entrepreneur has an outside option of 0. Hence, in the case of renegotiation the investor will receive a half of the surplus if it is greater than the liquidation value. No termination occurs in this case because it worsens the payoffs for both players.

If in some period  $t$  the liquidation value is greater than half of the surplus then there are two possible outcomes. First, if the liquidation value  $L_t$  is less than the surplus itself, then by threatening to liquidate the firm, the investor increases his share up to the liquidation value. The termination does not occur, because with termination nobody is better off.

Second, if the liquidation value is greater than the surplus in some period  $t$ , the entrepreneur cannot entirely compensate the investor and persuade him not to liquidate the firm. It means that the outcome of repudiation in such a period  $t$  is always liquidation of the firm.

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<sup>5</sup>Note that for convenience we sometimes refer to  $S_t(T)$  as  $S_t$ .

<sup>6</sup>This formulation is also used by Chiu (1998), Neher (1999) and Hart and Moore (1994).

Thus, for every  $1 \leq t \leq T$  the present value to the investor of renegotiation in period  $t$  is equal to

$$U_t = \max[L_t, S_t/2]. \quad (4)$$

The type of financial arrangement between the the entrepreneur and the investor discussed in this section is summarized below.

**Summary.** *The investor provides all the rounds of investment in the initially specified investment path  $I_1, I_2, \dots, I_T$  and returns the future discounted value of this path at the end of period  $T$ . In the initial contract the investor breaks even. The entrepreneur works on the project for all  $T$  periods and receives the net return, that is the final return minus the payment to the investor. If the entrepreneur repudiates the contract in some period  $t$ , then the control over the physical assets is transferred to the investor. Following renegotiation a new contract can be renegotiated or the investor can liquidate the assets for  $L_t$ . If the new contract is specified, the project starts from the same place where it was before the entrepreneur repudiated. The investor provides the next rounds of investment and at the end of period  $T$  he receives some other value specified in the new contract.*

### 3 Solving the model

The entrepreneur maximizes the net present value of his final return with respect to the number of periods  $T$  and the investment path  $\{I_t\}_{t=1}^{t=T}$

$$\beta^T R - \sum_{t=1}^T \beta^{t-1} I_t, \quad (5)$$

where  $\beta^T R$  is the present value of the project return, and  $\sum_{t=1}^T \beta^{t-1} I_t$  is the present value of the investment path. The entrepreneur will only be involved in the project that provides a non-negative return; that is

$$\beta^T R - \sum_{t=1}^T \beta^{t-1} I_t \geq 0. \quad (6)$$

Given the entrepreneur cannot commit not to repudiate the initial contract we restrict our attention to incentive-compatible paths. On such paths, even if the entrepreneur initiates the renegotiation, he cannot make the investor worse off than in the case of the initial contract. Consequently, the entrepreneur does not have any incentive to trigger renegotiation along these paths as he is not better off repudiating the contract in any period.

**Definition 1.** *Incentive-compatible investment paths are paths along which the entrepreneur will not trigger renegotiation.*

To solve for the incentive-compatible investment path start from period  $T$  and solve backwards. The incentive-compatibility constraint for period  $T$  is

$$\sum_{i=1}^T \beta^{i-1} I_i \leq \beta^{T-1} U_T. \quad (7)$$

The left-hand side is the present value of the payment to the investor as specified in the initial contract. The right-hand side is the present value to the investor of the outcome of the renegotiation in period  $T$ . The left-hand side must be not greater than the right-hand side for period  $T$ , otherwise the entrepreneur has an incentive to decrease the payment to the investor by repudiating.

Next, consider the incentive-compatibility constraint for period  $T - 1$ . Note that the outcome in period  $T - 1$  is not affected by the outcome in period  $T$  because the investor knows that the repudiation in period  $T$  will not occur. The incentive-compatibility constraints for periods  $t = 1, \dots, T - 2$  are constructed in the same way. All the incentive-compatibility constraints are summarized by the following formula

$$\sum_{i=1}^T \beta^{i-1} I_i \leq \beta^{t-1} U_t + \sum_{i=t+1}^T \beta^{i-1} I_i, \quad 1 \leq t \leq T. \quad (8)$$

The present value of the payment to the investor specified in the initial contract cannot be greater than the present value of the payment to the investor in the case of repudiation in period  $t$ ; this consists of the net surplus for the investor in period  $t$  plus the investment to be made by the investor in periods after  $t$ , both in present value terms.

After simplifying the inequality (8) we get

$$\sum_{i=1}^t \beta^{i-t} I_i \leq U_t, \quad 1 \leq t \leq T. \quad (9)$$

This inequality has the same meaning as inequality (2) in Hart and Moore (1994). The payoff to the investor in the case of repudiation must be at least as much as the expenses the investor has already incurred. In other words, investments should be completely covered by the liquidation value or by the future return. If at some time  $t$  this condition does not hold, the entrepreneur has an incentive to repudiate the initial contract at that time. Because the final return for a given investment path is fixed, if in the case of the repudiation the investor gets less, the entrepreneur necessarily gets a higher return than in the case of the initial contract.

Now let us present the full maximization problem. For a given discount factor  $\beta$ , the final return  $R$  and total amount of the physical capital investment  $K$ , the following utility function is maximized by the entrepreneur with respect to the number

of periods  $T$  and investment path  $\{I_t\}_{t=1}^{t=T}$

$$\max_{T, \{I_t\}_{t=1}^{t=T}} \beta^T R - \sum_{t=1}^T \beta^{t-1} I_t \quad (10)$$

subject to (1), (6) and (9), namely

$$\sum_{t=1}^T I_t = K, \quad \beta^T R - \sum_{t=1}^T \beta^{t-1} I_t \geq 0 \quad \text{and} \quad \sum_{i=1}^t \beta^{i-t} I_i \leq U_t, \quad 1 \leq t \leq T.$$

Equation 10 states that the entrepreneur wishes to maximize the return of the project provided that the project is completed ( $\sum_{t=1}^T I_t = K$ ), provides a non-negative return and the initial contract is incentive compatible (as defined in Definition 1).

Before we present solution to this problem let us introduce two definitions.

**Definition 2.** *Investment path  $I_1, I_2, \dots, I_T$  is feasible if and only if it satisfies conditions (1), (6) and (9).*

**Definition 3.** *Investment path  $I_1, I_2, \dots, I_T$  is optimal if and only if it solves problem (10).*

The following proposition gives necessary and sufficient conditions for the optimal investment path.

**Proposition 1.** *A feasible investment path is optimal if and only if the following 4 conditions are satisfied<sup>7</sup>*

$$U_t = L_t, \quad 2 \leq t \leq T, \quad (11)$$

$$\sum_{i=1}^t \beta^{i-t} I_i = U_t, \quad 2 \leq t \leq T, \quad (12)$$

$$U_1(T) \geq I_1(T) \quad \text{and} \quad U_1(T-1) < I_1(T-1), \quad (13)$$

and the Minimal Cost Condition MCC described below.

**Minimal Cost Condition.** *If there is only one investment path that satisfies conditions (11), (12) and (13), this path satisfies the MCC. If more than one investment path satisfies conditions (11), (12) and (13), there is a unique path that satisfies the MCC with the following pecking order property:*

- of all the possibilities select the path with the lowest value of  $I_1$ ;

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<sup>7</sup>Note, in (13)  $I_1(T)$  and  $U_1(T)$  are the investment and the present value to the investor of renegotiation in the first period when there are  $T$  periods. If  $T = 1$  the condition  $U_1(T-1) < I_1(T-1)$  is unnecessary. Note also that from (2) and (4) it follows that  $U_1 = S_1/2$ .

- of all the possibilities that satisfy the condition specified in the previous item select the path with the lowest value of  $I_2$ ;
- of all the possibilities that satisfy the conditions specified in the previous items select the path with the lowest value of  $I_3$ ;

and so on up until period  $T$ .

The technical proof of Proposition 1 can be found in the Appendix. Here we present some intuition for conditions (11), (12), (13) and the *MCC*.

Condition (11) states that for periods  $2 \leq t \leq T$ , the present value to the investor of the outcome of renegotiation in period  $t$  is equal to the liquidation value in period  $t$ . This means that for the optimal investment path, the liquidation value (2) is equal or higher than a half of the surplus (3). If in some period  $\tau > 1$  the liquidation value (2) is less than a half of the surplus (3), then investment from periods  $t = 1, 2, \dots, \tau$  can be added together and invested in period 1. The constructed investment path will satisfy all the feasibility constraints (because the original investment path satisfies them) and will be shorter, which means it will deliver a higher payoff to the entrepreneur.

Condition (12) states that the incentive compatibility constraints (9) for the optimal investment path in periods  $2 \leq t \leq T$  are binding. If one of these conditions is not binding in period  $t$ , then investment from earlier periods (for example, from period  $t - 1$ ) can be moved to period  $t$  and this change will increase the entrepreneur's payoff (same output, smaller net present value of costs) while all the incentive-compatible constraints remain satisfied ( $t$  constraint is non-binding, so a small change will leave it non-binding, while all other constraints can only be aided by this change).

Condition (13) is constructed from the incentive compatibility constraint (9) for  $t = 1$ . Due to the costs of delay this condition requires that the minimal  $T$  for which this constraint is satisfied is optimal.

The *MCC* chooses among the investment paths satisfying conditions (11), (12) and (13) a path that provides the smallest payment to the investor.<sup>8</sup> With the fixed number of periods, it is optimal to wait and invest as late as possible. Thus, an analogue of the Pecking Order is generated by this condition. Neher (1999) does not need the *MCC* because when the liquidation value  $L_t$  increases at a constant rate ( $s$ ) the problem with non-uniqueness does not appear.

Note that not every project will be financeable, which means that not always an optimal investment path exists. If, however, the project is financeable, Proposition 1 gives the necessary and sufficient conditions which this path must satisfy.

Now let us construct an algorithm for finding the optimal investment path: this investment path solves problem (10). For a moment assume that  $T$  is exogenously given. From conditions (11) and (12) we derive

$$I_t = L_t - \sum_{i=1}^{t-1} \beta^{i-t} I_i, \quad 2 \leq t \leq T \quad (14)$$

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<sup>8</sup>For a given  $T$  the problem of maximizing the entrepreneur's utility (10) is equivalent to minimizing the payment to the investor.

that depends only on the choice of  $I_1$ .<sup>9</sup> Next, we use equation (1) and find the value of  $I_1$ . The value of  $I_1$  solves

$$I_1 + I_2(I_1) + \dots + I_T(I_1) = K. \quad (15)$$

Conditions (11), (12) and (13) do not necessarily generate a unique path. However, these conditions along with the *MCC* generate a unique investment path. We use equation (14) and the *MCC* to derive the unique values of investments for  $2 \leq t \leq T$ . Thus, for any given  $T$  we can construct an investment path that is optimal when the number of periods is  $T$ . The next task is to find the optimal value of  $T$ . Condition (13) solves this task - this is the smallest number of periods for which the present value to the investor from renegotiation in the first periods is at least as large as the first period investment. With a fixed number of periods in the investment path, the *MCC* ensures that the cost of the investment in present terms is as small as possible by delaying the investment costs as much as possible. With a set number of periods ( $T$ ), the return of the project in present value terms is does not change; pushing the required investment further into the future, however, reduces the cost of financing the project in present value terms.

As summarized below, using the properties of the optimal path, one can use the Algorithm to find the optimal path.

**Algorithm.** *To derive the optimal investment path assume first that  $T = 1$ . Check condition (13). If this condition holds then the project is financed in one period with  $I_1 = K$ . If this condition does not hold then assume  $T = 2$ . Using equations (14), (15) and the *MCC*, derive the two-period investment path. Check condition (13). If this condition holds then the project is financed in two period. If not, then assume  $T = 3$ , etc.*

Note that if the project is not financeable, the Algorithm will not satisfy condition (13) for any  $T$ . In this case, it is never the case that the investor will be afforded sufficient collateral protection in order to ensure they receive a return that at least covers their investment costs.

Let us illustrate Algorithm in the following example.

**Example 1.** *Let  $\beta = 1$ ,  $K = 5$ , collateral accumulates according to  $f(x) = 0$  for  $x \leq 1$  and  $f(x) = 2(x-1)$  for  $x > 1$ . Figure 2 illustrates the accumulation of collateral for this project. Also, for simplicity let the final return be  $R = f(K) = 2(K-1) = 8$ . Using the Algorithm we can derive the optimal investment path for the entrepreneur.*

*First, note that the project will not be financed in one period. Condition (13) is not satisfied because  $U_1 = S_1/2 = f(K)/2 = K - 1 = 4$  is always less than  $I_1 = K = 5$ . In the event of renegotiation after the investment is sunk, the investor's renegotiated payment that arises out of the bargaining process is not large enough to cover her costs.*

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<sup>9</sup>The value of every next investment depends only on values of previous investments. For a given value of  $I_1$  we can find  $I_2$ , then  $I_3$  and so on.

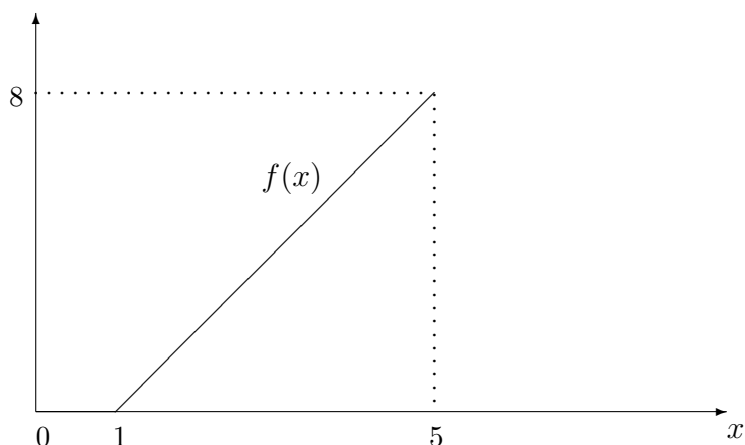


Figure 2: Collateral accumulation given by  $f(x)$

Second, let us check whether the project can be financed in two periods ( $T = 2$ ). Using equations (14) and (15) construct an optimal investment path.<sup>10</sup> Equation (14) is  $f(I_1) = I_1 + I_2$ ; while, equation (15) is  $I_1 + I_2 = K$ . Combine them and derive  $K = f(I_1)$ , where by definition  $f(I_1) = 2(I_1 - 1)$ . Thus,  $I_1 = K/2 + 1$  and  $I_2 = K/2 - 1$ . Now we apply condition (13), that is  $U_1 = S_1/2 = \frac{2(K-1)-I_2}{2} \geq I_1$ . The LHS of this equation is equal to 3.25, while the RHS is 3.5. Consequently, it is not possible to finance the project with two periods.

Now check whether the project can be financed in three periods. From equations (14)  $K = f(I_1 + I_2) = 2(I_1 + I_2 - 1)$ . Hence,  $I_1 + I_2 = \frac{K}{2} + 1$  and  $I_3 = \frac{K}{2} - 1$ . From (14),  $f(I_1) = I_1 + I_2$ . Combining  $f(I_1) = I_1 + I_2 = \frac{K}{2} + 1$  allows us to derive  $f(I_1) = \frac{K}{2} + 1$ . From this we can calculate  $I_1 = \frac{K}{4} + \frac{3}{2} = \frac{11}{4}$ ,  $I_2 = \frac{K}{4} - \frac{1}{2} = \frac{3}{4}$  and  $I_3 = \frac{6}{4}$ . This path is feasible if condition (13) is satisfied. This requires that  $S_1/2 = \frac{2(K-1)-I_2-I_3}{2} \geq I_1$ . This holds as  $\frac{S_1}{2} = \frac{23}{8} \geq \frac{11}{4}$ , so the project can be financed in three periods. This investment path is optimal.  $\square$

## 4 Extensions and comparisons

Here we consider several extensions to the model considered above. Importantly, we show that some of the results derived in Neher (1999) depend on the assumption of a constant rate of return (for the liquidation value) and that they do not necessarily hold in a more general model.

First, consider the relationship between the discount factor and the feasibility of

<sup>10</sup> Given the simplicity of function used in this example for collateral, it is not necessary to use the MCC in this case as the path is unique.

completing project with the commitment problem. Neher (1999) found that in the limit as  $\beta \rightarrow 1$  all profitable ventures become financeable. In the more general case presented here, this result does not necessarily hold. Consider the case when there is relatively little accumulation of tangible physical assets that can be liquidated at the beginning of the project. It is possible that insufficient collateral will accumulate, thus preventing adequate protection for the investor from repudiation. In this case, it is not possible for the entrepreneur credibly commit to the initial contract, and the investor could be unwilling to commence funding such a project. As a trivial example, consider a project in which no tangible collateral is produced until the whole project is completed; it follows that staging investment provides no more protection from repudiation to the investor than a one-off investment. Result 1 summarizes this discussion.

**Result 1.** *A lack of discounting does not ensure that all profitable projects are financed.*

Second, consider the case when the entrepreneur has wealth greater than zero. In this situation we have the following result.

**Result 2.** *Some of the profitable projects will not go forward even if an entrepreneur's wealth is greater than zero.*

Result 2 is different from the analogous result of Neher (1999), who found that all the profitable projects go forward if the entrepreneur's wealth is greater than zero. The intuition for this result is similar to the intuition for Result 1. One source of protection for the investor in the case of renegotiation comes from the collateral that has accumulated. As we noted in Result 1, if this collateral is not sufficiently large it will not afford the investor enough protection to ensure the project proceeds. Another potential source of collateral comes from the entrepreneur's own wealth. In this case, the entrepreneur puts their own money into the project, which will be added to the assets that can be liquidated by the investor in the event of termination. If, however, the combination of the entrepreneur's wealth and the collateral that accumulates during the development of the project is not sufficiently large, there may not necessarily be enough collateral to protect the investor in the case of repudiation. Consequently, even if the entrepreneur has some wealth, not all profitable projects will proceed.

To further illustrate the properties of the optimal investment path, we assume that  $\beta \rightarrow 1$  for the remainder of this section. Here we specify the necessary and sufficient conditions for the existence of the optimal investment path (in other words for the project to be financeable). In this special case the Algorithm can be used to help construct the optimal investment path graphically. In Figure 3 the liquidation value is characterized by  $f(x) = f(\Lambda_{t-1})$ . From equation 3, when  $\beta \rightarrow 1$  the one half of surplus ( $S_t/2$ ) can be represented by the line  $g(x) = \frac{1}{2}(x + R - K)$ . In addition, we draw a 45 degree line from the origin.

Now let us apply the Algorithm. For the project to be financeable in one period, its return  $R$  has to be not less than  $2K$  (condition 13). If this is not satisfied consider,

as specified by the Algorithm, whether the project can be financed in two periods. From equation 14 when  $\beta \rightarrow 1$ , the investment in the last ( $T$ -th) period has the property that  $f(\Lambda_{T-1}) = K$ . We take a point with coordinates  $(K, K)$  on the 45 degree line (labeled as point  $A$ ) and draw a horizontal segment until it intersects with  $f(x)$  function; label this point  $B$ . The distance between  $A$  and  $B$  is the investment in the final period for any  $T \geq 2$ . Let us check whether the project can be completed in two periods ( $T = 2$ ). Investment in the first period in that case would have to be equal to the horizontal distance between the origin and  $B$ ; the second-period investment is the distance between  $B$  and  $A$ . Consider  $g(x)$  - the value of  $S_1/2$  - when  $x = I_1 = K - I_T$ . In order for the project to be completed in two periods,  $g(K - I_T)$  needs to be above or on the 45 degree line (condition 13). If this condition is not met, as in our example, we continue the process for three investment periods as specified in the Algorithm. With three investment periods, from condition 14, the investment in the final period is the distance between  $B$  and  $A$ ; the second period investment is the horizontal distance between  $C$  and  $B$ ; and the investment in the first period is the horizontal distance between the origin and point  $C$ . The project can be financed in three periods if  $g(I_1) = g(K - I_T - I_{T-1})$  is above the 45 degree line. If  $g(I_1)$  lies below the 45 degree line, the project cannot be financed in three periods, we continue the process for four periods, and so on, as specified by the Algorithm and as illustrated in our example in Figure 3.

As shown graphically in Figure 3, the investment required in each period can be determined moving in a cobweb manner starting at point  $A$  so as to measure the horizontal distance between  $f(x)$  and the 45 degree line for each successive investment until we reach a point where condition 13 is satisfied; this will occur when  $x < R - K$ . From this we can find the condition when the Algorithm cannot provide a solution to the problem. This will occur when the cobweb never reaches a point where  $x < R - K$ . This will happen when  $f(x) \leq x$  for some  $x > R - K$ . In this case the cobweb will converge to some point  $x > R - K$ .<sup>11</sup> This discussion is summarized in Result 3.

**Result 3.** *When  $\beta \rightarrow 1$  the necessary and sufficient conditions for the project to be financeable in two or more periods ( $R < 2K$ ) is that  $f(x) > x$  for all  $R - K \leq x \leq K$ .*

This analysis allows us to clearly distinguish between the different roles of the final return  $R$  and the liquidation value in supporting investment in the project. The investment in the first period is supported by the final return  $R$ . Once the project has commenced, however, further investments are supported by the liquidation value. Provided the liquidation value is above the 45 degree line - the investor's collateral exceeds the cost of the investment they have already made - the investor can be assured that repudiation will not occur.

Further to this, the sufficient condition for the optimal sequence of investments to be increasing from the second period (as in Neher 1999) is that  $f'(x) \geq 1, \forall x$ .

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<sup>11</sup>Note that  $f(x)$  - the liquidation value - is a monotonic function. Further, if  $f(x)$  is not a continuous function we amend it by adding vertical segments at the points of discontinuity. The amended relationship is continuous, which is necessary and sufficient to prove the above claim.

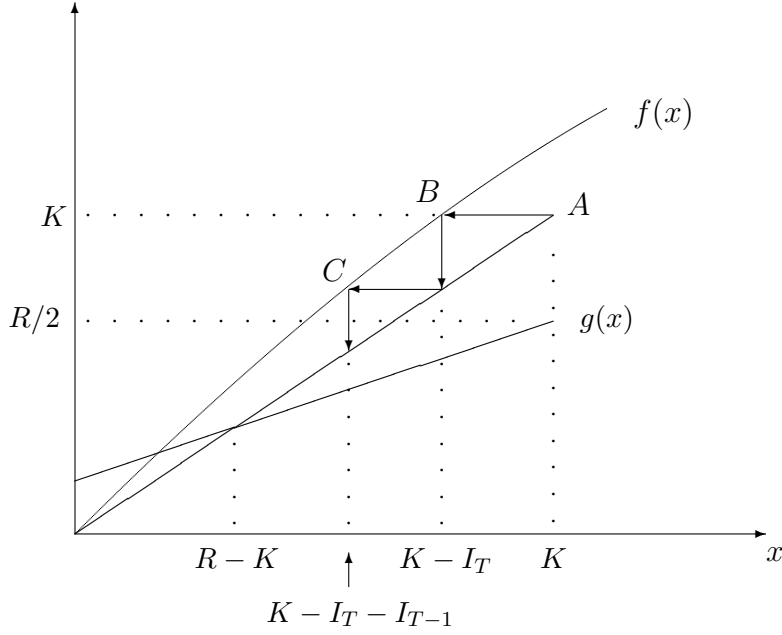


Figure 3: Graphical interpretation of staged financing when  $\beta \rightarrow 1$

As can be seen from the previous discussion, if the gap between  $f(x)$  and the 45 degree line is increasing as  $x$  increases, each subsequent investment (following the first investment) will be larger. This is summarized in Result 4.

**Result 4.** *When  $\beta \rightarrow 1$ , the sufficient condition for monotonically increasing sequence of investments from the second investment is that  $f'(x) \geq 1, \forall x$ .*

In this light, if  $f(x)$  gets closer to the 45 degree line - the gap between liquidation value and the 45 degree line is reduced - the optimal investment path will not necessarily be monotonically increasing. Consider the following example where the liquidation value  $L_t = f(\Lambda_{t-1})$  is described by the following function:

$$f(x) = \begin{cases} 2x, & x \leq 1; \\ 1.4 + 0.6x, & x \in (1, 2]; \\ 2x - 1.4, & x \in (2, 2.6]. \end{cases} \quad (16)$$

Figure 4 shows both  $f(x)$  and the 45 degree line. Again we assume that  $\beta \rightarrow 1$  and let the final return of the completed project be  $R = 3.8$ .

Let us show that investment path  $I_1 = \$1, I_2 = \$1$  and  $I_3 = \$0.6$  is optimal. First, the project is not financed in one period because  $S_1(1)/2 = R/2 = \$1.9 < K = \$2.6$ .

Second, the project is not financed in two periods. Investments  $I_1 = \$2$  and  $I_2 = \$0.6$  satisfy condition (14),  $L_2 = f(I_1) = \$2.6 = I_1 + I_2 = \$2.6$ . Let us examine condition (13):  $S_1(2)/2 = (R - I_2)/2 = \$1 < I_1 = \$2$ . Condition (13) is not satisfied, which means that the project is not financed in two periods.

Now we take three-period investment path  $I_1 = \$1, I_2 = \$1, I_3 = \$0.6$  and show

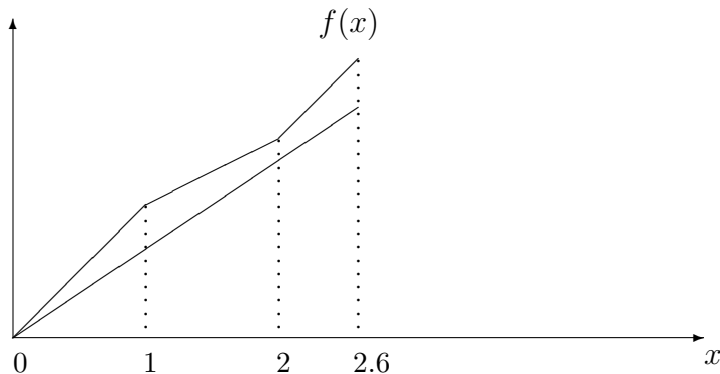


Figure 4: The accumulation of collateral  $f(x)$

that it is optimal. Condition (14) is satisfied,  $L_2 = f(I_1) = \$2 = I_1 + I_2 = \$2$  and  $L_3 = f(I_1 + I_2) = \$2.6 = I_1 + I_2 + I_3 = \$2.6$ . Condition (13) is also satisfied,  $S_1(3)/2 = (R - I_2 - I_3)/2 = \$1 = I_1 = \$1$ .

Thus, the three-period investment path is optimal. One can see that this path is decreasing.<sup>12</sup>

This result differs from the finding of Neher (1999) that investments, starting from the second investment, will be monotonically increasing; that is, every next investment after the second one is bigger than the previous investment. The explanation for the difference is that in the general case the values of investments are determined by collateral.<sup>13</sup> If the values of collateral for new investments are decreasing, the sequence of investments for such functions will be decreasing as well. A real-life example of this is the case of Federal Express, discussed in Gompers (1995): the first investment in Federal Express was £12.25 million, the second was £6.4 million and the third was £3.88 million. Clearly, the sequence of investments is decreasing.

## Appendix

### Proof of Proposition 1

The proof consists of two sections. In the first section we only consider investment paths that satisfy  $S_t \geq L_t$  for any  $t$ ; that is the surplus in any period is at least as large as the liquidation value. In the second section we relax this assumption.

<sup>12</sup>Note that we could have used the graphical approach to determine the sequence of investments, which would give the same results.

<sup>13</sup>In the constant rate of return case these values are always increasing, otherwise the project is unprofitable.

## Section 1

To proceed, assume that condition (13) holds and find which path is optimal in this case. Here we show that such a path has to satisfy conditions (11), (12) and the *MCC*.

(13)  $\implies$  (11)

First, we prove that from condition (13), condition (11) follows. That is, for the minimal feasible  $T$

$$L_t \geq S_t(T)/2 \quad \forall t = 2, 3, \dots, T.$$

This statement can be proved by contradiction. Assume that for optimal investment path  $I_1, \dots, I_T$  for some  $t \geq 2$  this property does not hold.<sup>14</sup> Let us construct some other shorter path with  $T' = T - t + 1$ , where  $I'_1 := I_t, I'_2 := I_{t+1}, \dots, I'_{T-t+1} := I_T$ . We now examine whether the incentive constraints are satisfied for the new path. The feasibility constraints (1) and (6) are satisfied by construction. Let us prove that the incentive compatibility constraint (9) with respect to  $I'_1$  holds, that is  $I'_1 = I_1 + \dots + I_t \leq S'_1(T - t + 1)/2 = U'_1$ , where  $S'_1(T - t + 1)$  is a surplus to be bargained over in period one. It is straight forward to show that this surplus is the same as  $S_t(T)$  for the initial path. From the assumption that condition (11) does not hold in period  $t$  and from the incentive compatibility constraint (9) for  $I_t$  it follows that  $S_t(T)/2 = U_t \geq I_t + \dots + I_1/\beta^{t-1} \geq I_1 + \dots + I_T$ . This proves the first incentive constraint for the new path. If  $t = T$ , there is only one incentive constraint, which has just been shown. If  $t < T$ , there are some other incentive constraints that need to be considered.

Let us prove that the incentive compatibility constraint with respect to  $I'_2$  holds. We can modify inequality (9) for period  $t + 1$  to get<sup>15</sup>

$$I'_2 = I_{t+1} \leq L_{t+1} - I_1/\beta^t - \dots - I_t/\beta \leq L_{t+1} - (I_1 + \dots + I_t)/\beta = L'_2 - I'_1/\beta.$$

The same way we prove the incentive compatibility for any  $I'_i$ , where  $2 \leq i \leq T - t + 1$

$$\begin{aligned} I'_i &= I_{i+t-1} = L_{i+t-1} - I_1/\beta^{i+t-2} - \dots - I_{i+t-2}/\beta < \\ &< L_{i+t-1} - (I_1 + \dots + I_t)/\beta^{i-1} - I_{t+1}/\beta^{i-2} - \dots - I_{i+t-2}/\beta = \\ &= L_{i+t-1} - I'_1/\beta^{i-1} - I'_2/\beta^{i-2} - \dots - I'_{i-1}/\beta. \end{aligned}$$

So far we have shown that the constructed  $T - t + 1$  path is feasible. Let us show the new path allows the entrepreneur to get higher utility than the original path. To do this we introduce another  $T$  period path, that has zero investments during the first  $t - 1$  periods, and then coincides with the  $T - t + 1$  path, that is

<sup>14</sup>If there is a set of indexes where condition (11) does not hold then  $t$  is the largest index in this set.

<sup>15</sup>Note, that in period  $t + 1$  condition (11) holds because  $t$  is chosen to be the largest among indexes for which that condition does not hold.

$I_i'' = 0 \forall i = 1, \dots, t-1$ ,  $I_t'' = I_1 + \dots + I_t$ ,  $I_i'' = I_i \forall t+1 \leq i \leq T$ . Now we show that this additional  $T$ -period path  $\{I_i''\}_{i=1}^{i=T}$  delivers higher utility than the original  $T$ -period path and lower utility than the new  $(T-t+1)$  path, see Figure 5.

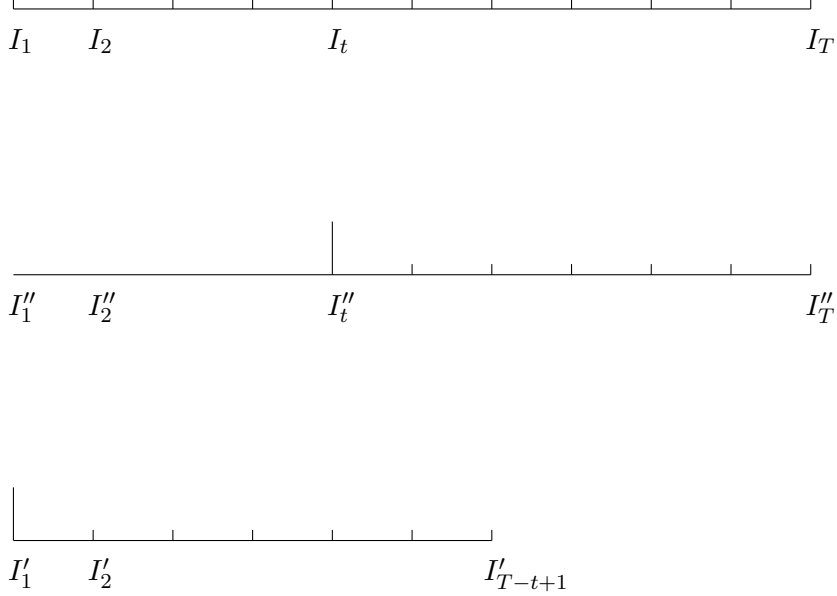


Figure 5: Investment paths  $\{I_i\}_{i=1}^{i=T}$ ,  $\{I_i''\}_{i=1}^{i=T}$  and  $\{I_i'\}_{i=1}^{i=T-t+1}$

Comparing the additional and the original paths, both paths generate the same return  $\beta^T R$ , while the original path has higher costs of delay because some of the investments were made earlier. Consequently,

$$\beta^T R - \sum_{t=1}^T \beta^{t-1} I_t \leq \beta^T R - \sum_{t=1}^T \beta^{t-1} I_t''.$$

The new  $T$ -period path is actually  $T-t+1$ -period path that starts after waiting for  $t-1$  periods. Because of costs of delay the utility from the additional  $T$ -period path will be  $1/\beta^{t-1}$  smaller than the utility from the  $T-t+1$ -period path

$$\beta^T R - \sum_{t=1}^T \beta^{t-1} I_t'' = \beta^{t-1} (\beta^{T-t+1} R - \sum_{t=1}^{T-t+1} \beta^{t-1} I_t') \leq \beta^{T-t+1} R - \sum_{t=1}^{T-t+1} \beta^{t-1} I_t'.$$

Thus, we have showed that the constructed  $T-t+1$ -period path is both feasible and delivers higher utility than the optimal  $T$ -period path. This is a contradiction.

**(13) and (11)  $\implies$  (12)**

Second, we show that condition (12) is also satisfied when conditions (13) and (11) hold. We again prove this statement by contradiction. Assume that for the optimal investment path  $I_1, \dots, I_T$  for some  $t \geq 2$ , the following strict inequality holds  $\sum_{i=1}^t \beta^{i-t} I_i < U_t$ . To find a contradiction we construct an additional investment path by moving  $\epsilon$  investment from  $I_{t-1}$  to  $I_t$  and show that all the feasibility constraints hold and the new path is preferred by the entrepreneur.<sup>16</sup> The feasibility constraints (1) and (6) are satisfied by construction.

With respect to incentive constraints for periods after  $t$  this change is feasible because it only diminishes the costs of the previous investments. From condition (11) the right-hand side of (9) is  $U_i = L_i \forall i = 2, \dots, T$ . These liquidation values are the same after the change, while the left-hand sides of (9) diminish because  $\epsilon$  investment was moved to a later period. Thus after the change the incentive compatibility constraints for periods after  $t$  are satisfied.

The constraint for  $t$  is satisfied because  $\epsilon$  is chosen to be small enough.

The constraints for periods before  $t$  are satisfied because they were satisfied before the change was made. Both the left- and the right-hand sides of (9) are unchanged.

The change is beneficial for the entrepreneur because it only diminishes the costs of the investment. Thus, this is a contradiction because the original investment path was optimal.

**(11), (12) and (13)  $\implies$  MCC**

Compare investment path  $I_1, \dots, I_T$  that satisfies conditions (11), (12), (13) and the *MCC* and some other investment path  $I'_1, \dots, I'_T$  that satisfies only (11), (12) and (13), but does not satisfy the *MCC*. Let us prove that  $\Lambda_t \leq \Lambda'_t \forall t = 1, 2, \dots, T-1$ .

The inequality for  $t = 1$  follows from the *MCC*:  $\Lambda_1 = I_1$  and  $I_1$  is the minimum.

The inequality for  $t = 2, \dots, T-1$  can be proved by contradiction. Assume  $\Lambda_t > \Lambda'_t$  for some  $t = 2, \dots, T-1$ . Let us construct some other path  $\{I''\}_{t=1}^{t=T}$  that has  $\Lambda''_t = \min[\Lambda_t, \Lambda'_t] \forall t = 2, \dots, T$ . This path satisfies all the incentive compatibility constraints because they are satisfied along both original paths. The feasibility constraints (1) and (6) are satisfied by construction. On the other hand, this path delivers higher utility than any of  $\{I_t\}_{t=1}^{t=T}$  and  $\{I'_t\}_{t=1}^{t=T}$  investment paths because it has smaller costs. Thus, we show that condition (12) is not necessary for an optimal investment path, which contradicts to previous sections of the proof.<sup>17</sup>

Next step is to show that path  $\{I_t\}_{t=1}^{t=T}$  that has the property  $\Lambda_t \leq \Lambda'_t \forall t = 1, 2, \dots, T-1$  delivers higher utility than  $\{I'_t\}_{t=1}^{t=T}$ . Note, that both investment paths have the same return  $\beta^T R$ . Let us show that the costs for the investment path  $\{I_t\}_{t=1}^{t=T}$  are smaller.

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<sup>16</sup> $I_{t-1}$  has to be strictly positive, otherwise the original investment path is not optimal as the  $(t-1)$ -th period can be removed from the investment path.

<sup>17</sup>If investment paths  $\{I_t\}_{t=1}^{t=T}$  and  $\{I'_t\}_{t=1}^{t=T}$  are different then at least in some period  $t$   $I''_t < I_t$ , which means that condition (12) is not satisfied on investment path  $\{I''_t\}_{t=1}^{t=T}$ , because by construction it is satisfied on  $\{I_t\}_{t=1}^{t=T}$ .

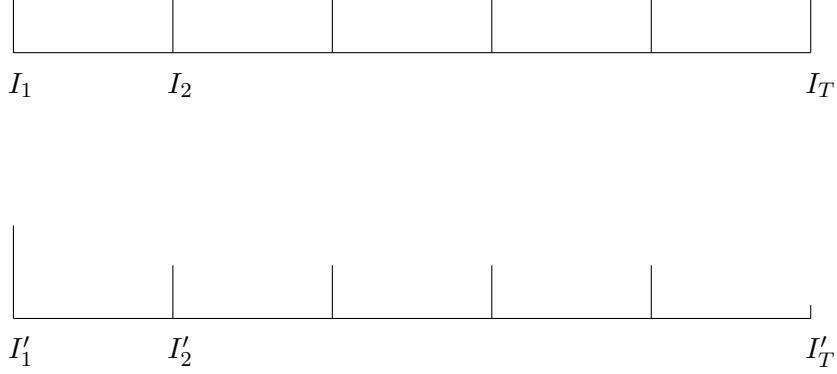


Figure 6: Investment paths  $\{I_t\}_{t=1}^{t=T}$  and  $\{I'_t\}_{t=1}^{t=T}$

Let us first compare costs of  $I_T$  investment for both paths. Note, that  $I_T \geq I'_T$  because  $\sum_{t=1}^{t=T} I_t = \sum_{t=1}^{t=T} I'_t = K$  and  $\Lambda_{T-1} \leq \Lambda'_{T-1}$ . It means that for  $\{I_t\}_{t=1}^{t=T}$  path the whole  $I_T$  was invested in the last period, while for  $\{I'_t\}_{t=1}^{t=T}$  path only some part of  $I_T$  was invested in the last period and the rest was invested in period  $T - 1$ . So,  $I_T$  has higher costs for path  $\{I'_t\}_{t=1}^{t=T}$ . Next, we compare costs of  $I_{T-1}, \dots, I_1$  investments for both paths and using the same approach get similar results. Thus, the costs of  $\{I_t\}_{t=1}^{t=T}$  investment path are smaller, which means that it delivers higher utility than the  $\{I'_t\}_{t=1}^{t=T}$  investment path, and consequently we prove that condition *MCC* is necessary.

To summarize, we have proved that for the minimal feasible  $T$  the optimal path satisfies conditions (11), (12) and the *MCC*. Now the next part of the proof, we show that this path, that is the path satisfying conditions (11)-(13) and the *MCC*, yields to the entrepreneur a higher utility than any feasible path for some larger  $T$ .

**(11)-(13) and *MCC* gives higher utility than any path with a greater number of periods**

Consider an investment path that satisfy (11)-(13) and *MCC* with  $T$  periods, namely  $I_1, \dots, I_T$ , and any other path with  $T + i$  periods, namely  $I'_1, \dots, I'_{T+i}$ . Let us show that the path with  $T + i$  during the first  $i + t$  periods invests not less than  $\Lambda_t$ .

First, to find a contradiction assume that  $\Lambda'_{i+1} < I_1$ . Construct another  $T$ -period path with  $\Lambda''_t = \min[\Lambda_t, \Lambda'_{i+t}]$ ,  $\forall t = 1, \dots, T$ . The feasibility constraints (1) and (6) are satisfied by construction. Let us prove that this path is incentive compatible. The incentive compatibility constraints for  $t = 2, \dots, T$  follow from the fact that the costs for the new path are smaller than in any of the two original paths. To prove the incentive compatibility of the  $t = 1$  constraint rewrite the first incentive compatibility

constraint for  $I_1, \dots, I_T$  path in the following way

$$I_1 \leq \beta^T (R - I_T/\beta - \dots - I_2/\beta^{T-1} - I_1/\beta^T).$$

The first incentive compatibility constraint for the new path is satisfied because  $I_1'' < I_1$  and the costs for the new path are smaller. Thus, the new path is feasible and it delivers higher utility than the original  $T$ -period path, which is optimal among  $T$ -period paths. This is a contradiction.

Second, to find a contradiction with  $2 \leq t \leq T$  assume that  $\Lambda_{i+t} < I_t$ . We use the same  $T$ -period path  $\Lambda_t'' = \min[\Lambda_t, \Lambda_{i+t}']$ ,  $\forall t = 1, \dots, T$ . We show that the new path is feasible and delivers higher utility than the original  $T$ -period path. This allows us to find a contradiction.

The property  $\Lambda_t \leq \Lambda_{t+i}'$  ensures that the investment path  $I_1, \dots, I_T$  delivers higher utility than  $I_1', \dots, I_{T+i}'$ . This observation concludes this part of the proof.

Note, that the investment path satisfying conditions (11)-(13) and the *MCC* is unique by construction.

### Property $S_t \geq L_t, \forall t$ is satisfied on optimal path

To finish this subsection we prove that  $S_t \geq L_t$  is always satisfied on the constructed investment path. First note that this property holds for  $t = 1$ .  $S_1$  is positive because the project is profitable and  $L_1 = 0$  by construction, that is

$$S_1 > L_1.$$

Next, using modified equations (3), (14) and (13) we prove the property holds for  $t = 2$ , that is

$$S_2 = S_1/\beta + I_2, \quad L_2 = I_1/\beta + I_2 \quad \text{and} \quad I_1 \leq S_1/2 \implies S_2 \geq L_2.$$

Now, prove by induction that  $S_t \geq L_t$ . Starting with  $S_2 \geq L_2$ , as shown above, using modified equations (3) and (14) it follows that

$$S_t = S_{t-1}/\beta + I_t, \quad L_t = L_{t-1}/\beta + I_t \implies S_t \geq L_t \quad \forall t = 3, \dots, T.$$

### Section 2

Here we show that any incentive-compatible path for which the property  $S_t \geq L_t$  for all  $t$  is not satisfied is either: dominated by the optimal investment path constructed using the algorithm. Of course, any project that at best delivers the entrepreneur negative payoff will not be chosen.

Consider an investment path that has

$$S_t < L_t$$

for some  $2 \leq t \leq T$ . Let us prove that in that period condition (9) is satisfied. To prove this inequality we use the fact that the project delivers the entrepreneur a

non-negative return, that is

$$\beta^T R - \sum_{i=1}^{i=T} \beta^{i-1} I_i \geq 0.$$

We divide the inequality above by  $\beta^{t-1}$  and using equation (3) derive

$$S_t \geq \sum_{i=1}^t \beta^{i-t} I_i.$$

Combining this inequality with the assumption that  $L_t > S_t$ , we prove condition (9) for period  $t$ . It means that in all periods where  $L_t > S_t$  condition (9) is satisfied. In other periods it is also satisfied because the investment path is incentive compatible. This means that the investment path is feasible for optimization problem (10). Once it is feasible it is dominated by the optimal investment path constructed by the Algorithm.

## References

- [1] Admati, A. and M. Perry 1991, 'Joint Projects Without Commitment', *Review of Economic Studies*, vol. 58, no. 2, pp. 259-76.
- [2] Admati, A. and P. Pfleiderer 1994, 'Robust Financial Contracting and the Role of Venture Capitalists', *The Journal of Finance*, vol. 49, pp. 371-402.
- [3] Bolton, P. and D. Scharfstein 1990, 'A Theory of Predation Based on Agency Problems in Financial Contracting', *American Economic Review*, vol. 80, no. 1, pp. 93-106.
- [4] Chiu, S. 1998, 'Noncooperative Bargaining, Hostages, and Optimal Asset Ownership', *American Economic Review*, vol. 88, no. 4, pp. 882-901.
- [5] Gompers, P. A. 1995, 'Optimal Investment, Monitoring, and the Staging of Venture Capital', *Journal of Finance*, vol. 50, pp. 1461-1489.
- [6] Hart, O. and J. Moore 1994, 'A Theory of Debt Based on Inalienable Human Capital', *Quarterly Journal of Economics*, vol. 109, no. 4, pp. 841-79.
- [7] Neher, D. 1999, 'Staged Financing: An Agency Perspective', *Review of Economic Studies*, vol. 66, no. 2, pp. 255-74.
- [8] Roberts, K. and M. Weitzman 1981, 'Funding Criteria for Research, Development and Exploration Projects', *Econometrica*, vol. 49, pp. 1261-1268.
- [9] Sahlman, W. A. 1988, 'Aspects of Financial Contracting in Venture Capital', *Journal of Applied Corporate Finance*, vol. 1, pp. 23-36.

- [10] Sahlman, W. A. 1990, 'The Structure of Governance of Venture-Capital Organizations', *Journal of Financial Economics*, vol. 27, pp. 473-521.
- [11] Shaked, A. and Sutton, J. 1984, 'Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model', *Econometrica*, vol. 52, no. 6, pp. 1351-64.