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Modeling airport capacity choice with real options

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This study analyzes optimal choice of the airport capacity to invest immediately (the prior capacity) and the size of real option to acquire for possible future expansion. Facing demand uncertainty, an airport first chooses its prior capacity and real option, and then later chooses its final capacity and airport charge once demand is observed. Our analytical results show that if demand uncertainty is low and capacity and real option costs are relatively high, an airport will not acquire a real option. Otherwise, an airport can use a real option to improve its expected profit or social welfare. Both the magnitude of profit or welfare gain and the optimal size of the real option increase with demand uncertainty. A higher real option cost leads to a larger prior capacity and smaller real option, whereas a higher capital cost leads to lower prior capacity. A profit-maximizing airport would choose a smaller prior capacity and real option than a welfare-maximizing airport. Competition in the airline market promotes airport capacity investments and the adoption of real options by profit-maximizing airports, whereas airport commercial services increase prior capacity but not real option.

KEY WORDS: Airport capacity choice; Capacity investment; Real option; Demand uncertainty

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1. Introduction

Airport capacity investment is lumpy and requires lengthy planning and construction process. Moreover, the investment decision is made facing significant demand uncertainty (Oum and Zhang 1990, Maldonaldo 1990, De Neufville and Barber 1991). As a result, capacity at many airports is either under- or over-invested, and non-optimal airport capacity leads to huge losses in social welfare (De Neufville and Odoni 2003). In this paper, we show that airports can use real options to improve the efficiency of their capacity investment and discuss the economic and policy implications of applying real options in airport capacity investment.

This paper is motivated by business strategies adopted by airports to deal with demand uncertainty in making investment decisions. One strategy used by airports is to reserve nearby land for possible future expansion, a practice called ‘land banking’. For example, UPS worked with Louisville airport to secure space for the future expansion of its WorldPort sorting center. Although the Seoul Incheon airport had only two runways when it opened, large areas of land were reserved for a possible future expansion to a maximum of five runways; with the increase in traffic, part of the reserved land has already been converted to capacity. In another example, Taipei Taoyuan International Airport reserved nearby farmland for extended periods before it was designated for airport expansion in 2011. Another strategy used by airports is to use flexible designs in operations and planning. The flexible designs allow airports to have flexibility to adjust their capacity under new circumstances. For example, the Hong Kong International Airport adopted a self-propelled intra-airport passenger transport system which costs more than a cable-driven system at low traffic volume but provides long-term flexibility in capacity expansion. These strategies – land banking and flexible designs, are the applications of “real options” (Transportation Research Board (TRB) 2012) in airport capacity planning; by making investments at an early stage, a real option provides the right, but not the obligation, to take a certain course of action at later stages when more information become available.

Motivated by these business strategies, we build a real option model for airport capacity investment and investigate the effects of using real options on consumer surplus and profits of airlines and airports. The real option in our paper is modeled as the reserved land for possible future airport capacity expansion, although the conclusions should hold for other real option applications. The built model has the following key features. First, capacity investment at an airport follows a multi-stage decision process; the airport first decides the initial capacity (‘prior capacity’ hereafter) and the size of the real option, and then decides to exercise a proportion of the real option given the realization of random demand. Second, the model incorporates airports’ pricing decision; the charge at an airport is made after the total capacity (prior capacity plus the exercised option) is invested. Third, we consider both welfare-maximizing and profit-maximizing airports, which serve as benchmark cases to model the effect of airport ownership. Finally, the model accounts for the vertical relationship between airlines and airports; airlines engage in Cournot competition given airport capacity and charge, and equilibrium outcomes of airline competition determine the traffic volume of airport.

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1 The Montreal Mirabel Airport, new Denver International Airport, and Kansai International Airport are examples of over-investment that led to capacity under-use, either in the short and/or long term. The Newark airport in New York also experienced extended periods of under-use (De Neufville and Odoni 2003). The Amsterdam Schiphol Airport and Hong Kong International Airport (Chek Lap Kok) are examples of under-investment that led to capacity shortage much earlier than expected.

2 Applications of real options in airport capacity planning identified in the TRB report include: land banking, reservation of terminal space, modular or incremental development, linear terminal design and centralized processing facilities, swing gates or spaces, non-load-bearing or glass walls and self-propelled people movers.
This paper is related to three broad streams of literature: literature on modeling airport capacity investment, literature on the effects of demand uncertainty on capacity choice and pricing in transportation sectors and literature on using real options in transport infrastructure investment. Studies modeling airport capacity investment mostly consider deterministic demand. A few studies have modeled the effects of demand uncertainty on airport congestion management (Czerny 2008, 2010), on highway pricing and capacity choice (Kraus 1982, D’Ouville and McDonald 1990) and congestible infrastructure in general (Basso and Zhang 2007, Proost and Van der Loo 2010). Different from these papers in which capacity and price are chosen simultaneously, we explicitly account for the features of large sunk cost and long project cycle of airport capacity investment by extending the analytical model in Xiao et al. (2013), which considers capacity investment and pricing at an airport as sequential decisions. Demand uncertainty matters especially for investment decisions when sunk costs are high and project cycle is long, as in the case of airport capacity investment. Lumpy investments and long project cycle make real options valuable tools for airports to invest effectively. The application of real options in transportation sectors has been studied under the modeling framework in Dixit and Pindyck (1994). These studies focused on valuation and pricing of real options, and the choice of timing given the possibility of deferred investments. The study most related to our paper is Smit (2003), which analyzed optimal investment at an airport when the airport’s future free cash-flow is uncertain. Heuristic solutions and numerical simulations are used in this paper to draw conclusions under very specific market conditions. In comparison, our paper focuses on understanding the effects of using real options on the efficiency of airports with different objectives (welfare maximizing vs. profit maximizing), and policy implications to government regulation when airports use real options in their capacity investment. As such, we develop a model which allows us to draw clear insights from analytical results.

We find that if demand uncertainty is relatively low but capacity and real option costs are relatively high, an airport will not acquire a real option. Otherwise, an airport can use a real option to improve its expected profit or social welfare. A profit-maximizing airport always chooses a smaller prior capacity and a smaller real option than a welfare-maximizing airport. Competition in the airline market promotes capacity investment and the adoption of real options by profit-maximizing airports, whereas commercial services promote prior capacity investments but not real options. In general, our study suggests that real option can be a value tool for the aviation industry to battle uncertainty.

The remainder of this paper is organized as follows. Section 2 introduces the economic model and provides analytical solutions for a profit-maximizing airport. The case of a welfare-maximizing airport is studied in Section 3. Section 4 compares and discusses the modeling results of different cases, and possible complicating factors in practical evaluation of business strategies and regulatory policies. The last section concludes the paper and discusses its limitations and possible future extensions.

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4 Examples of these studies include De Neufville (2003, 2008), Law et al. (2004), Chambers (2007), Martins (2013), Martins et al. (2014), Morgado et al. (2013), Gao and Driouchi (2013), Galera and Soliño (2010), Chow and Regan 2011a, b) and Smit (2003).
2. Economic model and the case of a profit-maximizing airport

In this section we introduce the economic model of real option application, and use the model to analyze capacity investment of a profit-maximizing airport.

2.1. Economic Model

We extend the model in Xiao et al. (2013) to consider a single, risk-neutral airport which invests in capacity to accommodate future traffic volume. The travel demand of airlines is $P = X - bQ$ ($b > 0$), where $P$ is the price paid by travelers (i.e., the airfare), $Q$ is the traffic volume, and $X$ is a random variable that captures the demand forecast’s margin of error (the demand shifters that cannot be precisely estimated). There are $N$ airlines competing in Cournot. Each airline produces an output (traffic volume) of $q_i$ ($i = 1 \ldots N$), such that $Q = \sum_{i=1}^{N} q_i$. The airlines are symmetric in the sense that they all have a constant marginal cost $c$. Let $f(x)$ be the density function of $X$ and $F(x)$ be the corresponding distribution function. For tractability, $X$ is assumed to follow a uniform distribution in the interval $(\bar{x}, \bar{x})$, so $f(x) = 1/(\bar{x} - \bar{x})$ and $(\bar{x} - \bar{x})$ measures the degree of demand uncertainty or variability. The airport sets a service charge per passenger, $w$, and derives concession revenue, $h$, from each passenger. The associated consumer surplus is $v$. Both $h$ and $v$ are assumed to be positive and exogenously determined. \(^5\) Without loss of generality, the marginal cost of the airport is normalized to zero.

We model capacity investment of the airport as a multi-stage game to account for the airport’s pricing decision and vertical relationship between the airport and airlines. In Stage 1, based on the distribution of the demand shifter $f(x)$, the airport decides its “prior capacity” $K_F$, which is immediately usable (e.g., the number of runways that are built immediately), and its “real option” $K_R$ (e.g., the land it reserves for possible future expansion). The unit cost of capacity is $r$ and the unit cost of the real option is $r^*$. The airport investment cost at this first stage is thus $rK_F + r^*K^r$. In stage 2, the airport observes the actual traffic demand pattern $X$ and then exercises a certain amount of real option (i.e. part of / a proportion of the real option $K^r$ purchased in Stage 1) to add the “extra capacity” $K (K \leq K^r)$ with a unit capacity cost $r$. Capacity invested at a later stage can be more costly due to inflation. Such an increase in investment cost is not explicitly modeled due to mathematical tractability. Intuitively, ceteris paribus, higher capacity cost in the future should increase immediate investment (i.e. prior capacity) but reduce future capacity investment thus also real option purchased. In Stage 3, conditional on the observed demand shifter $X$ (and thus the actual observed demand), and the total available capacity $K_F + K$, the airport sets its airport charge $w$ to maximize its profit or total social welfare. Finally, in Stage 4, airlines compete in Cournot to maximize their own profits and equilibrium outcomes from airline competition determine traffic volume at the airport. The multi-stage game is summarized as follows:

- Stage 1: The airport decides its prior capacity $K_F$ and real option $K^r$.
- Stage 2: The airport observes the actual traffic demand pattern $X$ and exercises part of its real option to add extra capacity $K (K \leq K^r)$.
- Stage 3: The airport sets its charge $w$ to maximize its profit or total social welfare.
- Stage 4: The airlines compete in Cournot to maximize their respective profits.

Under the multi-stage game framework we compare the capacity investment decision made by a profit-maximizing airport, with the one made by a welfare-maximizing airport. The comparison is under the assumption that $r^* \leq r/4$, which is likely to hold in practice because the capacity cost is usually much greater than the real option cost. For example, the cost of building an airport is usually much higher than the cost of reserving the land needed to build it on. This assumption does not affect solutions from the model but makes the comparison between a profit-maximizing and a welfare-maximizing airport

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\(^5\) For alternative modeling of concession services, see Czerny (2006, 2013) and Yang and Zhang (2011).
feasible. Moreover, we assume that $x \geq c + r - h$, which ensures that the airport capacity is non-negative for any $X \in (x, \bar{x})$.

This game is solved backward to obtain the sub-game perfect Nash equilibrium. In the last-stage of the sequential game, an airline $i$ chooses output to maximize its profit $\pi_i$ given the available airport capacity $K_f + K$, airport service charge $w$ and observed demand shifter $X$. The profit maximizing problem is:

$$\text{Max}_{q_i} \pi_i |K_f, K, w, X = (X - bQ - c - w)q_i, \text{ s.t. } Q = \sum_{i=1}^{N} q_i \leq K_f + K \ (i = 1 \ldots N). \quad (1)$$

Solving the profit-maximizing problem, we obtain the following results on airport’s derived demand.

**Lemma 1.** Conditional on the available airport capacity $K_f + K$, airport service charge $w$, and observed demand pattern $X$,

(a) if the airport charge is low and travel demand is high, such that $w \leq X - c - \frac{N+1}{N}b(K_f + K)$, the airport capacity will be fully used and its derived demand is

$$Q(K_f, K, w, X) = K_f + K; \quad (2.1)$$

(b) if the airport charge is high and travel demand is low, such that $X - c - \frac{N+1}{N}b(K_f + K) < w < X - c$, the airport capacity will be partially used and its derived demand is

$$Q(K_f, K, w, X) = \frac{(X-c-w)N}{b(N+1)}. \quad (2.2)$$

### 2.2. Capacity and real option choices by a profit-maximizing airport

In Stage 3 of the sequential game, the airport sets its service charge $w$ to maximize its own objective given the available airport capacity $K_f + K$ and airport’s derived demand $Q(K_f, K, w, X)$, which is given in Lemma 1. In this section, we first consider a profit-maximizing airport which solves the following problem:

$$\text{Max}_w \Pi|K_f, K, X = (w + h)Q - r(K_f + K) - r^*K_r. \quad (3)$$

Lemma 2 summarizes the results obtained from solving the maximization problem in Eq. (3).

**Lemma 2.** For a profit-maximizing airport with derived airport demand $Q(K_f, K, w, X)$, available airport capacity $K_f + K$, and observed demand pattern $X$,

(a) if $c - h < X < c + \frac{2(N+1)}{N}b(K_f + K) - h$, the optimal airport charge is

$$w^{opt}|K_f, K, X = \frac{X-c-h}{2}; \quad (4.1)$$

(b) if $X \geq c + \frac{2(N+1)}{N}b(K_f + K) - h$, the optimal airport charge is

$$w^{opt}|K_f, K, X = X - c - \frac{N+1}{N}b(K_f + K). \quad (4.2)$$
Given the available airport capacity $K_f + K_r$, when the observed travel demand $X$ is sufficiently large, such that $X \geq c + \frac{2(N+1)}{N}b(K_f + K_r) - h$, the airport should set a high service charge at which the airport capacity is just fully used. If the travel demand is low, the optimal airport service charge will lead to only partial capacity use.

Moving backward, in stage 2 of the sequential game the airport decides how much of its real option to be exercised. Let $K^*$ denote the optimal amount of the real option to be exercised and $K^{opt}\vert X$ denote the optimal airport capacity conditional on observed demand $X$, we have

$$K^{opt}\vert X = \frac{N}{2(N+1)b}(X - c - r + h). \quad (5)$$

With reference to the solution in equation (5), we can solve how much of the real option should be exercised in the following scenarios.

**Scenario I.** If the airport’s prior capacity is no less than the optimal capacity, such that $K^{opt}\vert X \leq K_f$ or $X \leq c + r - h + \frac{2(N+1)}{N}bK_f$, there is no need to exercise real option. Therefore, no extra capacity will be invested in, or $K^* = 0$. The airport’s profit can be solved as the following.

a. If $X < c - h + \frac{2(N+1)}{N}bK_f$, a profit-maximizing airport will set its service charge at $w^* = \frac{X - c - h}{2}$, leading to the derived demand $Q^* = \frac{(X - c - w^*)N}{b(N+1)}$. The airport’s profit is

$$\Pi_{1a}\vert K_f, K_r, X = \frac{N(X - c + h)^2}{4b(N+1)} - rK_f - r^*K_r. \quad (6.1)$$

b. If $c - h + \frac{2(N+1)}{N}bK_f \leq X \leq c + r - h + \frac{2(N+1)}{N}bK_f$, a profit-maximizing airport will set its service charge at $w^* = X - c - \frac{N+1}{N}bK_f$, leading to the derived demand $Q^* = K_f$. The airport’s profit is

$$\Pi_{1b}\vert K_f, K_r, X = (X - c - r + h - \frac{N+1}{N}b)K_f - r^*K_r. \quad (6.2)$$

**Scenario II.** If $K_f < K^{opt}\vert X < K_f + K_r$, it is implied that $c + r - h + \frac{2(N+1)}{N}bK_f < X < c + r - h + \frac{2(N+1)}{N}b(K_f + K_r)$. The airport will exercise part of real option. The final available capacity will be $K^{opt}\vert X$ and the invested extra capacity is $K^* = K^{opt}\vert X - K_f$. The airport will set its service charge at $w^* = \frac{1}{2}(X - c + r - h)$, leading to the derived demand $Q^* = K^{opt}\vert X$. The airport’s profit is

$$\Pi_{II}\vert K_f, K_r, X = \frac{N}{4(N+1)b}(X - c - r + h)^2 - r^*K_r. \quad (7)$$

**Scenario III.** If $K^{opt}\vert X \geq K_f + K_r$, it is implied that $X \geq c + r - h + \frac{2(N+1)}{N}b(K_f + K_r)$. The airport will then exercise all of its real option. The invested extra capacity is $K^* = K_r$. The airport will set its service charge at $w^* = X - c - \frac{N+1}{N}b(K_f + K_r)$, leading to the derived demand $Q^* = K_f + K_r$. The airport’s profit is

$$\Pi_{III}\vert K_f, K_r, X = [X - c - r + h - \frac{N+1}{N}(K_f + K_r)](K_f + K_r) - r^*K_r. \quad (8)$$

Some of the relevant results on airport capacity and real option use are summarized in Proposition 1.
Proposition 1. A profit-maximizing airport with observed travel demand $X$, invested prior capacity $K_f$ and real option $K_r$ will exercise the following quantity of its real option.

(a) If $X \leq c + r - h + \frac{2(N+1)}{N} bK_f$, the airport will not exercise its real option. No extra capacity will be invested in, or $K^* = 0$.

(b) If $c + r - h + \frac{2(N+1)}{N} bK_f < X < c + r - h + \frac{2(N+1)}{N} b(K_f + K_r)$, the airport will exercise part of its real option. The extra capacity invested in is

$$K^* = \frac{N}{2(N+1)b} (X - c - r + h) - K_f.$$  \hspace{1cm} (9.1)

(c) If $X \geq c + r - h + \frac{2(N+1)}{N} b(K_f + K_r)$, the airport will exercise all of its real option. The extra capacity invested in is

$$K^* = K_r.$$  \hspace{1cm} (9.2)

The intuition of Proposition 1 is as follows: when the observed travel demand $X$ is low and the prior capacity $K_f$ is high, such that $X \leq c + r - h + \frac{2(N+1)}{N} bK_f$, there is no need to exercise the real option to add more capacity. If the observed travel demand $X$ is high but the prior capacity $K_f$ and real option $K_r$ are low, such that $X \geq c + r - h + \frac{2(N+1)}{N} b(K_f + K_r)$, all of the real option will be used and $K^* = K_r$. In the intermediate case when $c + r - h + \frac{2(N+1)}{N} bK_f < X < c + r - h + \frac{2(N+1)}{N} b(K_f + K_r)$, part of the real option will be exercised to give an optimal final available capacity.

In the final step of the backward induction, the airport chooses its prior capacity $K_f$ and real option $K_r$ to maximize its expected profit $\Pi$. The airport’s profit-maximizing problem is formally:

$$\max_{K_f, K_r} E\Pi = E[ (w^* + h)Q^* - r(K_f + K^*) - r^*K_r ],$$  \hspace{1cm} (10)

where $w^*$, $Q^*$, and $K^*$ are the optimal airport charge, airport derived demand, and optimal extra capacity invested in Stage 2, respectively. As $X \in (\bar{x}, \check{x})$, the prior capacity $K_f$ must be in the range $[K_f^{opt}, \bar{x}]$ and the real option $K_r$ must be in the range $[0, K_r^{opt}]$. It can be shown that the airport’s expected profits have different specifications for Case I. when $r < \check{x} - \bar{x}$ and Case II. when $\check{x} \geq \bar{x} - r$. By solving the maximization problem as specified in Eq. (10) for these two cases, following proposition 2 can be obtained. A sketched proof is provided in Appendix A.

Proposition 2. For a profit-maximizing airport, the optimal prior capacity and real option are as follows.

1) If $r < \frac{\check{x} - x}{2}$ and $r^* \leq \frac{r^2}{2(\check{x} - \bar{x})}$, the optimal prior capacity and real option are

$$K_f^{opt} = \frac{N}{2b(N+1)} (\check{x} - c + h - r + \sqrt{(\check{x} - \bar{x})2r^*}),$$  \hspace{1cm} (11.1)

$$K_r^{opt} = \frac{N}{2b(N+1)} (\check{x} - \bar{x} - 2\sqrt{(\check{x} - \bar{x})2r^*}).$$  \hspace{1cm} (11.2)

2) If $r < \frac{\check{x} - x}{2}$ and $\frac{r^2}{2(\check{x} - \bar{x})} < r^*$, the optimal prior capacity and real option are
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Proposition 2 suggests that a profit-maximizing airport’s optimal prior capacity and real option are influenced by many factors, including the capital cost \( r \), real option cost \( r^* \), and demand uncertainty as measured by \( \bar{x} - \tilde{x} \). If the cost of the real option is sufficiently low, such that \( r^* \leq \frac{\bar{x} - \tilde{x}}{\theta} \), the airport will always invest in some real option. Otherwise, whether real option will be acquired depends on the relative size of the capital cost \( r \) and the variability \( \bar{x} - \tilde{x} \) of future demand. If both the capital cost and real option cost are relatively high but the demand uncertainty is low, such that \( r \geq \frac{\bar{x} - \tilde{x}}{2} \) and \( r^* \geq \frac{\bar{x} - \tilde{x}}{8} \), no real option will be invested.

3. Welfare-maximizing airport

In this section we model the prior capacity and real option choices of an airport that aims to maximize expected social welfare. We use the welfare-maximizing case as the benchmark to be compared with the profit-maximizing case.

The social welfare \( SW \) is the sum of the consumer surplus, airlines’ and airport’s profits as follows:

\[
SW = \left[ \int_0^Q P(\varphi, X) d\varphi - PQ + vQ \right] + \left[ \sum_i (P - c - w) q_i \right] + \left[ (w + h) Q - r(K_f + K) - r^*K_r \right]
\]

\[
= -\frac{b}{2} Q^2 + (X + h + v - c) - r(K_f + K) - r^*K_r,
\]

where \( Q \) is the derived demand of the airport solved in Lemma 1 and \( v \) denotes the consumer surplus derived from the consumption of commercial services per passenger.

Given solutions for Stage 4 as summarized in Lemma 1, in Stage 3 the airport sets its service charge \( w \) to maximize expected welfare conditional on its available capacity \( K_f + K \). We have the following results.
Lemma 3. For a welfare-maximizing airport, with derived airport demand $Q(K_f, K, w, X)$, available airport capacity $K_f + K$, and observed travel demand pattern $X$:

(a) if $c - h - v < X < c + b(K_f + K) - h - v$, the optimal airport charge is

$$w^{opt}|K_f, K, X = -\frac{X - c}{N} + \frac{(N+1)(h+v)}{N},$$  \hspace{1cm} (16.1)

(b) if $X \geq c + b(K_f + K) - h - v$, the optimal airport charge is

$$w^{opt}|K_f, K, X = X - c - \frac{N+1}{N}b(K_f + K).$$  \hspace{1cm} (16.2)

That is, given the available airport capacity $K_f + K$, when the observed travel demand pattern $X$ is sufficiently large such that $X \geq c + b(K_f + K) - h - v$, the airport sets its service charge at $X - c - \frac{N+1}{N}b(K_f + K)$. The airport capacity will then be used fully. Any service charge that is not larger than this value will induce full capacity use and lead to maximum welfare. If the travel demand is not very large, the optimal airport charge will lead to partial capacity use.

Now consider Stage 2, in which the airport decides how much of its real option to be exercised. Let $K^*$ be the optimal quantity of the real option to be exercised and $K^{opt}|X$ be the optimal airport capacity conditional on observed demand $X$, we have

$$K^{opt}|X = \frac{X - h + v - c - r}{b}. \hspace{1cm} (17)$$

With reference to the solution in equation (17), we can solve how much of the real option $K^*$ should be exercised in the following scenarios.

Scenario I. If the airport’s prior capacity is no less than the optimal capacity, such that $K^{opt}|X \leq K_f$ or $X \leq c - h - v + bK_f + r$, there is no need to exercise the real option such that $K^* = 0$. The social welfare in this scenario is then

a. If $X < c - h - v + bK_f$, a welfare-maximizing airport will set its service charge at $w^* = -\frac{X - c}{N} + \frac{(N+1)(h+v)}{N}$, leading to the derived demand $Q^* = \frac{(N+1)(c-w)}{b(N+1)}$. The corresponding welfare is

$$SW_{la}|K_f, K_r, X = \frac{x^2 + 2xh + c^2 + v^2 + hv - 2cX + 2hX - 2chX - 2cv}{2b} - rK_f - r^*K_r. \hspace{1cm} (18.1)$$

b. If $c - h - v + bK_f \leq X \leq c - h - v + bK_f + r$, a welfare-maximizing airport will set its service charge at $w^* = X - c - \frac{N+1}{N}bK_f$, leading to the derived demand $Q^* = K_f$. The corresponding welfare is

$$SW_{lb}|K_f, K_r, X = -\frac{b}{2}K_f^2 + (X + h + v - c - r)K_f - r^*K_r. \hspace{1cm} (18.2)$$

Scenario II. If $K_f < K^{opt}|X < K_f + K_r$, it is implied that $c - h - v + bK_f + r < X < c - h - v + b(K_f + K_r) + r$. The airport will exercise some of its real option, thus that the final available capacity is $K^{opt}|X$. The extra capacity to be invested in is $K^* = K^{opt}|X - K_f$. The airport will set its service
charge at $w^* = -\frac{X-c}{N} - \frac{(N+1)(h+v-r)}{N}$, leading to the derived demand $Q^* = K^{opt}|X$. The corresponding welfare is

$$SW_{II}|K_f, K_r, X = \frac{1}{2b} (X - c - r + h + v)^2 - r^* K_r.$$  

(19)

Scenario III. If $K^{opt}|X \geq K_f + K_r$, it is implied that $X \geq c - h - v + b(K_f + K_r) + r$. The airport will exercise all of its real option, thus that the extra capacity is $K^* = K_r$. The airport will set its charge at $w^* = X - c - \frac{N+1}{N} b(K_f + K_r)$ or any lower price, leading to the derived demand $Q^* = K_f + K_r$. The corresponding welfare is

$$SW_{III}|K_f, K_r, X = -\frac{b}{2} (K_f + K_r)^2 + (X + h + v - c - r)(K_f + K_r) - r^* K_r.$$  

(20)

Relevant results on airport capacity and real option use are summarized in Proposition 3.

**Proposition 3.** A welfare-maximizing airport with observed travel demand pattern $X$, invested prior capacity $K_f$, and real option $K_r$ will exercise the following portion of its real option.

(a) If $X \leq c - h - v + b K_f + r$, the airport will not exercise its real option such that $K^* = 0$.

(b) If $c - h - v + b K_f + r < X < c - h - v + b(K_f + K_r) + r$, the airport will exercise some of its real option. The extra capacity invested in is

$$K^* = \frac{x+h+v-c-r}{b} - K_f.$$  

(21.1)

(c) If $X \geq c - h - v + b(K_f + K_r) + r$, the airport will exercise all of its real option. The extra capacity invested is

$$K^* = K_r.$$  

(21.2)

Now consider Stage 1, in which the airport chooses its prior capacity $K_f$ and real option $K_r$ to maximize expected social welfare $ESW$. Its objective function is

$$\max_{K_f, K_r} ESW = E\left[-\frac{b}{2} Q^*^2 + (X + h + v - c)Q^* - r(K_f + K^*) - r^* K_r\right].$$  

(22)

where $w^*$, $Q^*$, and $K^*$ are the optimal airport charge, airport derived demand, and optimal extra capacity invested in Stage 2, respectively.

Using derivations similar to those used in Section 2, the optimal prior capacity and real option to be invested in Stage 1 can be solved. Proposition 4 summarizes the results. A sketched proof is provided in the Appendix B.

**Proposition 4.** For a welfare-maximizing airport, the optimal prior capacity and real option are as follows.

1) If $r < \frac{\bar{x} - \bar{x}}{2}$ and $r^* \leq \frac{r^2}{2(\bar{x} - \bar{x})}$, the optimal prior capacity and real option are

$$K_f^{opt} = (\bar{x} - c + h + v - r + \sqrt{2r^*(\bar{x} - \bar{x})})/b,$$  

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\[ K_r^{\text{opt}} = (\bar{x} - \bar{x} - 2 \sqrt{2r^*(\bar{x} - \bar{x})})/b. \]  

2) If \( r < \frac{\bar{x} - \bar{x}}{2} \) and \( \frac{r^2}{2(\bar{x} - \bar{x})} < r^* \leq \frac{r}{4} \), the optimal prior capacity and real option are:

\[ K_f^{\text{opt}} = (\bar{x} - c + h + v - r + \frac{r^*}{r}(\bar{x} - \bar{x}))/b. \] (24.1)

\[ K_r^{\text{opt}} = (\bar{x} - \bar{x} - \frac{r}{2} + \frac{r^*}{r}(\bar{x} - \bar{x}) - \sqrt{2r^*(\bar{x} - \bar{x})})/b. \] (24.2)

3) If \( r \geq \frac{\bar{x} - \bar{x}}{2} \) and \( r^* \leq \frac{\bar{x} - \bar{x}}{8} \), the optimal prior capacity and real option are:

\[ K_f^{\text{opt}} = (\bar{x} - c + h + v - r + \sqrt{2r^*(\bar{x} - \bar{x})})/b. \] (25.1)

\[ K_r^{\text{opt}} = (\bar{x} - \bar{x} - 2 \sqrt{2r^*(\bar{x} - \bar{x})})/b. \] (25.2)

4) If \( r \geq \frac{\bar{x} - \bar{x}}{2} \) and \( \frac{\bar{x} - \bar{x}}{8} \leq r^* \leq \frac{r}{4} \), the optimal prior capacity and real option are:

\[ K_f^{\text{opt}} = (\frac{\bar{x} + \bar{x}}{2} - c + h + v - r)/b. \] (26.1)

\[ K_r^{\text{opt}} = 0. \] (26.2)

The intuition for Proposition 4 is similar to that for the case of a profit-maximizing airport. Detailed discussions are provided in the following section.

4. Comparison of results and discussion

In previous sections, the optimal prior capacity and real option have been solved for both profit-maximizing and welfare-maximizing airports. As expected, prior capacity and real option choices are affected by many factors, including demand uncertainty, capacity and real option costs, airline competition, the airport's objectives (which are usually determined by its ownership form), and possibly commercial services. We now compare the results in the cases considered to identify the effects of the various factors.

4.1. Comparison of a profit-maximizing and welfare-maximizing airport

As reported in Propositions 2 and 4, the optimal prior capacity and real option vary across the different regions defined by the combinations of values of \( r \) and \( r^* \); in quite a few cases optimal solutions are obtained at the borders of these regions (i.e. corner solutions). As such, comparison between profit-maximizing and welfare-maximizing airports can only be made conditional on the regions of the values of \( r \) and \( r^* \). We summarize the general conclusions from the comparison as follows.

- Compared with the benchmark of a welfare-maximizing airport, a profit-maximizing airport always chooses a smaller prior capacity and a smaller real option. However, the under-investment issue can be alleviated by competition in the airline market; when number of airlines increases, a profit-maximizing airport increases both prior capacity and real options.

- The optimal prior capacity of a welfare-maximizing airport always increases in \( h \) and \( v \), whereas its real option choice is always independent of \( h \) and \( v \). The optimal capacity of a profit-maximizing airport increases in \( h \) but not in \( v \), whereas its real option choice is not affected by \( h \) or \( v \). Intuitively,
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a welfare-maximizing airport considers both concession profit and consumer surplus when making its prior capacity choices, whereas a profit-maximizing airport only considers profit. The new finding in this study is that real options are not affected by commercial services at all. An airport uses its real option to avoid downsides and exploit upsides. Therefore, the quantity of the real option chosen is only directly affected by demand variability, and the costs of capacity and real option.

- In general, unless both the capacity cost $r$ and real option cost $r^*$ are high ($r \geq \frac{\bar{x} - \bar{x}}{2}$ and $r^* \geq \frac{\bar{x} - \bar{x}}{b}$ in our model), an airport always invests in a real option. When there is greater demand uncertainty (as measured by $\bar{x} - \bar{x}$), a larger real option will be invested in. In addition, when real option is invested, a higher real option cost $r^*$ leads to a larger prior capacity but smaller real option. A higher capital cost $r$ leads to a smaller prior capacity.

4.2. Comparison with the case without real options

To identify the effects of real option on airport capacity and performance, we now compare the outcomes of airport capacity investments with and without real options. The decision process without real option only involves three stages: In Stage 1, the airport decides its capacity. In Stage 2, conditional on the observed demand shifter $X$, the airport sets its airport charge to maximize its profit or total social welfare. In Stage 3, the airlines compete in Cournot to maximize their respective profits. Using derivations similar to those in the previous sections, we can solve an airport’s optimal capacity when real options are not available. Lemma 4 summarizes the results.

Lemma 4. In the presence of future demand uncertainty, when real options are not available, the optimal capacities of a profit-maximizing airport and a welfare-maximizing airport are as follows.

(a) If $r < \frac{\bar{x} - \bar{x}}{2}$, a profit-maximizing airport and a welfare-maximizing airport will choose the following respective capacities:

$$R_{pro}^{opt} = \frac{N}{2b(N+1)} [\bar{x} - c + h - \sqrt{2r(\bar{x} - \bar{x})}], \quad (27.1)$$

$$R_{wel}^{opt} = \frac{1}{b} \left( \bar{x} - c + h + v - \sqrt{2r(\bar{x} - \bar{x})} \right). \quad (27.2)$$

(b) If $r \geq \frac{\bar{x} - \bar{x}}{2}$, a profit-maximizing airport and a welfare-maximizing airport will choose the following respective capacities:

$$R_{pro}^{opt} = \frac{N}{2b(N+1)} [\frac{\bar{x} - \bar{x}}{2} - c + h - r], \quad (28.1)$$

$$R_{wel}^{opt} = \frac{1}{b} \left( \frac{\bar{x} - \bar{x}}{2} - c + h + v - r \right). \quad (28.2)$$

The profit and welfare corresponding to the optimal capacities solved in Lemma 4 allow us to compare the outcomes with and without real options. For example, when $r^*$ is sufficiently small (in the sense that $r^* \leq \frac{r^2}{2(\bar{x} - \bar{x})}$), the gain in expected social welfare from using real option, which is denoted as $IEW_\leq$, can be derived as

$$IEW_\leq = \frac{1}{6b} \left( 3r^2 + 3d(r - 2r^*) - 4\sqrt{2d} \left( \frac{3}{r^2} - 2r^* \right) \right) \quad (29.1)$$

Note in this study it is assumed that $r^* \leq r/4$. If this assumption is relaxed, with very high cost of real option the airport would not invest in an option even if capital cost $r$ is low. We are grateful to Robin Lindsey for pointing out this to us.
When $r^*$ is relatively large (in the sense that $\frac{r^2}{2(x-x)} < r^* \leq \frac{r}{4}$), the gain in expected social welfare from using real option, which is denoted as $IEW_>$, can be derived as

$$IEW_> = \frac{1}{24\sigma d} \left( \frac{12r^3 d + 12r^2 d^2 + 12r^2 d^2 + 12r^* r^2 d - 24r^* r d^2}{-r^4 - 16\sqrt{2} r^3 (r^2 - r^*)} \right).$$

(29.2)

where $d = \bar{x} - x$. It can further be shown that $\frac{\partial}{\partial d} IEW_ > > 0$, $\frac{\partial}{\partial d} IEW_< > 0$, $IEW_ < > 0$ and $IEW_ > > 0$. Similar results are obtained when we compare airport profits between the cases with and without real options. We now summarize the main findings from the comparison between cases with and without real options as follows.

- Unless both the capacity cost $r$ and real option cost $r^*$ are high ($r \geq \frac{x-x}{2}$ and $r^* \geq \frac{x-x}{8}$ in our model), an airport always invests in a real option to maximize its expected profit or social welfare. Moreover, the sum of the prior capacity and real option is larger than the optimal capacity when a real option is unavailable, that is, $K_f^opt < \overline{R}$ and $K_f^opt + K_r^opt < \overline{R}$ for both profit-maximizing and welfare-maximizing airports.

- Real options are beneficial to both profit-maximizing and welfare-maximizing airports. The welfare or profit gain from using real options increases with the demand uncertainty. This probably explains why airports such as Seoul Incheon, Taipei Taoyuan, and the second airport in Sydney (at Badgerys creek) have carried out large-scale land-banking for extended periods.

4.3. Implications of alternative distribution of demand uncertainty

In our analysis, the demand shifter $X$ is assumed to follow a uniform distribution. Although such a specification has been used and discussed in previous investigations including the Transportation Research Board (TRB 2012), Xiao et al. (2013) and Xiao et al. (2015a), few studies have examined the possible limitation of using a particular distribution. Most distributions used in industry analysis as reviewed by the Transportation Research Board (TRB 2012) are symmetric. To examine the reliability of our modeling results, a binominal distribution specification of the demand shifter is tested for its asymmetry and mathematical tractability. Note all analytical results obtained for Stages 2-4 of the four-stage game continue to hold, since they are based on observed demand $X$. Therefore, we will focus on the analysis of Stage 1.

We consider the case when the demand shifter $X$ follows a binominal distribution, where $X$ takes the value of $x$ with probability $p$ and takes the value of $\bar{x}$ with probability $1-p$. For a profit-maximizing airport, its objective function in Stage 1 is specified as

$$Max_{K_f,K_r} E \Pi = E[(w^* + h)Q^* - r(K_f + K_r^*) - r^* K_f],$$

(30)

where $w^*, Q^*$ and $K^*$ are the optimal airport charge, derived demand and extra capacity invested (i.e. real option utilized) in Stage 2, respectively. As discussed earlier, it is assumed that $x \geq c + r - h$, thus that $K^opt |X > 0$ for both $X = \bar{x}$ and $X = x$. With similar derivation process as reported in Section 3, following proposition 5 can be obtained. A sketched proof is provided in Appendix C.

Proposition 5. For a profit-maximizing airport, its optimal prior capacity and real option are as follows.

1) If $r^* \leq \min(p(1-p)(\bar{x} - x), rp)$, the optimal prior capacity and real option are

$$K_f^{opt} = \frac{N}{2(N+1)b} (x - c + h - r + \frac{r^*}{p})$$

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\[ K_r^{opt} = \frac{N}{2(N+1)b} (\bar{x} - x - \frac{r^*}{p(1-p)}) \]  

(31.2)

2) If \( p(1-p)(\bar{x} - x) \leq r^* < r \) and \( r > (1-p)(\bar{x} - x) \), the optimal prior capacity and real option are

\[ K_f^{opt} = \frac{N}{2(N+1)b} \left( (1-p)\bar{x} + px - c + h - r \right) \]

\[ K_r^{opt} = 0 \]  

(32.1)

(32.2)

3) If \( rp \leq r^* < r \) and \( r \leq (1-p)(\bar{x} - x) \), the optimal prior capacity and real option are

\[ K_f^{opt} = \frac{N}{2(N+1)b} (\bar{x} - c + h - \frac{r}{1-p}) \]

\[ K_r^{opt} = 0 \]  

(33.1)

(33.2)

The objective function of a welfare-maximizing airport can be specified similarly as follows.

\[ \text{Max}_{K_f, K_r} E SW = E \left[ -\frac{b}{2} Q^*^2 + (X + h + v - c)Q^* - r(K_f + K^*) - r^*K_r \right] \]  

(34)

It can be shown as summarized in Appendix C that the following proposition 6 can be obtained.

**Proposition 6.** For a welfare-maximizing airport, its optimal prior capacity and real option are as follows.

1) If \( r^* \leq \min\{p(1-p)(\bar{x} - x), rp\} \), the optimal prior capacity and real option are

\[ K_f^{opt} = \frac{1}{b} \left( x - c + h + v + \frac{r^*}{p} \right) \]

\[ K_r^{opt} = \frac{1}{b} (\bar{x} - x - \frac{r^*}{p(1-p)}) \]  

(35.1)

(35.2)

2) If \( p(1-p)(\bar{x} - x) \leq r^* < r \) and \( r > (1-p)(\bar{x} - x) \), the optimal prior capacity and real option are

\[ K_f^{opt} = \frac{1}{b} \left( (1-p)\bar{x} + px - c + h + v - r \right) \]

\[ K_r^{opt} = 0 \]  

(36.1)

(36.2)

3) If \( rp \leq r^* < r \) and \( r \leq (1-p)(\bar{x} - x) \), the optimal prior capacity and real option are

\[ K_f^{opt} = \frac{1}{b} \left( \bar{x} - c + h + v - \frac{r}{1-p} \right) \]

\[ K_r^{opt} = 0 \]  

(37.1)

(37.2)

With Propositions 2, 4, 5 and 6, it is clear that virtually all conclusions as summarized in Section 4.1 remain valid in the case of binominal distribution. The only notable difference is that in the case of binominal distribution, if real option is very costly, then even if capital cost is not very high (i.e., \( rp \leq r^* < r \) and \( r \leq (1-p)(\bar{x} - x) \) in our model), no real option shall be acquired. With reference to Lemma 4, Propositions 5 and 6, it is clear that the conclusions summarized in Section 4.2 continue to hold qualitatively. In summary, whereas different specifications of the demand shifter do change analytical solutions, the general conclusions appear to be fairly robust. Of course, such results are obtained with the rest of the model specifications remain unchanged. Further studies are needed to validate these conclusions in extended models.
4.4. Results interpretation and implications to the aviation industry

In general, our analytical results suggest that real option can be very valuable to the airport industry, where substantial demand uncertainty has been identified. Maldonaldo (1990) found that 5-year forecasts of airport traffic could deviate from the actual volume by -36% to 96%, whereas 15-year forecasts could deviate by -34% to 210%. De Neufville and Barber (1991) analyzed the traffic volatility for the 38 largest US airports in the 1968-1988 period and concluded that traffic volatility clearly increased after the 1978 deregulation. De Neufville and Odoni (2003) studied airport forecasts errors in the United States, Japan, and Australia, and found significant biases even in aggregate forecasts at the national level. Although our study does not focus on calculating the monetary values of real options, such benefits are likely to be substantial. If Taipei and Sydney had not conducted land-banking, it is likely that alternative sites will have to be sought for new airports. This will not only be costly, but may be infeasible due to constraints such as air space, geographic conditions, community development etc.

With significant benefits at stake, it is puzzling why policy guidance and notable examples of real option application have not been many. This may be ascribed to various issues including higher initial investments involved, limited government regulation and intervention, and complications in practical decisions due to additional dynamic factors.

- Despite higher airport profit and social welfare expected, the acquisition of real option involves higher initial investment. Since airport projects tend to be large and lumpy, short term financial constraints can override long term benefits brought by real options. Cooperation with airlines may be a solution. For example, airlines may serve as the guarantors of airport bonds to reduce interest costs (Fu et al. 2011). Although most studies on airport-airline vertical arrangements found gains in airport profits and social welfare (Fu and Zhang 2010, Zhang et al. 2010, Barbot 2011, D’Alfonso and Nastasi 2012, Barbot et al. 2013, Yang et al. 2015), a dominant airline may strategically influence airport capacity to achieve competitive advantage over rivalry airlines (Xiao et al. 2015b). Since an increasing number of airports are required to be free from government support, even worthy projects may not be implemented timely.

- Airport capacity is one core issue in long term planning but it is not routinely reviewed by regulators. Even in master planning for expansion projects, much regulatory attention is devoted to environmental and finance issues rather than capacity optimality. Our study suggests that a profit-maximizing airport chooses a smaller prior capacity and real option than a welfare-maximizing airport. The optimal choice of real option needs a good understanding of future demand (both the trend/mean and variability). Policy-makers often lack the expertise and information to regulate the choices of capacity and real option, especially for private airports favoring under-investment.

- Additional complicating factors are present in practical decision making. Our study suggests that airline competition tends to promote investment in both prior capacity and real option. However, the number of airlines may be endogenous and dependent on both airline competition and airport

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7 Some analytical solutions are defined as in Eqs.(29.1) and (29.2). A numerical example can be found in Smit (2003).
8 Alternative sites for Sydney’s second airport have been evaluated by various studies and organizations. It is generally agreed that the current site at Badgerys Creek, which has been reserved for decades, is the best choice overall when multiple factors are considered.
9 Many airports monitor congestion and delay performances continuously. However, instead of designing the right capacity, the purpose is generally to improve operation efficiency, or to allocate existing capacity (i.e. airport slots) more efficiently (Verhoef, 2010, Fukui, 2010, Li et al., 2010, Czerny, 2010, European Commission, 2011a, b, Shen et al. 2015). Service quality at many airports in UK and Australia are regularly monitored, and a few measures are linked to facility availability (e.g. fixed electrical ground power serviceability, jetty serviceability, departure lounge seat availability, flight information - for more discussions see, for example, Civil Aviation Authority 2009, Yang and Fu 2015). However, these measures do not measure the core (runway) capacity which is the major constraint of airport throughput (De Neufville and Odoni, 2003).
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As discussed above airport-airline vertical arrangements are increasingly used which could have mixed implications to airline competition and airport operations. Airport pricing and operation may affect low cost carriers and full service airlines differently (Fu et al. 2006, Oum and Fu 2007). These two types of carriers tend to have different requirements on airport facility and operation, and bring varying revenue streams to airports (Dresner et al. et al. 1996, Windle and Dresner 1999, Gillen and Lall 2004, Barrett 2004). All these factors might complicate the choices of airport capacity and real option, and call for more comprehensive analysis on this important issue.

For practical decisions related to regulatory policy and business strategy, additional factors may need to be considered in an extended model. Two notable areas for future research is the consideration of investment timing and airport competition. In our study, it is modeled that demand shifter is observed after prior capacity is invested. In practice, better information of demand can be obtained over time but the true growth pattern may never be perfectly identified. Therefore, it will be meaningful to consider investment timing and information accumulation over time (see for example, Bernanke, 1983). Early investment leads to a timely supply of capacity in the case of high demand. However, a better forecast of demand growth pattern can be obtained with more observations of actual traffic volumes. Such a trade-off calls for the identification of the optimal investment timing. In certain cases, there is a need to consider investment timing and competition simultaneously. The industrial organization literature suggests that firms need to balance two countervailing considerations: on the one hand, timely investment may bring first-mover advantage since strategic commitment can influence competitors’ actions. On the other hand, postponing costly investments or acquiring flexibility can be valuable during times of market uncertainty (Ferreira et al. 2009). Therefore, it is useful to consider an integrated model of real option and competition, an approach referred as “option game” (Smit and Ankum, 1993, Balduresson, 1998, Smit and Trigeorgis, 2004, Ferreira et al. et al. 2009, Chevalier-Roignant et al. 2011). In our model airline competition influences traffic volume and thus the airport’s choices of capacity and real option. Such a vertical relationship between the airport and its airlines are however different from horizontal airport rivalry. It will be meaningful to consider airport competition and real option in an integrated option game model. Many metropolitan areas already have or are planning secondary airports. Airports in the same region may be owned/managed by the same authority (e.g. New York, Tokyo, Shanghai) or competing/separate decision-makers (e.g. London, Washington-Baltimore area, San Francisco area, the Pearl River delta area which includes five airports in mainland China, Macau and Hong Kong). Extending our model to the case of multi-airport systems is of great theoretical/practical importance.

Complications in practical decisions are not unique to the airport industry. Slade (2001) noted that although options are routinely considered by financial analysts, most practitioners in the mining industry still rely on traditional discounted-cash flow or net-present value methods which are only appropriate for valuing safe assets. She argued that lack of good data was the most important reason for such a large gap between theory and practice. Mixed findings have been obtained from empirical investigations. Carruth et al. (2000) reviewed eight empirical studies on the relationship between uncertainty and investment. Four of them found a strong negative relationship, whereas the other four found very weak or no-relationship. Henriques and Sadorsky (2011) investigated oil price volatility and strategic investments by large US firms. They argued that a U shaped relationship could be identified which is consistent to the growth options literature. These findings suggest that further investigations are needed for a better understanding of investment under uncertainty in general and real option use in particular.

Although our model incorporates some distinctive features of the airport industry, due to possible complications as discussed above, caution should be exercised when generalized recommendations are made based on one analytical study. We hope our modeling work on real option could lead to extended models and comprehensive investigations, so that appropriate policies and business strategies can be formed. What regulators can do immediately is to encourage airports to explore the opportunities of using real option, and facilitate such applications. Indeed, without government intervention, it would
have been impossible for cities such as Seoul, Taipei and Sydney to reserve the lands needed for future airport expansion. As Slade (2001) pointed out, “routine application of real-option theory to practical investment problems is not likely to become the norm in the near future. Nevertheless, the qualitative insights that the theory offers are undoubtedly valuable.”

5. Summary and conclusions

In this paper we develop a multi-stage game model that identifies the optimal capacity to be invested immediately (i.e. prior capacity) and the size of a real option to be acquired for possible future expansion of airports. The model analyzes airports’ capacity investment, which has large sunk-costs and significant demand uncertainty accounting for airports’ pricing decision, airline competition and alternative airport objectives (profit maximization vs. welfare maximization). Analytical results from the model offer fresh insights on capacity investment at airports and have important policy implications.

In general, our findings indicate that using real options in capacity planning can be a valuable tool for airports to battle uncertainty, especially for capacity investment when demand uncertainty is high. Our findings suggest also market competition is essential for the success of privatizing airports; competition in the airline market promotes capacity investment and the adoption of real options by profit-maximizing airports.

Although we tried to explicitly model the fundamental factors in airport capacity choice with real option, additional issues may need to be examined in extended models. The effects of airport-airline vertical arrangements and dominant airlines’ influence could be substantial in oligopoly aviation markets. Other issues, such as investment timing, airport competition and possible government regulation may have significant implications to the optimal choices of airport capacity and real option. A few simplifying assumptions were imposed in our analysis to ensure mathematical tractability. For example, the assumptions of a linear travel demand can be restrictive. It would be good to test general functional forms in the future, so that the robustness of our conclusions can be validated. Moreover, in our analysis the cost of real option \( r^* \) is modeled as an exogenous parameter. In practice, it is possible that such a cost is negotiated with the supplier of the real option (e.g. a landlord in the case of land banking), or it is dependent on the amount of real option to be acquired (e.g. quantity discount when a larger real option is secured). Explicitly considering these factors may offer fresh insights into the overall economic effects of real options, thus a “global” optimal outcome may be identified for the airport and its suppliers. These extensions and improvements will contribute to the burgeoning literature of real option application in transportation. We hope that our model can lead to more advanced studies on this important topic in analysis of aviation industry.
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Zhang, A., Zhang, Y., 2010. Airport capacity and congestion pricing with both aeronautical and commercial operations. Transportation Research Part B 44(3), 404-413.
Appendix A. Proof of Proposition 2

For ease of notation and derivation, we define the following variables:

\[
X_1 = c - h + \frac{2(N+1)}{N} bK_f, \quad X_2 = c + r - h + \frac{2(N+1)}{N} bK_f,
\]
\[
X_3 = c + r - h + \frac{2(N+1)}{N} b(K_f + K_r), \quad KB_1 = K^{opt}\begin{vmatrix} \times \end{vmatrix} = \frac{N}{2(N+1)b} (\bar{x} - c - r + h), \quad (A1)
\]
\[
KB_2 = K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} = \frac{N}{2(N+1)b} (\bar{x} - c - h), \quad KB_3 = \frac{N}{2(N+1)b} (\bar{x} - c + h).
\]

It is easy to verify that \( KB_1 < KB_2 \leq KB_3 \) if \( r \geq \bar{x} - \bar{x} \), otherwise \( KB_1 < KB_3 < KB_2 \) if \( r < \bar{x} - \bar{x} \).

We now solve the airport’s profit maximization problem in Eq. (10) for Case I, \( r < \bar{x} - \bar{x} \), and Case II, \( r \geq \bar{x} - \bar{x} \).

Case I. \( r < \bar{x} - \bar{x} \)

In this case \( KB_1 < KB_2 < KB_3 \). Therefore, \( X_1 \leq \bar{x} \) when \( K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} \leq K_f \leq KB_3 \) and \( X_1 \geq \bar{x} \) when \( KB_3 \leq K_f \leq K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} \). Thus, when \( K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} \leq K_f \leq KB_3 \), the airport’s expected profit is calculated by

\[
E\Pi_x = \int_{\bar{x}}^{x_2} \Pi_{lb} f(x)dx + \int_{\bar{x}}^{x_3} \Pi_{ll} f(x)dx + \int_{\bar{x}}^{\bar{x}} \Pi_{lil} f(x)dx,
\]

where \( \Pi_{lb}, \Pi_{ll}, \) and \( \Pi_{lil} \) are defined as in Eqs. (6)-(8), respectively.

The first order conditions \( \frac{\partial E\Pi_x}{\partial K_f} = 0 \) and \( \frac{\partial E\Pi_x}{\partial K_r} = 0 \) imply that

\[
K_{f,s} = \frac{N}{2b(N+1)} \left[ \bar{x} - c + h + r + \sqrt{(\bar{x} - \bar{x})2r^*} \right], \quad (A3.1)
\]
\[
K_{f,s} = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - 2 \sqrt{(\bar{x} - \bar{x})2r^*} \right]. \quad (A3.2)
\]

The interior solution conditions \( K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} \leq K_{f,s} \leq KB_3, \quad K_{f,s} \geq 0, \) and \( K_{f,s} + K_{r,s} \leq K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} \) imply that

\[
r^* \leq \min \left\{ \frac{\bar{x} - \bar{x}}{2}, \frac{r^2}{2(\bar{x} - \bar{x})} \right\}. \quad (A4)
\]

When \( KB_3 \leq K_f \leq K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} \), the airport’s expected profit is calculated as follows:

\[
E\Pi_x = \int_{\bar{x}}^{\bar{x}} \Pi_{lb} f(x)dx + \int_{\bar{x}}^{\bar{x}} \Pi_{ll} f(x)dx + \int_{\bar{x}}^{\bar{x}} \Pi_{lil} f(x)dx. \quad (A5)
\]

The first order conditions \( \frac{\partial E\Pi_x}{\partial K_f} = 0 \) and \( \frac{\partial E\Pi_x}{\partial K_r} = 0 \) imply that

\[
K_{f,s} = \frac{N}{2b(N+1)} \left[ \bar{x} - c + h + r - \frac{r}{2} (\bar{x} - \bar{x}) \right], \quad (A6.1)
\]
\[
K_{r,s} = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - \frac{r}{2} - \frac{r^*}{r} (\bar{x} - \bar{x}) - \sqrt{(\bar{x} - \bar{x})2r^*} \right]. \quad (A6.2)
\]

By interior solution conditions \( K_{f,s} \geq KB_3, K_{r,s} \geq 0, \) and \( K_{f,s} + K_{r,s} \leq K^{opt}\begin{vmatrix} \bar{x} \end{vmatrix} \) it can be derived that when \( 0 < r < \frac{\bar{x} - \bar{x}}{2} \), the following condition must hold:

\[
\frac{r^2}{2(\bar{x} - \bar{x})} \leq r^* \leq \left( \sqrt{r - \frac{r}{\sqrt{2(\bar{x} - \bar{x})}}} \right)^2. \quad (A7)
\]
When \( \frac{(\bar{x}-\bar{x})}{2} \leq r \leq (\bar{x} - \bar{x}) \), there is no \( r^* \) that satisfies the interior conditions. However, a boundary solution may exist (i.e., when a solution of prior capacity or real option equals zero or is on the boundary of the feasible intervals). Therefore, we examine two subcases, Case I-I, \( 0 < r < \frac{(\bar{x}-\bar{x})}{2} \), and Case I-II, \( \frac{(\bar{x}-\bar{x})}{2} \leq r \leq (\bar{x} - \bar{x}) \).

**Case I-I.** \( 0 < r < \frac{X-x}{2} \)

In line with the specifications of the expected profit of the airport, in this sub-section two scenarios are examined, (1) when \( K_{opt}\bar{x} \leq K_f \leq KB_3 \), and (2) when \( KB_3 \leq K_f \leq K_{opt}\bar{x} \).

**Scenario I-I(1).** \( K_{opt}\bar{x} \leq K_f \leq KB_3 \)

As \( 0 < r < \frac{X-x}{2} \) in subcase I-I, \( \frac{r^2}{2(X-x)} < \frac{X-x}{8} \) holds. When \( r^* \leq \frac{r^2}{2(X-x)} \), the optimal prior capacity and real option are either interior solutions given by Eqs. (A3.1) and (A3.2), or boundary solutions. When \( r^* > \frac{r^2}{2(X-x)} \), the solutions can only be on the boundary. These solutions are identified as follows.

When \( r^* \leq \frac{r^2}{2(X-x)} \):
- The solution can be interior, which are defined by Eqs. (A3.1) and (A3.2).
- For the boundary solution when \( K_f = 0 \), the stationary point \( K_{fr} = \frac{N}{2b(N+1)} \left( \frac{X-x}{2} - c + h - r \right) \geq KB_3 \). Thus the optimum solutions for \( (K_f, K_r) \) are \( (KB_3, 0) \).
- For the boundary solution when \( K_f = K_{opt}\bar{x} \), the stationary point \( K_{rl} = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - \sqrt{\left( \bar{x} - \bar{x} \right)^2 r^*} \right] < K_{opt}\bar{x} - K_{opt}\bar{x} \), thus the optimum solutions are \( (K_{opt}\bar{x}, K_{rl}) \).
- For the boundary solution when \( K_r = K_{opt}\bar{x} - K_f \), the stationary point \( K_{rt} = \frac{N}{2b(N+1)} \left[ \bar{x} - c + h - r + \sqrt{\left( \bar{x} - \bar{x} \right)^2 r^*} \right] \leq KB_3 \), thus the optimum solutions are \( (K_{rt}, K_{opt}\bar{x} - K_{fr}) \).
- For the boundary solution when \( K_f = KB_3 \), the stationary point \( K_{r} = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - r - \sqrt{\left( \bar{x} - \bar{x} \right)^2 r^*} \right] \). It can be shown that \( 0 < K_{r} < K_{opt}\bar{x} - KB_3 \). The optimum solutions are \( (KB_3, K_{r}) \).

When \( r^* > \frac{r^2}{2(X-x)} \):
- For the boundary solution when \( K_r = 0 \), the stationary point \( K_{fr} = \frac{N}{2b(N+1)} \left( \frac{X-x}{2} - c + h - r \right) > KB_3 \). Thus the optimum solutions are \( (KB_3, 0) \).
- For the boundary solution when \( K_f = K_{opt}\bar{x} \), the stationary point \( K_{rl} = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - \sqrt{\left( \bar{x} - \bar{x} \right)^2 r^*} \right] < K_{opt}\bar{x} - K_{opt}\bar{x} \), thus the optimum solutions are \( (K_{opt}\bar{x}, K_{rl}) \).
- For the boundary solution when \( K_r = K_{opt}\bar{x} - K_f \), the stationary point \( K_{rt} = \frac{N}{2b(N+1)} \left[ \bar{x} - c + h - r + \sqrt{\left( \bar{x} - \bar{x} \right)^2 r^*} \right] > KB_3 \). Thus the optimum solutions are \( (KB_3, K_{opt}\bar{x} - KB_3) \).
For the boundary solution when \( K_f = KB_3 \), the stationary point \( K_{rR}' = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - r - \sqrt{(\bar{x} - \bar{x})2r^*} \right] \). It can be shown that \( 0 < K_{rR}' < K^{opt}[\bar{x} - KB_3] \). Thus the optimum solutions are \( (KB_3, K_{rR}') \).

By comparing the expected profits in the cases solved above, it can be shown that when \( K^{opt}[\bar{x}] \leq K_f \leq KB_3 \):

(i) if \( r^* \leq \frac{r^2}{2(\bar{x} - \bar{x})} \), the optimal prior capacity and real option are defined as in Eqs. (A3.1) and (A3.2), respectively; and

(ii) if \( r^* > \frac{r^2}{2(\bar{x} - \bar{x})} \), the optimal prior capacity and real option are \( (KB_3, K_{rR}') \).

Scenario I-I(2), \( KB_3 \leq K_f \leq K^{opt}[\bar{x}] \)

As \( 0 < r < \frac{\bar{x} - \bar{x}}{2} \) in subcase I-1, we have \( r^* \leq \frac{r}{4} \leq \left( \frac{r - \frac{r}{2(\bar{x} - \bar{x})}}{2(\bar{x} - \bar{x})} \right)^2 \). When \( r^* \geq \frac{r^2}{2(\bar{x} - \bar{x})} \), the optimal prior capacity and real option are either interior solutions given by Eqs. (A6.1) and (A6.2), or boundary solutions. When \( r^* < \frac{r^2}{2(\bar{x} - \bar{x})} \), the solutions can only be on the boundary. These solutions are identified as follows.

When \( r^* \leq \frac{r^2}{2(\bar{x} - \bar{x})} \):

- For the boundary solution when \( K_r = 0 \), the stationary point \( K_{fB} = \frac{N}{2b(N+1)} \left( \bar{x} - c + h - \sqrt{(\bar{x} - \bar{x})2r^*} \right) \in [KB_3, K^{opt}[\bar{x}] \]. Thus the optimum solutions are \( (K_{fB}', 0) \).

- For the boundary solution when \( K_f = KB_3 \), the stationary point \( K_{rL}' = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - r - \sqrt{(\bar{x} - \bar{x})2r^*} \right] \). It can be shown that \( 0 < K_{rL}' < K^{opt}[\bar{x} - KB_3] \). Thus the optimum solutions are \( (KB_3, K_{rL}') \).

- For the boundary solution when \( K_r = K^{opt}[\bar{x} - K_f] \), the stationary point \( K_{fT} = \frac{N}{2b(N+1)} \left[ \bar{x} - c + h - \frac{r}{2} \right] \leq KB_3 \). Thus the optimum solutions are \( (KB_3, K^{opt}[\bar{x} - KB_3]) \).

When \( r^* > \frac{r^2}{2(\bar{x} - \bar{x})} \):

- The solution can be interior, which are defined by Eqs. (A6.1) and (A6.2).

- For the boundary solution when \( K_r = 0 \), the stationary point \( K_{fB} = \frac{N}{2b(N+1)} \left( \bar{x} - c + h - \sqrt{(\bar{x} - \bar{x})2r^*} \right) \in [KB_3, K^{opt}[\bar{x}] \]. Thus the optimum solutions are \( (K_{fB}', 0) \).

- For the boundary solution when \( K_f = KB_3 \), the stationary point \( K_{rL} = \frac{N}{2b(N+1)} \left[ \bar{x} - \bar{x} - r - \sqrt{(\bar{x} - \bar{x})2r^*} \right] \). It can be shown that \( 0 < K_{rL}' < K^{opt}[\bar{x} - KB_3] \). Thus the optimum solutions are \( (KB_3, K_{rL}') \).

- For the boundary solution when \( K_r = K^{opt}[\bar{x} - K_f] \), the stationary point \( K_{fT} = \frac{N}{2b(N+1)} \left[ \bar{x} - c + h - \frac{r}{2} + \frac{r^*}{r} (\bar{x} - \bar{x}) \right] \leq KB_2 \). Thus the optimum solutions are \( (K_{fT}', K^{opt}[\bar{x} - K_{fT}]) \).
By comparing the expected profits in the cases solved above, it can be shown that when $KB_3 \leq K_f \leq K_{opt}|\bar{x}|$,

(i) if $r^* < \frac{r^2}{2(\bar{x}-\bar{x})}$, the optimal prior capacity and real option are $(K_B^3, K_{rl}^3)$; and

(ii) if $r^* \geq \frac{r^2}{2(\bar{x}-\bar{x})}$, the optimal prior capacity and real option are defined as in Eqs. (A6.1) and (A6.2), respectively.

**Case I-II.** $\frac{\bar{x}_3 - \bar{x}}{2} \leq r \leq (\bar{x} - \bar{x})$

In this section, two scenarios are examined: (1) when $K_{opt}|\bar{x}| \leq K_f \leq KB_3$, and (2) when $KB_3 \leq K_f \leq K_{opt}|\bar{x}|$. With derivations similar to those used in Case I-I, it can be shown that if $\frac{\bar{x}_3 - \bar{x}}{2} \leq r \leq (\bar{x} - \bar{x})$, then the following solutions can be obtained.

(i) When $r^* \leq \frac{\bar{x}_3 - \bar{x}}{8}$, the optimal prior capacity and real option are $\left( N \frac{2b(N+1)}{N} \left( \frac{x - c + h - \frac{r}{\sqrt{2(\bar{x}_3 - \bar{x})}} N \frac{2b(N+1)}{N} \left( \frac{x - c + h - \frac{r}{\sqrt{2(\bar{x}_3 - \bar{x})}} \right) \right) \right)$. 

(ii) When $r^* > \frac{\bar{x}_3 - \bar{x}}{8}$, the optimal prior capacity and real option are $\left( N \frac{2b(N+1)}{N} \left( \frac{\bar{x}_3 + \bar{x}}{2} - c + h - \frac{r}{\sqrt{2(\bar{x}_3 - \bar{x})}} \right) \right)$.

**Case II.** $r \geq \bar{x} - \bar{x}$

In this case $KB_1 < KB_2 \leq KB_3$. Therefore, $X_1 \leq \bar{x}$ when $K_{opt}|\bar{x}| \leq K_f \leq K_{opt}|\bar{x}|$. The airport’s expected profit is calculated by

\[
\Pi = \int_{\bar{x}}^{\bar{x}_1} \Pi_{ib}f(x)dx + \int_{\bar{x}_1}^{\bar{x}_2} \Pi_{Ii}f(x)dx + \int_{\bar{x}_2}^{\bar{x}_3} \Pi_{III}f(x)dx,
\]  

(A8)

where $\Pi_{ib}$, $\Pi_{Ii}$, and $\Pi_{III}$ are defined in Eqs. (6)-(8), respectively.

With derivations similar to those used in Case I, the optimal prior capacity and real option when $r > \bar{x} - \bar{x}$ can be obtained.
Appendix B. Proof of Proposition 4

Consider Stage 1, in which the airport chooses its prior capacity $K_f$ and real option $K_r$ to maximize the expected welfare $ESW$, thus its objective function is specified as in Eq. (22). As $X \in (x, \bar{x})$, the prior capacity $K_f$ must be in the range $[K_f^{opt}, \bar{x}, K_f^{opt}][\bar{x}]$, and real option $K_r$ must be in the range $[0, K_r^{opt}][\bar{x} - K_f]$. For ease of notation and derivation, we define some variables as follows:

\[
X_1 = c - h - v + bK_f, \quad X_2 = c - h - v + bK_f + r,
\]
\[
X_3 = c - h - v + b(K_f + K_r) + r, \quad KB_1 = K_f^{opt}[\bar{x}] = (x - c + h + v - r)/b, \quad KB_2 = K_f^{opt}[\bar{x}] = (\bar{x} - c + h + v + r)/b.
\] (B1)

It is easy to verify that $KB_1 < KB_2 \leq KB_3$ if $r \geq \bar{x} - x$, otherwise $KB_1 < KB_3 < KB_2$ if $r < \bar{x} - x$. We solve the welfare maximization problem Eq. (22) for Case I, $r < \bar{x} - x$ and Case II, $r \geq \bar{x} - x$ respectively as follows.

**Case I.** when $r < \bar{x} - x$

In this case $KB_1 < KB_2 \leq KB_3$. It is implied that $X_1 \leq x$ when $K_f^{opt}[\bar{x}] \leq K_f \leq KB_3$, and $X_1 \geq x$ when $KB_3 \leq K_f \leq K_f^{opt}[\bar{x}]$. Thus when $K_f^{opt}[\bar{x}] \leq K_f \leq KB_3$ the expected welfare is calculated as follows, where $SW_{ib}, SW_{il}$ and $SW_{il}$ are defined in Eq.(18)-(20) respectively.

\[
ESW_e = \int_{X_2}^{X_3} SW_{ib} f(x) dx + \int_{X_2}^{X_3} SW_{il} f(x) dx + \int_{X_3}^{\bar{x}} SW_{il} f(x) dx.
\] (B2)

The first order conditions $\frac{\partial ESW_e}{\partial K_f} = 0$ and $\frac{\partial ESW_e}{\partial K_r} = 0$ imply that

\[
K_f^* = (x - c + h + v - r + \sqrt{2r^* (\bar{x} - x)})/b, \quad (B3.1)
\]

\[
K_r^* = (\bar{x} - x - 2\sqrt{2r^* (\bar{x} - x)})/b. \quad (B3.2)
\]

The interior solution conditions $K_f^{opt}[\bar{x}] \leq K_f^* \leq KB_3, K_r^* \geq 0$ and $K_f^* + K_r^* \leq K_f^{opt}[\bar{x}]$ imply that

\[
r^* \leq \min\left\{ \frac{\bar{x} - x}{b}, \frac{r^2}{2(\bar{x} - x)} \right\}. \quad (B4)
\]

When $KB_3 \leq K_f \leq K_f^{opt}[\bar{x}]$ the airport’s expected profit is calculated as follows

\[
ESW_e = \int_{X_2}^{X_1} SW_{ia} f(x) dx + \int_{X_2}^{X_1} SW_{ib} f(x) dx + \int_{X_2}^{X_3} SW_{il} f(x) dx + \int_{X_3}^{\bar{x}} SW_{il} f(x) dx. \quad (B5)
\]

The first order conditions $\frac{\partial ESW_e}{\partial K_f} = 0$ and $\frac{\partial ESW_e}{\partial K_r} = 0$ imply that

\[
K_f^* = (x - c + h + v - r + \frac{r^*}{2} (\bar{x} - x)) / b, \quad (B6.1)
\]

\[
K_r^* = (\bar{x} - x - \frac{r^*}{2} (\bar{x} - x) - \sqrt{2r^* (\bar{x} - x)}) / b. \quad (B6.2)
\]

By interior solution conditions $K_f^* \geq KB_3, K_f^* \geq 0$ and $K_f^* + K_r^* \leq K_f^{opt}[\bar{x}]$ it can be derived that when $0 < r < \frac{(\bar{x} - x)}{2}$, the following condition must hold
the stationary point and Case I-II in subcase I-I, the stationary point holds. When 

When the solutions can only be on the boundary. These solutions are identified as follows:

When \( r^* \leq \frac{r^2}{2(\bar{x} - x)} \):
- The solution can be interior, which are defined by Eqs. (B3.1) and (B3.2).
- For boundary solution when \( K_f = 0 \), the stationary point \( K_f^b = \left( \frac{x + x}{2} - c + h + v - r \right) / b > KB_3 \). Thus the optimum solutions for \((K_f, K_r)\) are \((KB_3, 0)\).
- For boundary solution when \( K_f = K_{opt} | \bar{x} \), the stationary point \( 0 < K_f^* = (\bar{x} - x - \sqrt{2r^*(\bar{x} - x)}) / b < K_{opt} | \bar{x} - K_{opt} | x \), thus the optimum solutions are \((K_{opt} | \bar{x}, K_f^r)\).
- For boundary solution when \( K_f = K_{opt} | \bar{x} - K_f \), the stationary point \( K_f^* = (\bar{x} - c + h + v - r + \sqrt{2r^*(\bar{x} - x)}) / b \leq KB_3 \), thus the optimum solutions are \((K_f^* | \bar{x} - K_{opt} | \bar{x} - K_f^r)\).
- For boundary solution when \( K_f = KB_3 \), the stationary point \( K_f^* = (\bar{x} - x - r - \sqrt{2r^*(\bar{x} - x)}) / b \). It can be shown that \( 0 < K_f^* < K_{opt} | \bar{x} - KB_3 \). Thus the optimum solutions are \((KB_3, K_f^*)\).

When \( r^* > \frac{r^2}{2(\bar{x} - x)} \):
- For boundary solution when \( K_r = 0 \), the stationary point \( K_f^* = (\frac{x + x}{2} - c + h + v - r) / b > KB_3 \). Thus the optimum solutions are \((KB_3, 0)\).
- For boundary solution when \( K_f = K_{opt} | \bar{x} \), the stationary point \( 0 < K_f^* = (\bar{x} - x - \sqrt{2r^*(\bar{x} - x)}) / b < K_{opt} | \bar{x} - K_{opt} | x \), thus the optimum solutions are \((K_{opt} | \bar{x}, K_f^r)\).
- For boundary solution when \( K_f = K_{opt} | \bar{x} - K_f \), the stationary point \( K_f^* = (\bar{x} - c + h + v - r + \sqrt{2r^*(\bar{x} - x)}) / b > KB_3 \), thus the optimum solutions are \((KB_3, K_{opt} | \bar{x} - KB_3)\).
For boundary solution when $K_f = KB_3$, the stationary point $K_{rB}^* = (\bar{x} - c + h + v - \sqrt{2r(\bar{x} - \bar{x})})/b \in [K_B, K^{opt}\bar{x}]$. Thus the optimum solutions are $(K_{rB}^*, 0)$.

For boundary solution when $K_f = KB_3$, the stationary point $K_{rL}^* = (\bar{x} - c + h + v - \sqrt{2r(\bar{x} - \bar{x})})/b \in [K_B, K^{opt}\bar{x}]$. Thus the optimum solutions are $(K_{rL}^*, 0)$.

For boundary solution when $K_f = K^{opt}\bar{x} - K_f$, the stationary point $K_{rT}^* = (\bar{x} - c + h + v - \sqrt{2r(\bar{x} - \bar{x})})/b \in [K_B, K^{opt}\bar{x}]$. Thus the optimum solutions are $(K_{rT}^*, 0)$.

By comparing the expected welfare in cases solved above, it can be shown that when $K^{opt}\bar{x} - K_f \leq KB_3$, we have (i) if $r^* \leq \frac{r^2}{2(\bar{x} - \bar{x})}$, the optimal prior capacity and real option are defined as in Eq. (B3.1) and Eq. (B3.2), respectively. (ii) if $r^* > \frac{r^2}{2(\bar{x} - \bar{x})}$, the optimal prior capacity and real option are $(KB_3, K_{rL}^*)$.

Scenario I-I(2), when $KB_3 \leq K_f \leq K^{opt}\bar{x}$

Because $0 < r < \frac{\bar{x} - \bar{x}}{2}$ in subcase I-1, we have $r^* \leq \frac{r}{4} \leq \left(\sqrt{\frac{r}{2(\bar{x} - \bar{x})}}\right)^2$. When $r^* \geq \frac{r^2}{2(\bar{x} - \bar{x})}$, the optimal prior capacity and real option are either interior solutions given by Eqs. (B6.1) and (B6.2), or boundary solutions. Otherwise when $r^* < \frac{r^2}{2(\bar{x} - \bar{x})}$, the solutions can only be on the boundary. These solutions are identified as follows:

When $r^* \leq \frac{r^2}{2(\bar{x} - \bar{x})}$:

- For boundary solution when $K_r = 0$, the stationary point $K_{rB}^* = (\bar{x} - c + h + v - \sqrt{2r(\bar{x} - \bar{x})})/b \in [K_B, K^{opt}\bar{x}]$. Thus the optimum solutions are $(K_{rB}^*, 0)$.

- For boundary solution when $K_f = KB_3$, the stationary point $K_{rL}^* = (\bar{x} - c + h + v - \sqrt{2r(\bar{x} - \bar{x})})/b \in [K_B, K^{opt}\bar{x}]$. Thus the optimum solutions are $(K_{rL}^*, 0)$.

- For boundary solution when $K_f = K^{opt}\bar{x} - K_f$, the stationary point $K_{rT}^* = (\bar{x} - c + h + v - \sqrt{2r(\bar{x} - \bar{x})})/b \in [K_B, K^{opt}\bar{x}]$. Thus the optimum solutions are $(K_{rT}^*, 0)$.

By comparing the expected welfare in cases solved above, it can be shown that when $KB_3 \leq K_f \leq K^{opt}\bar{x}$, we have (i) if $r^* < \frac{r^2}{2(\bar{x} - \bar{x})}$, the optimal prior capacity and real option are $(KB_3, K_{rL}^*)$. (ii) if
r^* \geq \frac{r^2}{2(\bar{x}-\underline{x})}, the optimal prior capacity and real option are defined as in Eqs. (B6.1) and (B6.2) respectively.

**Case I-II** when \( \frac{\bar{x}-\underline{x}}{2} \leq r \leq (\bar{x} - \underline{x}) \).

In this section, two scenarios are examined: (1) when \( K_{opt} \mid \bar{x} \leq K_f \leq K_B \), and (2) when \( K_B \leq K_f \leq K_{opt} \mid \bar{x} \). With derivations similar to those used in Case I-I, it can be shown that if \( \frac{\bar{x}-\underline{x}}{2} \leq r \leq (\bar{x} - \underline{x}) \), then following solutions can be obtained: (i) when \( r^* \leq \frac{\bar{x}-\underline{x}}{8} \), the optimal prior capacity and real option are \( ((x - c + h + v - r + \sqrt{2r^*(\bar{x} - \underline{x}))}/b, (\bar{x} - \underline{x} - 2\sqrt{2r^*(\bar{x} - \underline{x}))}/b) \), and (ii) when \( r^* > \frac{\bar{x}-\underline{x}}{8} \), the optimal prior capacity and real option are \( ((\frac{\bar{x}_+\underline{x}}{2} - c + h + v - r)/b, 0) \).

**Case II.** when \( r \geq \bar{x} - \underline{x} \)

In this case \( K_B < K_2 \leq K_B \). It is implied that \( X_1 \leq \bar{x} \) when \( K_{opt} \mid \bar{x} \leq K_f \leq K_{opt} \mid \bar{x} \). The expected welfare is calculated as follows, where \( SW_{II} \), \( SW_{I} \), and \( SW_{III} \) are defined in Eqs.(26)-(28) respectively.

\[
ESW = \int_{\underline{x}}^{\bar{x}} SW_{I} f(x) dx + \int_{\underline{x}}^{\bar{x}} SW_{II} f(x) dx + \int_{\underline{x}}^{\bar{x}} SW_{III} f(x) dx \quad (B8)
\]

With derivations similar to those used in Case I, the optimal prior capacity and real option when \( r > \bar{x} - \underline{x} \) can be obtained. Combining the analytical results obtained in Case I and Case II leads to Proposition 4.
Appendix C. Sketched solutions when demand shifter follows a binomial distribution.

This appendix provides sketched solutions for an airport’s optimal choices of prior capacity and real option when the demand shifter $X$ follows a binomial distribution as defined in Section 4.3. Due to space limitation, only the key results are summarized. We will first analyze the case of a profit-maximizing airport, then a brief introduction for the case of a welfare-maximizing airport.

A profit-maximizing airport

For the profit maximization problem as specified in Eq. (30), it is assumed that $x \geq c + r - h$, thus that $K_{opt}^{|X| > 0}$ for both $X = \bar{x}$ and $X = \bar{x}$. Clearly, the airport’s prior capacity $K_f$ is in the range of $[K_{opt}^{|X| > 0}, K_{opt}^{|X| < 0}]$ whereas real option $K_r$ is in the range of $[0, K_{opt}^{|X - K_f|}]$. For ease of notion and derivation, we define some variables as follows:

\[
X_1 = c - h + \frac{2(N+1)}{N} b K_f, \quad X_2 = c + r - h + \frac{2(N+1)}{N} b K_f, \\
X_3 = c + r - h + \frac{2(N+1)}{N} b (K_f + K_r), \quad KB_1 = \frac{N}{2(N+1)b}(\bar{x} - c - r + h) \\
KB_2 = \frac{N}{2(N+1)b}(\bar{x} - c + h), \quad KB_3 = \frac{N}{2(N+1)b}(\bar{x} - c + h)
\]  

(C1)

It can be verified that $KB_1 < KB_2 < KB_3$ if $r \geq \bar{x} - x$, otherwise $KB_1 < KB_3 < KB_2$ if $r < \bar{x} - x$. With reference to $\Pi_{ta}, \Pi_{tb}, \Pi_{tt}$, and $\Pi_{tt^3}$, as defined in Eqs. (6)-(8), respectively, the airport’s expected profit can be specified for the following cases.

Case I: $r < \bar{x} - x$

In this case, $KB_1 < KB_3 < KB_2$. It can be derived that when $KB_1 \leq K_f \leq KB_3$ and $K_f + K_r \leq KB_2$, the conditions $X_1 \leq X_2 \leq X_3$ both hold, and thus the airport’s expected profit is $\Pi_1 = p \Pi_{ta} \Pi_{tb} \Pi_{tt} \Pi_{tt^3}$. When $KB_3 < K_f \leq KB_2$ and $K_f + K_r \leq KB_2$, the conditions $X_1 \leq X_2$ and $X_2 \geq X_3$ both hold, and thus the airport expected profit is $\Pi_2 = p \Pi_{ta} \Pi_{tb} \Pi_{tt} \Pi_{tt^3}$. The airport’s profit function and constraint conditions are all convex, and the following conclusions can be obtained.

Conclusion C1.1 When $KB_1 \leq K_f \leq KB_3$.

- If $r^* \geq (1 - p)(\bar{x} - x - r)$ and $r < (1 - p)(\bar{x} - x)$, the optimal prior capacity is $K_f = \frac{N}{2(N+1)b}(\bar{x} - c + h)$ and the optimal real option is $K_r = 0$.
- If $r^* \geq p(1 - p)(\bar{x} - x)$ and $r > (1 - p)(\bar{x} - x)$, the optimal prior capacity is $K_f = \frac{N}{2(N+1)b}((1 - p)\bar{x} + p\bar{x} - c + h - r)$ and the optimal real option is $K_r = 0$.
- If $r^* \geq rp$ and $r^* \leq (1 - p)(\bar{x} - x - r)$, the optimal prior capacity is $K_f = \frac{N}{2(N+1)b}(\bar{x} - c + h)$ and the optimal real option is $K_r = \frac{N}{2(N+1)b}(\bar{x} - x - r - \frac{r^*}{1 - p})$.
- If $r^* \leq rp$ and $r^* \leq p(1 - p)(\bar{x} - x)$, the optimal prior capacity is $K_f = \frac{N}{2(N+1)b}(\bar{x} - c + h - r + \frac{r^*}{p})$ and the optimal real option is $K_r = \frac{N}{2(N+1)b}(\bar{x} - x - r^*/p(1 - p))$.

Conclusion C1.2 When $KB_3 \leq K_f \leq KB_2$.

- If $r^* \geq (1 - p)(\bar{x} - x - r)$ and $r < (1 - p)(\bar{x} - x)$, the optimal prior capacity is $K_f = \frac{N}{2(N+1)b}(\bar{x} - c + h)$ and the optimal real option is $K_r = 0$. 

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- If $r^* \geq rp$ and $r \leq (1 - p)(\bar{x} - \bar{x})$, the optimal prior capacity is $K_f = \frac{N}{2(N+1)b}(\bar{x} - c + h - \frac{r}{1-p})$, and the optimal real option is $K_r = 0$.
- If $r^* \leq (1 - p)(\bar{x} - \bar{x} - r)$ and $r^* \leq rp$, the optimal prior capacity is $K_f = \frac{N}{2(N+1)b}(\bar{x} - c + h)$ and the optimal real option is $K_r = \frac{N}{2(N+1)b}(\bar{x} - \bar{x} - r - \frac{r^*}{1-p})$.

By comparing the expected profit for different value combinations of $r$ and $r^*$, the following proposition can be obtained.

**Proposition C1.1** When $r < \bar{x} - \bar{x}$,

I. If $r^* \geq p(1 - p)(\bar{x} - \bar{x})$, the optimal prior capacity and real option are $K^\text{opt}_f = \frac{N}{2(N+1)b}\left((1 - p)\bar{x} + p\bar{x} - c + h - r\right)$ and $K^\text{opt}_r = 0$, respectively.

II. If $r^* \leq p(1 - p)(\bar{x} - \bar{x})$ and $r^* \leq rp$, the optimal prior capacity and real option are $K^\text{opt}_f = \frac{N}{2(N+1)b}\left(\bar{x} - c + h - r + \frac{r^*}{p}\right)$ and $K^\text{opt}_r = \frac{N}{2(N+1)b}\left(\bar{x} - \bar{x} - r - \frac{r^*}{p(1-p)}\right)$, respectively.

III. If $r^* \geq rp$ and $r \leq (1 - p)(\bar{x} - \bar{x})$, the optimal prior capacity and real option are $K^\text{opt}_f = \frac{N}{2(N+1)b}\left(\bar{x} - c + h - \frac{r^*}{1-p}\right)$ and $K^\text{opt}_r = 0$, respectively.

Case II: $r \geq \bar{x} - \bar{x}$
In this case, $KB1 < KB2 \leq KB3$. The expected profit is $E\Pi = p\Pi_{\bar{x}=\bar{x}} + (1 - p)\Pi_{\bar{x}=\bar{x}}$. Using a similar approach as in Case I, the following proposition can be obtained

**Proposition C1.2** When $r \geq \bar{x} - \bar{x}$,

I. If $r^* \geq p(1 - p)(\bar{x} - \bar{x})$, the optimal prior capacity and real option are $K^\text{opt}_f = \frac{N}{2(N+1)b}\left((1 - p)\bar{x} + p\bar{x} - c + h - r\right)$ and $K^\text{opt}_r = 0$, respectively.

II. If $r^* < p(1 - p)(\bar{x} - \bar{x})$, the optimal capacity and real option are $K^\text{opt}_f = \frac{N}{2(N+1)b}\left(\bar{x} - c + h - r + \frac{r^*}{p}\right)$ and $K^\text{opt}_r = \frac{N}{2(N+1)b}\left(\bar{x} - \bar{x} - r - \frac{r^*}{p(1-p)}\right)$, respectively.

Combining the analytical results obtained in Proposition C1.1 and C1.2, Proposition 5 in the main text can be reached.

**A welfare-maximizing airport**

For the welfare maximization problem as specified in Eq. (34), the solutions are slightly more complex than the case of a profit-maximization airport. The analytical procedure is nevertheless similar. To save space, we will only present the results without explaining the derivation details.

With welfare functions defined in Eqs. (18)-(20), following conclusions can be obtained

**Conclusion C2.1** When $KB_1 \leq K_f \leq KB_3$,

- If $r^* \geq (1 - p)(\bar{x} - \bar{x} - r)$ and $r < (1 - p)(\bar{x} - \bar{x})$, the optimal prior capacity is $K_f = \frac{1}{b}(\bar{x} - c + h + v)$ and the optimal real option is $K_r = 0$.
- If $r^* \geq p(1 - p)(\bar{x} - \bar{x})$ and $r > (1 - p)(\bar{x} - \bar{x})$, the optimal prior capacity is $K_f = \frac{1}{b}\left((1 - p)\bar{x} + p\bar{x} - c + h + v - r\right)$ and the optimal real option is $K_r = 0$. 
Combining the analytical results obtained in Proposition C2.1 and C2.2, Proposition 6 in the main text can be reached.