“The Estimation of Psychic Returns for Cultural Assets”

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This paper presents procedures for evaluating psychic returns to cultural assets. Measuring the psychic return of art investments is an important issue in cultural economics. We focus on the psychic returns of art relative to equity using British data from 1895 to 2011. However, our arguments are entirely general. We take into account the substantial costs involved in art investment and also discuss the existing estimates of the psychic returns to art in the literature which are typically between 10 per cent to 30 per cent. Applying utility based models and equilibrium based models, we construct new estimates of psychic returns based on plausible portfolio weights and also trace the linkages of psychic returns of art to other markets by an examination of trade flows.

1.1 Introduction

The returns from owning art not only consist of expected price rises but also include psychic returns. Economic research on the financial returns to art has not satisfactorily regarded this aspect which distinguishes the art market from pure financial markets. The broadest definition of psychic returns covers any asset whereby there is a stream of positive or negative benefits conferred by ownership of a non-material kind. Psychic income has also been defined as the level of satisfaction derived from a job rather than the salary earned doing it (QFinance Dictionary, 2009). Another definition of psychic income is the subjective value of non-monetary satisfaction gained from an activity (Economic Essays, University of North Carolina). We do not distinguish between psychic income and psychic returns; the purpose of this paper is to suggest an improved methodology for estimating psychic returns for cultural assets. We treat these returns as a multiplicative gross process in our analysis.

Recent literature also focuses on investigating the role of these psychic returns. Pownall et al. (2009) investigate diversification of risk by investment in emotional assets, for which the motives for investing go beyond investment value or pecuniary benefits alone; these assets may also provide a consumption value, greater utility in the form of aesthetic value or act as a signal of the owner’s wealth. The authors find evidence that direct investors are willing to forgo financial returns to invest in certain emotional assets, such as art, wine, clocks, watches, atlases and stamps. Furthermore, the high consumption value of these assets provides an alternative attribute in portfolio optimisation because of their ability to transfer consumption over time. This results in greater utility for the investor when holding a set of emotional assets in his investment portfolio. As a result of this feature, these returns are often called psychic dividends in the literature.

Recently with the emergence of the behavioural field of finance, these concepts have sprung up in the financial literature for instance Statman (2011) points out that some
investors want more than financial return and are subjective to their own emotional needs such as status. Similarly green investing, corporate social responsibility are all investment profiles where there is an additional value over and above purely financial value.

However, we note that not all agents derive these psychic returns, so we can distinguish between investors and collectors. Pownall (In Mimeo) models investors as having a low marginal utility from consuming art and invest purely for financial reward whereas collectors have a high level of aesthetic value and thus higher utility from owning an individual piece of art; this is captured by their higher marginal utility from the consumption of art. The paper also discusses the trade off between the reduction in financial risk from diversifying the portfolio across assets and the marginal loss in the emotional value from direct ownership or access; this in turn depends on the utility function of the individual investor or collector. The author’s measure of emotional and aesthetic value can be attributed in part to the psychic return. For instance, collectors may purchase assets solely for the emotional value or the pleasure from owning the object. Other social and psychological benefits that the owner may accrue are represented in the form of social status, social acclaim, and prestige factors. At the other end of the spectrum, investors buy financial assets purely for financial return and choose investments which maximise the total monetary return, for the level of associated risk.

Several other studies distinguish between investing and collecting for instance Belk (1995) refers to collecting as an activity that involves passion. Satchell and Auld (2009) draw a distinction between the demand driven by purely financial motives and the demand driven by the intrinsic pleasures of ownership.

We differentiate between investors and collectors in this paper as those who invest in art purely for financial returns as opposed to those who derive psychic returns in addition to financial returns respectively. There are numerous sources in the literature which point to the issue of the financial rate of return on paintings being lower than for other types of investments in financial assets over comparable periods (see for example Baumol (1986); Pesando and Shum (2007); Buelens and Ginsburgh (1993); Goetzmann (1993); Mei and Moses (2002); Renneboog and Spaenjers (2009)). Frey and Eichenberger (1995) discuss some issues surrounding the determinants of the additional psychic returns for art. The authors reconcile the risk-to-return ratio on art by implying an additional positive return to the art investor above the financial return. They propose measuring the psychic benefit of art from either ‘rental fees’, or from ‘the marginal willingness to pay for viewing art in museums’, an issue we shall return to later.

We evaluate the psychic returns of art relative to equity using British data available to us from 1895 to 2011. We point out that our procedures could be applied to other asset classes which have a cultural element and thus yield psychic returns to the owner or investor. We take into account the substantial costs involved in art investment and also examine the existing estimates of the psychic returns to art which are typically between 10 per cent to 30 per cent.

Using utility based and equilibrium based models, we construct estimates of psychic returns based on plausible portfolio weights and also trace the linkages between the psychic returns to art and the equity market. Such assets whose attributes are likely to attain a personal or societal value over and above their financial value in the portfolio
are often classified as passion or emotional assets in the literature, a point we have mentioned earlier. The additional utility gained is also likely to be a function of wealth, as exemplified by the larger proportion of such assets held by investors with greater incomes.

Turning to the issue of estimation, the existing literature estimates psychic returns to art using a number of procedures which we discuss below; these include the rental value of art, hedonic modelling and inferences based on the intercept of the Capital Asset Pricing Model (CAPM). Whilst each of these methods seems quite plausible, they typically provide estimates of psychic returns of the order of 10 per cent to 30 per cent. Taking this range of estimates together with information about expected capital gains for art and expected total returns for equity, we can construct optimal portfolios for investors by allowing the psychic return parameter to vary. This approach enables us to get some notion of what might be a range of possible values.

A related question of interest is how psychic returns could be included in an equilibrium model of the art and equity markets. Baumol (1986) first addressed this by assuming a risk neutral market. As a consequence, the differential between equity returns and art numbers must be the psychic return of art. This is a powerful insight but not an accurate description of price determination. We extend his approach in several directions. We first allow for risk aversion. Secondly, we address the difficulty that arises because not every agent derives a psychic return from art, a feature which requires modelling of different agents’ demands or utilities. We show that this problem can be put into a generalised CAPM using the structure that individual investors have subjective beliefs about the payout distribution. The resulting equilibrium clarifies the relationship between psychic returns and the equilibrium price of art together with the impact of the psychic return to art on the equity market.

Furthermore, by using the market fraction CAPM (Chiarella et al. (2009)), we can ascertain the extent to which a given proportion of wealth invested in art relative to the proportion of wealth invested in equity, and for a given proportion of the investing population who experience psychic returns, will influence the overall determination of the psychic return.

In the next section 1.2, we provide a brief survey of the existing literature that discusses the presence of psychic returns in art and their estimation. Section 1.3 addresses the issue of the presence of transaction costs and cites the literature for plausible estimates. Our framework for estimating psychic returns using utility based arguments is presented in section 1.4 and the results are discussed. In section 1.5, we present equilibrium based analyses to estimate these returns and discuss the implications of our findings. Section 1.6 discusses the statistical significance of our estimates and section 1.7 concludes.

### 1.2 Literature Survey

In this section, we survey the literature on psychic returns to art. Baumol (Baumol (1986)) says, “The (financial) rate of return on paintings is lower than for investment in financial assets (given higher risk in the former market) because paintings also yield a psychic return from owning and viewing the paintings”. This is an important theme in this paper and we try to measure these psychic returns that motivate agents to
invest in art. Frey and Eichenberger (1995) present a number of interesting points about measuring psychic returns. They assume that the psychic benefits from art are derived from the difference of financial returns on other markets as compared to the art markets. They also discuss the determinants of psychic benefits and suggest rental fees and willingness to pay studies as a possible way to analyse and estimate the psychic benefits from art. Their paper documents an overview of the major studies that have looked into psychic returns to paintings and other artworks (see Frey and Eichenberger (1995), Table 1, page 531).

Stein (1977) (see page 1029) suggests that paintings yield a flow of non-pecuniary viewing services which can be attributed as psychic returns. Using rental fees in the art rental market as an upper bound on the return from viewing services, the author finds that average annual rental charges are at about 11 per cent of the appraised value. Furthermore, Atukeren and Seckin (2007) present two arguments that set the psychic return to art to be about 28 per cent. The first argument is based on the rental value for art, where the authors make use of the prices from a Canadian fine art company that offers art rental and leasing services and derive an estimate of the extent of psychic returns. The second argument is based upon the CAPM together with transaction costs which are claimed to be of the order of 10 per cent to 30 per cent (see Atukeren and Seckin (2007), page 8)). We find that these studies differ greatly with respect to the period covered and its length, as well as many other factors such as the length of the holding period. We discuss this later in the paper.

The sense of the arguments in the literature is that psychic returns are in the 10 per cent to 30 per cent range; in what follows we shall argue that this is probably a large overestimate. We shall demonstrate this through the use of portfolio models and asset pricing models but our initial criticism will address the use of rental values and the CAPM. Rental services as a proxy for psychic returns were suggested by Stein (1977) and criticised by Frey and Eichenberger (1995) since it failed to take into account the aspect of ownership. This aspect could be interpreted in terms of luxury goods or status goods. This criticism whilst valid, would suggest that psychic returns are even higher than the barely plausible numbers suggested above. We will argue that they should be much lower because the rental market offers a rather different service. Renting art usually comes with a great deal of expert advice and is an ideal service for those who want the status of art but lack any expertise or appreciation of art. Thus the rather inflated rent is a rent of human capital as well as psychic returns to art. Given the desire by many to outsource these services to external consultancies, the potential value of these services could be substantial. Thus, it does not seem entirely appropriate to use rental values as a measure of psychic returns.

We now turn to the use of the CAPM and the estimated intercept which is seen as a measure of constant service yield. Stein (1977) finds an estimate of service yield of 1.6 per cent and treats this correctly as a cost relative to any psychic return that might be found separately. A number of other authors have followed this approach including Chanel et al. (1994); Hodgson and Vorkink (2004). Atukeren and Seckin (2007) discuss many previous authors who use this approach and some of whom have tried to interpret the intercept as the net quantity of service yield minus psychic returns. They argue that since this quantity is invariably close to 0 and insignificant it is reasonable to set psychic returns equal to costs. By setting cost at about 28 per cent they conclude that psychic returns are 28 per cent corroborating the value they find from rental value. This is an ingenious argument but its difficulty in terms of art indices is that it fails
to take into account the length of the holding period. Whilst the 28 per cent rental value corresponds to an annual amount, the 28 per cent auction cost corresponds to an indeterminate holding period. If the holding period were 28 years one might argue that the auction cost is 1 per cent per annum. We shall return to this point later in the paper when we discuss transaction costs.

Our concerns about such an approach are to do with models of asset pricing. Contemporary finance now looks at numerous factors in explaining returns in asset pricing; a minimal set would be the market, size, value and momentum. Commercial pricing models would add in factors like sector and region; many recent contributions are focused on liquidity and perhaps many other variables. Since these discussions are widely cited elsewhere we shall omit references. However, we do note that regardless of the nature of these factors, unless they are orthogonal to the market, they will lead to biases in estimates of the intercept and so arguments based around this quantity seem incomplete.

The research closest to our approach is that of Candela et al. (2013). In their paper the authors use mean variance analysis to ascertain an implied psychic return to art. They take the investment proportion as fixed but measure psychic risk as, “the incremental risk an investor should face to get the same return as the risk free rate with a portfolio that includes a predetermined investment in art” (see Candela et al. (2013), page 5). They use this measure of psychic risk, scaled by the Sharpe ratio of the market to define a psychic return. This is an interesting measure which produces much more plausible numbers but suffers from some weaknesses for instance, whilst psychic returns are assumed to be positive, the confidence intervals given in table 1 of their paper, allow for negative numbers. Our work can be seen as an extension of their approach in that we define psychic returns in terms of economic decision making; we take into account investor heterogeneity and risk aversion.

Finally, we do not try to estimate the public good aspects of cultural returns. Whilst it may be possible to do this by comparing prices that museums will pay relative to private sector purchases, estimating the price of public goods involves a great deal of added complexity. It is well known that efficiency requires each individual to pay a price that reflects her marginal evaluation of the public good at the efficient quantity. This is a widely cited result in the literature (Samuelson (1954, 1976); Lindahl (1919); Buchanan (1967, 1999); Coase (1974); Lee (1977, 1982) amongst others).

If these goods were efficiently priced, all the public good supplied will be consumed at the sum of each agent’s marginal evaluation, which would be reflected in the price. However, the problem with this result is that once provided, because of the non rival and non excludable nature of public goods, the cost of additional consumption is zero and agents do not have an incentive to truly reveal their preference; even zero prices will be insufficient to motivate the efficient provision of the public good. Therefore, decision makers or social planners would not be able make an efficient decision based on the assumption that prices convey true marginal valuations. So for instance, when museums charge fees for viewing art works or such, this would not accurately reflect each agent’s willingness to pay as the agent has a wide discretion on the nature of utility she would derive once the price is paid. However, because of the all-or-none nature of the choice being made, entry charges that even roughly reflect marginal evaluations can still be consistent with efficient consumption decisions. This is not to say that we could not estimate the public good benefits. Fullerton (1991) provides an interesting analytical framework which could be used to measure “public good”
returns in an equilibrium context. Therefore, using utility and equilibrium based frameworks and the analytical tools of portfolio theory, we propose methods to measure psychic returns, which are not affected by the common limitations that characterise this literature. We apply our measure to estimate psychic return to art investments, but our measure can be applied, in principle, to all forms of investment driven by passion as noted earlier.

1.3 Transaction Costs for Equity and Art

The issues of transaction costs are clearly very subtle and we cannot address them in full complexity. As a financial example, Anderson et al. (2012) look at equity and bond investment from 1926-2010. They assume trading costs of 1 per cent during 1926-1955, 0.5 per cent during 1956-1970 and 0.1 per cent during 1971-2010 in stocks and bonds. Renneboog and Houtte (2002) note that stock transaction costs for an individual investor amount to around 1-1.5 per cent (brokerage fees and, in some countries, local stock exchange taxes) for domestic shares and to around (maximally) 2.5 per cent for shares traded in foreign stock exchanges. They note that art transaction costs amount to more than 25 per cent.

Frey and Pommerehne (1989) assess transaction costs in art to decrease the annual real rate of return by 0.4 per cent for the period 1961-1987. Taking into account other costs like insurance costs which amount to about 1 per cent as noted by Worthington and Higgs (2003), total costs become considerable. The key points to note are that results are sensitive to the exact starting date and conditions and that it is necessary to assess the actual transaction costs in a time dependent manner. To find such information on UK auctions over our sample period is very challenging as commissions depend upon the value of the painting being sold. Furthermore, commission structure changes with time (for instance buyer’s premium was installed after 1975 by Christie’s) and typical commissions vary between 2 per cent and 35 per cent. Factors that can change the commission structure include commission charged to the seller, commission charged to the buyer, sales tax and possibly others.

To be conservative we can amortise transaction costs to be 1 per cent per annum. For equity, transaction costs of 1 per cent might seem about right as an average number over a long period of time. Again, the costs vary depending upon whether one has to pay sales tax (this has been 50 basis points in the UK in recent years but some exemptions exist for market makers). Historically, holding times for equity may have been close to a year, of course in recent years high frequency trading has reduced the average holding time to seconds. If we compare buying a painting and buying shares and holding them both for 20-30 years then an annual cost differential of between 50 basis points to 1 per cent seems plausible.

1.4 Utility Based Arguments

A starting point for the evaluation of psychic returns might be based on such a concept as a utility function for the representative agent. We could define this psychic return as a willingness to pay concept relative to an alternative investment in equity. If we
treat the differential transaction costs per year of art (denoted by $A$) relative to equity (denoted by $E$) as $c$ and the psychic returns to art per year as $p$, then the utility equalising rate of psychic returns could be defined by the following equations and definitions. Given an utility function $U$ and initial wealth $W_0$, fixed costs $c$, and art and equity returns $r_A$ and $r_E$ respectively, we can define $p$ additively or multiplicatively given by equations (Equation 0.1) and (Equation 0.2) respectively;

\[
E(U(W_0(1 + r_A + p - c))) = E(U(W_0(1 + r_E))) \tag{0.1}
\]

\[
E(U(W_0(\exp(1 + r_A + p - c)))) = E(U(W_0(\exp(r_E)))) \tag{0.2}
\]

To separate the value of $c$ from the level of $W_0$, which seems sensible, if only because of data availability issues, we propose homogeneous (in wealth) utility functions such as power utility; so let $U(W) = \frac{W^{1-\alpha}}{1-\alpha}$; using the multiplicative form equation (Equation 0.2) and letting $m(.)$ represent the symbol for a moment generating function, we arrive at the following expression for psychic returns in the equation below

\[
E(\frac{W^{1-\alpha}}{1-\alpha}) = E((W_0(\exp(1 + r_A + p - c))^{1-\alpha})) = E(W_0(\exp(r_E))^{1-\alpha})
\]

which implies that

\[
\exp((1 - \alpha)(p - c)m_A(1 - \alpha)) = m_E(1 - \alpha)
\]

or

\[
(1 - \alpha)(p - c) = \ln(m_E(1 - \alpha)) - \ln(m_A(1 - \alpha))
\]

Therefore, simplifying the above equation, we can write $p$ as

\[
p = c + \frac{\ln(m_E(1 - \alpha)) - \ln(m_A(1 - \alpha))}{(1 - \alpha)}
\]

Assuming normality, and denoting the mean of art and equity as $\mu_A$ and $\mu_E$ respectively and the variance of art and equity as $\sigma_A^2$ and $\sigma_E^2$ respectively, we can write $m_E(1 - \alpha)$ and $m_A(1 - \alpha)$ as follows

\[
m_E(1 - \alpha) = \exp((1 - \alpha)\mu_E + \frac{1}{2}(1 - \alpha)^2\sigma_E^2)
\]
\[ m_A(1 - \alpha) = \exp((1 - \alpha)\mu_A + \frac{1}{2}(1 - \alpha)^2\sigma_A^2) \]

Thus,

\[ p = c + \mu_E - \mu_A + \frac{1}{2}(1 - \alpha)(\sigma_E^2 - \sigma_A^2) \]  

(0.3)

\[ \alpha \] is the coefficient of relative risk aversion. If \( 0 < \alpha < 1 \), we are less averse than \( \ln(W) \) utility, for example \( U(W) = W^{1/2} \). If \( \alpha = 1 \), then utility is logarithmic. If \( \alpha > 1 \), then we are more risk averse than log utility. We can also see that given in our data \( \sigma_A^2 > \sigma_E^2 \),

\[ \frac{\partial p}{\partial \alpha} = -\frac{1}{2}(\sigma_E^2 - \sigma_A^2) > 0 \]

Increasing our risk aversion increases our psychic returns which are certain as it also lowers the value, relatively speaking, of the riskier asset, art. As an exercise, we can treat costs as fixed. If everything else is iid as it is in the above equation, and we assume costs \( c(t) \) to be time dependent, then if we amortise a fixed cost of \( K \) per cent over a holding period of \( t \) years, \( c(t) = \frac{K}{t} \), then \( \frac{\partial c(t)}{\partial t} < 0 \).

The above definition in equation (Equation 0.3) can be criticised as it depends only upon the marginal distributions of art and equity and not their joint distributions. We can rectify this by using a portfolio construction approach which we outline in the next section. Before we do, we note an alternative procedure based on compensatory risk premia involving the covariance between art and equity which could be used.

\[
E(U(W_0(1 + \theta r_A + (1 - \theta)r_E + p - c))) = E(U(W_0(1 + r_E)))
\]

\[
E(U(W_0(exp(\theta r_A + (1 - \theta)r_E + p - c)))) = E(U(W_0(exp(r_E))))
\]

Here the interpretation is that we could measure net psychic returns of an optimised portfolio of art and equity relative to a portfolio only of equity. This however seems somewhat cumbersome and we shall approach this problem in a more general framework.

Having noted the deficiencies of the approaches we suggest above, we now turn to a portfolio construction approach for two risky assets, art and equity. We assume we have returns \( r_{t+1}(N \times 1) \), a mean vector \( \mu_{t+1}(N \times 1) \) and a covariance matrix \( \Sigma(N \times N) \). The joint density function of \( r_{t+1} \) is assumed to be multivariate normal, which we denote as \( r_{t+1} \sim N(\mu, \Sigma) \). Consider the constant absolute risk aversion (CARA) utility function, subject to a budget constraint; we can transform the optimisation problem to the expression below given by equation:

\[ U = \omega'\mu - \frac{1}{2}\omega'\Sigma\omega - \theta(\omega'\Omega - 1) \]  

(0.4)
where $\theta$ is the Lagrange multiplier and where $\lambda$ is the coefficient of absolute risk aversion, $\omega$ is a vector of portfolio weights chosen to maximize the above equation and $1$ is a vector of ones. We note the following result (see Allen et al. (2012), Proposition 5). The optimal mean-variance weights in the presence of a budget constraint with known parameters is given by

$$\omega = \frac{1}{\lambda} \sum^{-1} \mu - \frac{\beta - \lambda}{\lambda \gamma} \sum^{-1} 1$$

(0.5)

where $\beta = \mu' \sum^{-1} 1$ and $\gamma = 1' \sum 1$. Let $\delta = \mu \sum^{-1} \mu$, then the expected utility associated with this case is given by, substituting equation (Equation 0.5) into equation (Equation 0.4) and simplifying,

$$V = E(U) = \frac{\delta \gamma - (\beta - \lambda)^2}{2 \lambda \gamma}$$

If we ignore the budget constraint and consider only equity and art, then the optimal portfolio becomes

$$\omega = \frac{1}{\lambda} \sum^{-1} \mu$$

(0.6)

and

$$V = \frac{\delta}{2 \lambda}$$

(0.7)

where $\omega$ consists of $\omega_A$ and $\omega_E$, the optimal portfolio weights of art and equity respectively. The result given by equation (Equation 0.5) assumes all assets are risky. However, this can be simplified if the third asset is cash (riskless). Then we could adjust the formula above by replacing $\mu$ by $\mu - r_f$ where $r_f$ is the riskless real rate of interest and $1$ is a 2 by 1 vector of 1s. However, since over one hundred years the discrepancy between the riskless rate and real rate taken over a 20 year holding period is likely to be very small, we shall omit the results of the recalculation here.

Now, we conduct the following exercise; we shall assume that $p$ is net of costs in what follows and drop the $c$ term. We assume that $\mu = \left( \begin{array}{c} \mu_A + p \\ \mu_E \end{array} \right)$. We substitute in the recorded values for psychic returns to see what invested proportions result from our UK data. These values have been discussed in the previous section.

Simplifying using the optimal portfolio values in equations (Equation 0.6) and (Equation 0.5), cancelling common factors, we see that the ratio of art investment to equity investment is given by

$$\frac{\omega_A}{\omega_E} = \frac{\sigma^2_E (\mu_A + p) - \sigma_{AE} \mu_E}{\sigma_A^2 \mu_E - \sigma_{AE} (\mu_A + p)}$$

(0.8)
where $\sigma_{AE}$ is the covariance of art and equity. Equation (Equation 0.8) will lead to a consistent estimation of the fixed parameter $p$, assuming that the relative proportions are known. This is because we can solve equation (Equation 0.8) for $p$ as a function of sample moments (see Appendix B). Under fairly general assumptions, we can assume that these are consistent. In fact the assumptions of our model are that the data are generated by iid Gaussian processes and under these assumptions, invoking Mann and Wald and Slutsky theorems, we can prove consistency and asymptotic normality. Further calculations using the delta method will enable us to calculate asymptotic standard errors of $p$. These involve derivatives of $p$ with respect to the covariance matrix elements (see Appendices 4.C and 4.D) and the variance covariance matrix of these elements.\(^1\) We shall deal with this in greater detail later.

### 1.4.1 Data and Results

The data we use are the annual FTSE All Share total returns equity index, the London All Art price and an inflation index (Annual Consumer Price Index data from the Office of National Statistics from 1895-2003). The London All Art price index uses the London sales from the European Art Database and is constructed using the repeat sales approach used in Mei and Moses (2002), who use comparable data for the US art market. Sales of identical artworks are identified and art returns are constructed using the purchase and sales price over the holding period. The data include over 10,000 repeat sales pairs for constructing the repeat sales index for London sales only. The London art market has been historically and still remains, a very important auction market for the major collecting art classes currently representing 23 per cent of the global market and about two-thirds of the European market (TEFAF report, 2012).\(^2\)

The repeat sales model is specified as follows,

$$\ln \left( \frac{p_{i,t+\tau}}{p_{i,t}} \right) = \sum_{i=1}^{T} \pi_t D_{i,t} + u_{i,t}$$

where $p_{i,t}$ and $p_{i,t+\tau}$ are the transaction prices for the artwork $i$, sold at time $t$ and resold at time $t + \tau$ respectively. The set of $D_{i,t}$s are time dummy variables which take the value 1 in the period that the resale occurs, and -1 in the period of the previous sale, and otherwise are 0. Mei and Moses (2002) acknowledge a number of limitations from the use of repeat sales for estimating art indices. There is naturally a sample selection bias. Only those sales which are resold at auction are included in the sample, so that there may be an over representation of high quality artworks. Since only those artworks are included which have appeared twice or more at auction are included a large proportion of art auction sales are not included in the sample. If more expensive artworks are traded less frequently, then this will cause the error terms to be heteroskedastic. We use repeated least squares to correct for heteroskedasticity in the data. The repeat sales methodology has been shown to be representative of all auction sales, which may in itself not be fully representative of the total art market,

\(^1\)See the following link http://www.statlect.com/wishart_distribution.htm

\(^2\)See "Observations of the Art Trade over 25 Years", The European Fine Art Foundation Art Market Report 2012, prepared by Clare McAndrew
since over 50 per cent of sales are transacted through galleries, where price data are not usually available.

That art started being considered as an investment commodity from the late nineteenth to the twentieth century receives support from art historians. So the period we are looking at, and have data available for, seems appropriate. For example, Robert Hughes (1984) explains how it was only during the beginning of this period that art dealers started to matter. This new class of entrepreneur was very much interested in art prices and using it as an investment commodity in addition to art being bought for pleasure or status. This was, he explains, a result of greater liquidity and credit availability. Also, at this time, sale of paintings also included reproduction rights, which greatly affected the price of pictures. Thus, at least initially, the purchase of art entitled the owner to a stream of income just like equity.

Campbell (2008, 2009) gives a description of the art market and points out that there is a high probability that art funds who also act more like private dealers than auction houses, adopt a very similar strategy to private dealers to take advantage of the inefficiencies in the market through their insider knowledge and expertise. These features are likely to produce art market returns which could be greater than the benchmarks used in this analysis.

Using real geometric rates of return for equity and art, we can reduce the ratio of art to equity held to the simple expression given by equation (Equation 0.8). We consider a range of values for $p$, from one percent to thirty percent annualised. We see the corresponding proportion in art go from 7.6 per cent up to 64 per cent. The current value of the UK art market (TEFAF report (ibid)) is approximately 9-10 billion pounds as a flow at the time of writing. In fact this seems to be the sum of 3 categories, fine art, decorative art, and antiques.

We know from our art index that the time of resale for art is approximately 30 years; Stein (1977) sets it at a generation (a somewhat unclear concept but possibly 30 years). Clearly a proportion of paintings are bought but never resold and paintings in museums are effectively excluded from the market value in the same way that family owned firms are excluded from the value of the equity market. This suggests a stock value of the UK art market at about roughly 200 billion pounds. The current value of the UK FT All Share is approximately 1.8 trillion pounds (Office for National Statistics, 2010).\(^3\) There are differences in definition, the World Bank list for market capitalisation for all listed companies on national exchanges\(^4\) gives the value of the UK market in 2010 at 2.018 trillion pounds based on average exchange rates. This implies that the current relative proportions of holdings in the UK art (broadly defined) and the UK equity are about 10 per cent and 90 per cent. This of course ignores the effects of trade in art; it is highly likely that much art sold in the UK is exported.

By comparison, we note that the French stock market in 2010 is approximately 1.25 trillion pounds in capitalisation and that TEFAF report (ibid) gives the sales of art in France at approximately 2 billion pounds. Using the same assumptions about holding periods the stock of tradable art in France might be worth 40 to 60 billion pounds.

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\(^4\)World Bank Market Capitalisation of Listed Companies by Nation can be seen here: http://data.worldbank.org/indicator/CM.MKT.LCAP.CD
This would mean that the current relative proportions of holdings of French art versus French equity will be closer to 4 per cent and this may well reflect the UK position once imports and exports have been taken into account. However, the figures provided by TEFAF report (ibid, page 79, table 3f) suggest that France is a substantial net exporter whilst UK, surprisingly, is a modest importer. Whilst these figures clearly are subject to great annual variation (UK imports were up by 40 per cent from 2010, French exports up by 54 per cent from 2010), they do not suggest that the proportions need to be radically changed.

Given this amount, real returns to both asset classes and estimated parameter values for $\mu_A, \mu_E, \sigma_A, \sigma_E,$ and $\sigma_{AE}$, we can use equation (Equation 0.8) to find the proportion $\frac{\omega_A}{\omega_E}$. As discussed above, the proportion we wish to match is approximately 10 per cent to 90 per cent. The mean and variance of real geometric art equity returns are 0.018, 0.202 and that of equity returns are 0.0416, 0.035 respectively and their covariance is 0.007. The mean inflation rate is 0.04 in our sample and the variance is 0.003. All measurements are per annum.

Table 4.1 shows us that this corresponds to a value of $p$ of about 1.5 per cent. Since this is net of the service charge yield, we can propose a value of about 3 per cent to 3.1 per cent as the psychic return to art. If we wish to measure this as a nominal quantity, a number of about 6.5 per cent sounds plausible. It is also clear that a nominal amount for psychic returns of 28 per cent would imply that the relative proportion of art to equity would be something in excess of 100 per cent; this seems quite implausible.

1.5 Equilibrium Based Arguments

In the previous section we treated art as providing psychic returns to the representative investor. To the extent that this is an equilibrium model, it might be construed that all investors in the economy enjoy art. It is worth noting that Baumol (1986, 2007) models that all expected returns should be the same; this can be seen as a simple equilibrium model assuming risk neutrality. We would disagree with this risk neutral assumption as it seems quite inappropriate in such a portfolio choice problem.

In this section we consider how the presence of psychic returns in art might influence the return to art and the return to equity in a heterogeneous environment. In this world, we can define a CAPM where different types of agents have different beliefs about the future returns distribution. In a world of mean-variance, these beliefs can be manifested in different means, variances and covariances. Agents can also have different utilities.

Turning to expected returns, we shall subtract costs from expected rates of return, assuming a specific holding period. We shall assume that there are two groups of investors in the economy, collectors (denoted by $C$), who value art (we add $p$, psychic returns, to their expected rate of return) and investors (denoted by $I$) who do not. Both groups are endowed with constant absolute risk aversion utility functions, initial wealth, and face a choice between three assets, namely, equity, art and cash. We shall allow their coefficients of absolute risk aversion to differ. We denote the coefficient of absolute risk aversion as $\lambda_i$ where $i = C, I$. We can use the differing expected returns as essentially subjective beliefs about expected rates of return but we assume agreement about covariances and variances. This framework has been advanced by
1.5.1 CAPM with Heterogeneous Beliefs

Before we present our framework, it is useful to outline in this section, the model which introduces heterogeneous beliefs into the mean-variance framework of the standard CAPM, in contrast to the standard approach which assumes homogeneous beliefs. Heterogeneity arises by assuming that agents form optimal portfolios based upon their heterogeneous beliefs about conditional means and/or variances and covariances of the risky asset returns.

The impact of heterogeneous beliefs among investors on the market equilibrium price has been an important focus in the CAPM literature. Chiarella et al. (2009) give an extensive overview of this literature including works of Lintner (1969); Huang and Litzenberger (1988); Detemple and Murthy (1994); Basak (2000) amongst others. For instance, Williams (1977); Varian (1985) consider a portfolio of many risky assets and one risk free asset, investors are assumed to be heterogeneous in their risk preferences and expected payoffs or returns of the risky assets. Various heterogeneous agent models have been developed to characterise the dynamics of financial asset prices resulting from the interaction of heterogeneous agents with different attitudes towards risk and different expectations about the future evolution of asset prices. This framework has successfully explained different types of market behaviour, such as the long term swing of market prices from the fundamental price, asset bubbles and market crashes. Thus, such a framework seems appropriate for our purposes. Our contribution is towards this relatively unexplored issue of the impact of heterogeneity on the art market equilibrium and in markets with other risky assets and heterogeneous investors.

Following Chiarella et al. (2009), consider an economy with many agents who invest in portfolios consisting of a riskless asset and N risky assets as mentioned before in section 4.4. We note that N = 2 in our case, i.e we consider art and equity. Following the standard CAPM setup, we assume that the returns of the risky assets are multivariate (conditionally) normally distributed and the utility function $U_i(.)$ of the agents twice differentiable, concave and strictly increasing, again as before. Let $W_0^i$ be the initial wealth of agent $i$ and suppose there are H investors. Let $w_{ij}$ be the wealth proportion of agent $i$ invested in asset $j$ where $j = 1, 2...N$. Then the end-of-period wealth, $\tilde{W}_i$, of agent $i$ is given by $\tilde{W}_i = W_0^i \left( 1 + r_f + \sum_{j=1}^{N} w_{ij} (r_j - r_f) \right)$ where $r_j$ is the rate of return on asset $j$. The maximisation of the expected utility of the portfolio wealth of agent $i$ is characterized by the first order condition as given in Chiarella et al. (2009), which can be written as follows

$$E_i \left[ U_i (\tilde{W}_i) \right] E_i [r_j - r_f] = -E_i \left[ U_i'' (\tilde{W}_i) \right] \text{cov} \left( \tilde{W}_i, r_j \right)$$

(0.9)

where $E_i (.)$ is the conditional mean of agent $i$, characterizing the heterogeneity of the agents in their beliefs. By defining the global absolute risk aversion of agent $i$ as

$$\lambda_i = \frac{-E_i \left[ U_i'' (\tilde{W}_i) \right]}{E_i \left[ U_i (\tilde{W}_i) \right]}$$

and noting the definition of $\text{cov} \left( \tilde{W}_i, r_j \right) = W_0^i \sum_{k=1}^{N} w_{ik} \text{cov} \left( r_k, r_j \right)$, it
follows that we can write equation (Equation 0.9) as

$$\lambda_i^{-1} E_i [r - r_f \mathbf{1}] = W_i^0 \sum \omega_i$$

(0.10)

where $r$ is the vector of returns, $\mathbf{1}$ is the vector of ones, $\omega_i$ is the vector of wealth proportions for agent $i$ as before. $\sum$ is assumed to be a positive definite and thus invertible covariance matrix such that $\sum = [\sigma_{jk}]_{N \times N}$ where $\sigma_{jk} = \text{cov}(r_j, r_k)$.

From equations (Equation 0.9) and (Equation 0.10), we can write the optimal portfolio $\omega_i$ of agent $i$ as

$$\omega_i = \frac{1}{W_i^0} \lambda_i^{-1} \sum_j^{-1}(E_i [r] - r_f \mathbf{1})$$

(0.11)

Now, if we take the total societal wealth as $W_m^0 = \sum W_i^0$ and $\omega$ to be the proportions of the total wealth in the economy invested in the risky assets, then the market is in equilibrium when $W_m^0 \omega = \sum_{i=1}^H \omega_i$.

To build a consensus belief in the expected return vector, let us assume $\lambda = (\lambda_C^{-1} + \lambda_I^{-1})^{-1}$, following Proposition 1 in Chiarella et al. (2009), we can write the consensus belief as follows

$$E_m [r] = \lambda \sum_{i=1}^H \sum_j \lambda_i^{-1} \sum_j^{-1} E_i [r]$$

(0.12)

Further, when the market is in equilibrium, the vector proportions $\omega$ of the total wealth in the economy invested in the risky assets is given by

$$\omega = \frac{1}{W_m^0} \lambda^{-1} \sum_j^{-1}(E_m (r) - r_f \mathbf{1})$$

(0.13)

We can extend this framework now to examine a fractional market CAPM framework. Assume that the $H$ investors can be grouped into a finite number of agent-types, indexed by $h$ such that $h \in H$, where the agents within the same group are homogeneous in their beliefs $E_h [r]$ as well as in the risk aversion coefficient $\lambda_h$. Following Chiarella et al. (2009), instead of using the aggregate risk aversion as defined before as $\lambda = (\lambda_C^{-1} + \lambda_I^{-1})^{-1}$, the average risk aversion is now given by $\lambda_m = \left( \sum_{h \in H} n_h \lambda_h^{-1} \right)^{-1}$.

Thus, we can write the aggregate expected return vector as

$$E_m [r] = \lambda_m \sum_{h \in H} n_h \lambda_h^{-1} \sum_j^{-1} E_h [r]$$

(0.14)
1.5.2 Psychic Returns in this Framework

In this section, we apply the above framework to our problem formally. We derive the optimal portfolio where \( i = C, I \) for collectors and investors respectively. We treat \( j = A, E \), i.e there are two risky assets art \( (A) \) and equity \( (E) \).

\[
E_C(r) = \left( \frac{\mu_A + p}{\mu_E} \right), \\
E_I(r) = \left( \frac{\mu_A}{\mu_E} \right) \text{ and } \sum = \begin{pmatrix} \sigma_A & \sigma_{AE} \\ \sigma_{AE} & \sigma_E \end{pmatrix}
\]

where the symbols have the same meanings as before. Following equation (Equation 0.11), the optimal portfolio is given by;

\[
\omega_i = \frac{1}{W_0} \lambda_i^{-1} \sum^{-1} (E_i(r) - r_f) 
\]

(0.15)

This is the same result as given in equation (Equation 0.6) except that individuals now differ in terms of initial wealth, absolute risk aversion and expected rates of return. Defining societal wealth as \( W_{m0} \) as before such that \( W_{m0} = W_C^0 + W_I^0 \), the societal investment in the different asset classes is equal to \( \omega \) where \( \lambda = (\lambda_C^{-1} + \lambda_I^{-1})^{-1} \) is the societal risk aversion and therefore,

\[
\omega = \frac{1}{W_{m0}} \sum^{-1} \left( \frac{1}{\lambda_C} E_C(r) + \frac{1}{\lambda_I} E_I(r) \right) - r_f 
\]

(0.16)

This result follows from specialising Proposition 1 in Chiarella et al. (2009) and equation (Equation 0.13).

Following the same argument as above, we could treat aggregate wealth as equal to 1 so that each investor’s contribution to the overall portfolio proportions depends upon the relative magnitude of the absolute risk tolerance (defined as the inverse of their absolute risk aversion). To proceed further we need to have some notion of the relative magnitudes of their absolute risk aversion.

A number of authors have addressed these issues in the finance literature; in Appendix A we take an argument from Allen et al. (2012). Summarising, this argument suggests that high risk tolerant investors would use a value of allocation of the aggressive investor. For details, see Appendix A. The authors estimate the coefficient of absolute risk aversion using their definitions of allocations of the conservative, balanced and aggressive investors. The resultant levels of \( \lambda \) are 0.045, 0.025, and 0.015 respectively. These effectively weight the aggressive and conservative investors by ratios of one to three. Thus aggression here means more risk tolerant. Moving this argument forward, should art investors be more or less conservative than conventional investors? We have no evidence on this but creative activity has always been seen as riskier than more conventional ways of earning a living and we shall assume that collectors are aggressive investors whilst investors are conservative investors.

Another argument in favour of this assumption is that the above modelling is an approximation of a more general situation where both investors have decreasing absolute risk aversion (DARA). We note that in a sense by varying \( \lambda \) across agents, we could expect to observe DARA here even though the utility function is CARA. If in aggregate, collectors are high net worth individuals relative to investors, then we would come to the same conclusion.
If we follow this assumption, we can establish new values for psychic returns which we detail below. Using equation (Equation 0.14), we can use the same calculations as above, where \( \omega_A \) is the proportion of wealth invested in art and \( \omega_E \) is the proportion of wealth invested in equity in the society in equilibrium.

Following equations (Equation 0.16) and (Equation 0.13), the proportional investment can be derived as,

\[
\frac{\omega_A}{\omega_E} = \frac{\sigma_A^2 d_1 - \sigma_{AE} d_2}{\sigma_A^2 d_2 - \sigma_{AE} d_1}
\]  

(0.17)

where \( d_1 = (\lambda_C^{-1} + \lambda_I^{-1})\mu_A + \frac{\mu_A}{\lambda_I} - r_f \) and \( d_2 = (\lambda_C^{-1} + \lambda_I^{-1})\mu_E - r_f \). Solving for \( p \) (where \( k = \frac{\omega_A}{\omega_E} \), see Appendix B for details),

\[
p = \frac{(k\sigma_A^2 + \sigma_{AE})}{(\sigma_E^2 + k\sigma_{AE})} \left( \left( 1 + \frac{\lambda_C}{\lambda_I} \right) \mu_E - \lambda_C r_f \right) - \left( \left( 1 + \frac{\lambda_C}{\lambda_I} \right) \mu_A - \lambda_C r_f \right)
\]  

(0.18)

Some immediate comparative statics indicate that an increase in \( p \) will increase \( \frac{\omega_A}{\omega_E} \) as long as \( \sigma_{AE} \) is positive which it is in our sample; in fact the correlation between the two real rate of return series is 0.09. In the case that \( \sigma_{AE} \) is negative, it would follow that art would be a direct hedge for equity. It is then conceivable but unlikely that an increase in \( p \) would increase \( d_1 \) sufficiently that the proportional increase in the denominator would be greater than the proportional increase in the numerator leading to a fall in the proportion invested. This would only happen if the volatility of art would be huge compared to the volatility of equity.

There is another case of considerable interest worth mention here. Suppose that the psychic returns for collectors were to increase exogenously in equilibrium. From equation (Equation 0.18) and calculations in Appendix C, we can see that \( \frac{\partial p}{\partial \mu_A} = -\left( 1 + \frac{\lambda_C}{\lambda_I} \right) \). So, \( \frac{\partial \mu_A}{\partial p} = -\frac{\lambda_I}{\lambda_I + \lambda_C} \) which is negative and less than 1. For investors, we note that an increase in psychic returns leads to a fall in total expected returns to art (given that \( \frac{\partial \mu_A}{\partial p} < 0 \)). Now if we define the total expected rate of return to art \( \mu_T \) to the collector as \( \mu_A + p \), it follows that \( \frac{\partial \mu_T}{\partial p} = \frac{\partial \mu_A}{\partial p} + 1 = \frac{\lambda_C}{\lambda_I + \lambda_C} \) which is positive but less than 1. Thus, an increase in psychic returns leads to an increase in expected total returns for the collector. So for instance if we take values for \( \lambda_C, \lambda_I \) to be 0.015 and 0.045 respectively, then \( \frac{\partial \mu_T}{\partial p} = 0.25 \). This means that a 1 percent increase in psychic returns increases the total expected returns for collectors by 25 basis points. For the same values, \( \frac{\partial \mu_A}{\partial p} = -0.75 \) which means that an increase in psychic returns by 1 per cent leads to a decrease in the investors’ total expected returns by 75 basis points.

Table 4.2 reports the relative portfolio proportions in art \( \omega_A \), and equity \( \omega_E \), with varying \( p \). These results show that decomposing investors into two types will reduce the proportion invested in art for the same level of psychic returns. In order to capture the proportions of about 10 per cent to 90 per cent, which seem to roughly accord with the value of the stock invested in tradable art and equity as we noted earlier, psychic returns increase from 1.5 per cent per annum (our result from equation (Equation 0.8) from the model in section 4.4) to 2 per cent per annum in this model.

The prevailing view is that to encourage investors to invest in the art market the required rate of return hence appears to be higher than for collectors alone. This is in
line with the current literature on the estimation of psychic returns, or other types of aesthetic or emotional returns which individuals value. Our results provide an important contribution to the literature as to the societal value from psychic returns. In our model, this would require increasing the proportion of investors whilst investment proportions are fixed as well as the expected return to equity. Unfortunately increasing the proportions of investors reduces the proportion of collectors, and even with the ceteris paribus assumptions made above, it is difficult to verify this effect. However, at an intuitive level, an increase in the proportion of investors at the expense of collectors will lower the demand for art, and therefore lower the price of art and hence increase the return.

In the current argument, we act as if there is one collector and one investor and wealth effects have been excluded by our choice of utility function. We can recapture the wealth effects to some extent by noting that each category of investor has the same wealth to hand and then capturing proportions of wealth by using the dynamic market fraction CAPM discussed in Chiarella et al. (2009). By treating the agents as each having access to the same amount of wealth we can use the market fraction of agents of a particular type as a measure of market share and influence. The intuition as to why this works follows from the fact that if we sum aggregate constant absolute risk premium investors we sum their risk tolerances. As discussed above, this model allows for groups of investors with identical characteristics which in our case would mean the same risk aversion and belief about psychic returns amongst groups of collectors and investors respectively. It transpires that exactly the same relationships go through except that these fractions scale the risk tolerances of the relevant groups (see Chiarella et al. (2009), page 10).

Using the results from Chiarella et al. (2009) together with a trivial extension, we can define proportions below using the additional terms $n_C$ and $n_I$ which are the proportions of each investor type. $W_{m0}$ can be defined in terms of these proportions but relative investment will not depend upon $W_{m0}$ such that

$$\omega = \frac{1}{W_{m0}} \sum_{-1} \left( \left( \frac{n_C}{\lambda_C} E_C(r) + \frac{n_I}{\lambda_I} E_I(r) \right) - r_f \right)$$

(0.19)

We note that the aggregate expected return vector as seen in equation (Equation 0.14) depends on the average risk aversion. In our case, the average risk aversion is given by $\lambda_m = \frac{n_C}{\lambda_C} + \frac{n_I}{\lambda_I}$. This is also referred to as the weighted average risk tolerance (WART) in the literature (Levy (2011)). WART is important in explaining equilibrium returns, since aggregate agents’ risk-aversion determines the required rate of return of the risky asset.

We can do a similar calculation as before on the impact of changes in psychic returns in equilibrium. By taking into account now the weights of both the groups of investors and collectors, $\frac{\partial \mu_A}{\partial p} = -\frac{\lambda_I n_C}{n_I (n_I + n_C)} = -\frac{n_C \lambda_I}{n_I (n_I + n_C)}$ and similarly $\frac{\partial \mu_T}{\partial p} = \frac{\lambda_C n_I}{n_I (n_I + n_C)} = \frac{n_I \lambda_C}{n_I (n_I + n_C)}$.

We can also see from these expressions that if the proportion of investors $n_I$ or $\lambda_C$ tends to 0, i.e the proportion of investors become negligible or the collectors become more risk tolerant, then $\frac{\partial \mu_A}{\partial p} \approx -1$. In this case, a 1 per cent increase in $p$ will lead to a 1 per cent decrease in expected returns for investors whilst a 1 per cent increase in $p$ will lead to no change in $\mu_T$. $\frac{\partial \mu_T}{\partial p} \approx 1$ when either $n_c$ or $\lambda_I$ tend to 0, i.e when investors
become extremely risk tolerant or the proportion of collectors shrinks substantially. In this case, an increase in \( p \) by 1 per cent will lead to a simultaneous increase in expected total returns for collectors.

In the above example, a different but equivalent interpretation might be that everyone has the same risk aversion but that there are 75 per cent collectors and 25 per cent investors. Thus, if we take the value of equity to art as being broadly in the proportion of 90 to 10, and the weight of money of investors to that of collectors as being a variable proportion, the finding of a given psychic return will depend upon what that variable proportion is.

Ceteris paribus, a society dominated by investors will need a much higher psychic return to sustain a social 10 per cent holding in art then a society dominated by cultivated people. It is possible to do the same calculation for prices; we can use this model to express the dependence of art and equity prices on psychic dividends and on market proportions. Since the results are fairly predictable, we omit details. However, to apply this sort of analysis to a comparative study of different countries would require that we are able to value each countries art market and equity market and this would exclude a number of interesting examples. TEFAF report (ibid, page 25), does give current sales of art for a range of countries and in principle we could compare France, for example, with the UK under varying assumptions of investor proportions. This would lead to potentially different psychic returns.

### 1.6 Computing the Statistical Significance of Psychic Returns

In the previous section, we estimated psychic returns to be about 2 per cent. We want to test our estimate of psychic return, \( p \). One way to do this is by using the delta method. The delta method allows us to calculate the asymptotic distribution of the parameter estimate of \( p \). In essence, it expands a function of a random variable about its mean, usually with a one-step Taylor approximation, and then takes the variance. Let \( \phi = (\mu_A, \mu_E, \sigma_A^2, \sigma_E^2, \sigma_{AE}) \), then we can write the following using asymptotic theory where the symbol \( ~ \) means “asymptotically distributed as”;

\[
\sqrt{n}(\hat{p} - p) \sim AN \left(0, \sigma_p^2 = \frac{\partial p'}{\partial \phi} \sum \phi \frac{\partial p}{\partial \phi} \right)
\]

\[
\sqrt{n} \left( \frac{\hat{p} - p}{\sigma_p} \right) \sim AN(0, 1)
\]

\[
n \left( \frac{(\hat{p} - p)^2}{\sigma_p^2} \right) \sim \chi^2(1)
\]

We derive \( p = f(\phi) \) in Appendix B where \( \mu_A, \mu_E \) are means of art and equity returns respectively. \( \sigma_A^2 \) and \( \sigma_E^2 \) are the variances of art and equity respectively and \( \sigma_{AE} \) the
covariance. In order to calculate the variance covariance matrix, we begin by calculating the partial derivatives matrix \( \frac{\partial p'}{\partial \phi} \) given in Appendices 4.B and 4.C. To get the matrix \( \sum_{\phi} \), we assume art and equity are iid normal and their joint distribution is bivariate normal (this will lead to simplifications in the covariances of sample moments), which is given in Appendix D. We have all the estimates from our data to calculate the matrix.

We use our estimate of \( \hat{p} = 0.02 \) as reported in section 4.5. From the delta method, our estimate of \( \sigma^2_p, \hat{\sigma}^2_p = 0.397 \). \( \hat{\sigma}_p \) is therefore found to be 0.199. Now, we want to test whether \( \hat{p} \) is significantly different from 0. The number of observations in our data is \( n = 108 \). Thus, the test statistic is given by \( \sqrt{108} \ast \left( \frac{0.02}{0.199} \right) = 1.047 \). This is not significantly different from 0 at 5 per cent level of significance.

The problem with using the delta method is that we do not take care of the interdependence between art and equity. We have already established this interdependence in Pownall et al. (2013) such that \( \text{cov}(r_{A,t}, r_{E,t-1}) \) is non-zero. More details can be found in Appendix E. This will have an impact on the magnitude of the \( t \) statistics which in the above example is 1.047. Typically, ignoring autocorrelation will increase the magnitude of the volatility without necessarily changing the magnitude of the point estimate (since in many situations the point estimate remains consistent albeit inefficient). To address this, we can use repeated random sampling to test for \( p \). We can find the standard error of our estimate by using a bootstrap procedure. In order to obtain errors, we estimate a \( \text{VAR}(2) \) for art and equity returns. The results of the \( \text{VAR} \) also indicate causality from equity to art which confirm the findings in Pownall et al. (2013). We can specify the \( \text{VAR}(2) \) model as follows

\[
\begin{pmatrix}
r_{A,t} \\
r_{E,t}
\end{pmatrix} =
\begin{pmatrix}
a_A \\
a_E
\end{pmatrix} + \sum_{j=1}^{2} \Lambda_j \begin{pmatrix}
r_{A,t-j} \\
r_{E,t-j}
\end{pmatrix} + \begin{pmatrix}
u_t \\
v_t
\end{pmatrix}
\]

where

\[
\Lambda_1 = \begin{pmatrix}
\hat{b}_1^1 & \hat{b}_1^2 \\
\hat{b}_1^3 & \hat{b}_1^4
\end{pmatrix}, \quad \Lambda_2 = \begin{pmatrix}
\hat{b}_2^1 & \hat{b}_2^2 \\
\hat{b}_2^3 & \hat{b}_2^4
\end{pmatrix}, \quad \begin{pmatrix}
u_t \\
v_t
\end{pmatrix} \sim \begin{pmatrix}
0 \\
0
\end{pmatrix}, \Omega
\]

Table 4.3 gives the estimates of \( \Lambda_1, \Lambda_2 \) using the VAR results. We can bootstrap with replacement from the estimated residuals to generate art and equity series (using initial values from our sample). We do \( n = 1000 \) replications to calculate \( p \) in each iteration, \( \hat{p}_1, \hat{p}_2, ..., \hat{p}_n \).

From this, we can obtain a distribution of \( p \) to test for the significance of our original estimate.

\[
\hat{\sigma}^2_p = \frac{\sum_{i=1}^{n} (p_i - \bar{p})^2}{n - 1} = \frac{\sum_{i=1}^{n} p_i^2 - n(\bar{p}^2)}{n - 1}
\]

\[
E(\bar{p}) = \mu_p
\]
\[ E(p^2_i) = \mu_p^2 + \sigma_p^2 \]

The mean in our bootstrap for \( p \) is 0.022 (compared with 0.020), our estimate of \( \sigma_p^2 \) is 0.002. The resulting estimate for \( se(\hat{p}) \) is 0.039. Therefore, \( \sqrt{n}\frac{\hat{p}}{se(\hat{p})} = 5.77 \). Therefore, once we take care of the interdependence in the data, our estimate of \( p \) becomes significant.

### 1.7 Concluding Remarks

Using data on London art prices over the long term, we find evidence in this paper that when estimating the cultural return to assets, the psychic component of the return is statistically significant. Our psychic return estimates for art data are lower than what is typically quoted in the existing literature. We use a utility based approach to estimate these returns to investors. This approach helps us estimate psychic returns in a manner consistent with an equilibrium in which agents differ. Our estimates are positive and significant once we take into account temporal correlations between art and equity markets. We use a fractional market CAPM to identify psychic returns in a world where both investors and collectors are present and such a model enables us to allow for heterogeneity both in risk attitudes and proportions of “market influence”. Our methodology also allows us to vary these proportions and thus proxy wealth differentials in this manner.

The presence of significant psychic returns can be captured in a higher psychic return required by collectors than investors. To encourage investors to invest in the art market the required rate of return hence appears to be higher than for collectors alone. This is in line with the current literature on the estimation of psychic returns, or other types of aesthetic or emotional returns which individuals value. Our results provide an important contribution to the literature on the societal value of psychic returns. The contribution of this paper is in terms of extending procedures for evaluating psychic returns which have a broad applicability to other asset classes similar to art. In future research, these methods can be applied to other problems like ethical investments.

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\(^5\)We can also check the robustness of this result by ignoring the coefficients that have a \( t \) statistic less than 1. From our results, values for \( a_A, b_1^1, b_1^1 \) and \( b_2^2 \) are taken to be 0 in this iteration. In this case, the new mean is 0.008 and the new estimate of \( \sigma_p^2 \) is 0.002. Thus, the test statistic is given by \( \sqrt{\hat{p}/(se(\hat{p}))} = 2.23 \). This is still significant but the statistic is lower than the previous case.
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Appendix A Derivation of the Coefficient of Risk Aversion

This discussion closely follows arguments presented in Allen et al (2012). Following them, three levels of risk aversion are used, representing a conservative, a balanced and an aggressive investor. They also cite Kritzman (2011) who shows how we can infer the level of risk aversion using actual investor allocations, albeit in a different setting. For an investor that allocates investment to bonds \( b \) and equities \( e \), the constant absolute risk aversion utility function can be expressed as (where the symbols have the usual meanings):

\[
E(U) = x_e\mu_e + x_b\mu_b - \frac{\lambda}{2}(\sigma_e^2 w_e + \sigma_b^2 w_b + 2\rho\sigma_e\sigma_b w_e w_b)
\]

The marginal utility is maximised by equating the partial derivatives:

\[
\frac{\partial E(U)}{\partial x_e} = \mu_e - \lambda(\sigma_e^2 w_e + \rho\sigma_e\sigma_b w_b)
\]

\[
\frac{\partial E(U)}{\partial x_e} = \mu_b - \lambda(\sigma_b^2 w_b + \rho\sigma_e\sigma_b w_e)
\]

\[
\lambda = \frac{\mu_e - \mu_b}{\sigma^2 w_e + \rho\sigma_e\sigma_b w_b - \sigma_b^2 w_b - \rho\sigma_e\sigma_b w_e}
\]

To calibrate the above, Allen et al (2012) use long-dated US government bonds and the S&P500 since 1927. The average allocation for balanced US mutual funds is 50 per cent equities and 50 per cent fixed income and cash. They use the definitions from Morningstar, the fund rating service, which define a conservative (aggressive) fund as a 20 to 50 per cent (70 to 90 per cent) allocation to equities, and a 50 to 80 per cent (10 to 30 per cent) allocation to fixed income and cash. They use the mid-points of these ranges to give the allocations of the conservative and aggressive investors. The resulting levels of \( \lambda \) are 0.015, 0.025, and 0.045 respectively.

Appendix B Derivation of \( p \)

Solving for equation (0.17), let \( \frac{\omega_A - \omega_E}{\omega_E} = k \), we can then rewrite

\[
k \left( \sigma_A^2((\lambda_C^{-1} + \lambda_I^{-1})\mu_E - r_f) - \sigma_{AE}((\lambda_C^{-1} + \lambda_I^{-1})\mu_A + p/\lambda_C - r_f) \right) = \\
\sigma_E^2((\lambda_C^{-1} + \lambda_I^{-1})\mu_A + p/\lambda_C - r_f) - \sigma_{AE}((\lambda_C^{-1} + \lambda_I^{-1})\mu_E - r_f)
\]
\[ \sigma_E^2 p \frac{p}{\lambda C} + k \sigma_{AE} \frac{p}{\lambda C} = k \sigma_A^2 (\lambda C^{-1} + \lambda I^{-1}) \mu_E - \sigma_E^2 ((\lambda C^{-1} + \lambda I^{-1}) \mu_A - k \sigma_A^2 r_f) \]
\[ - k \sigma_{AE} (\lambda C^{-1} + \lambda I^{-1}) \mu_A + k \sigma_{AE} r_f + \sigma_{AE} (\lambda C^{-1} + \lambda I^{-1}) \mu_E - \sigma_{AE} r_f \]

\[ \frac{p}{\lambda C} (\sigma_E^2 + k \sigma_{AE}) = (k \sigma_A^2 + \sigma_{AE}) \left( (\lambda C^{-1} + \lambda I^{-1}) \mu_E - r_f \right) - (\sigma_E^2 + k \sigma_{AE}) \left( (\lambda C^{-1} + \lambda I^{-1}) \mu_A - r_f \right) \]

\[ p = \frac{(k \sigma_A^2 + \sigma_{AE})}{(\sigma_E^2 + k \sigma_{AE})} \left( (1 + \frac{\lambda C}{\lambda I}) \mu_E - \lambda C r_f \right) - \left( (1 + \frac{\lambda C}{\lambda I}) \mu_A - \lambda C r_f \right) \]

**Appendix C Delta Method**

We can obtain the gradient vector with the partial derivatives of \( p \) with respect to \( \phi = (\hat{\mu}_A, \hat{\mu}_E, \hat{\sigma}_A^2, \hat{\sigma}_E^2, \hat{\sigma}_{AE}) \) which will be denoted by \( G' \):

\[
G' = \begin{pmatrix}
\frac{\partial p}{\partial \hat{\mu}_A} \\
\frac{\partial p}{\partial \hat{\mu}_E} \\
\frac{\partial p}{\partial \hat{\sigma}_A^2} \\
\frac{\partial p}{\partial \hat{\sigma}_E^2} \\
\frac{\partial p}{\partial \hat{\sigma}_{AE}}
\end{pmatrix}
\]

From equation (0.18), we can obtain this matrix as follows:

\[ \frac{\partial p}{\partial \hat{\mu}_A} = - \left( 1 + \frac{\lambda C}{\lambda I} \right) \]

\[ \frac{\partial p}{\partial \hat{\mu}_E} = \frac{(k \sigma_A^2 + \sigma_{AE})}{(\sigma_E^2 + k \sigma_{AE})} \left( 1 + \frac{\lambda C}{\lambda I} \right) \]

\[ \frac{\partial p}{\partial \hat{\sigma}_A^2} = k \left( \frac{1 + \frac{\lambda C}{\lambda I}}{\sigma_E^2 + k \sigma_{AE}} \right) \mu_E - \lambda C r_f \]

\[ \frac{\partial p}{\partial \hat{\sigma}_E^2} = - \frac{(k \sigma_A^2 + \sigma_{AE})}{(\sigma_E^2 + k \sigma_{AE})^2} \left( \left( 1 + \frac{\lambda C}{\lambda I} \right) \mu_E - \lambda C r_f \right) \]

\[ \frac{\partial p}{\partial \hat{\sigma}_{AE}} = \frac{1}{(\sigma_E^2 + k \sigma_{AE})} \left( 1 - k \frac{(k \sigma_A^2 + \sigma_{AE})}{(\sigma_E^2 + k \sigma_{AE})} \right) \left( \left( 1 + \frac{\lambda C}{\lambda I} \right) \mu_E - \lambda C r_f \right) \]
Appendix D The Matrix $\Sigma \phi$ 

\[
\begin{pmatrix}
\hat{\mu}_A & \hat{\mu}_E & \hat{\sigma}^2_A & \hat{\sigma}^2_E & \hat{\sigma}_{AE} \\
\hat{\mu}_A & 0 & 0 & 0 & 0 \\
\hat{\mu}_E & 0 & 0 & 0 & 0 \\
\hat{\sigma}^2_A & 0 & 0 & 0 & 0 \\
\hat{\sigma}^2_E & 0 & 0 & 0 & 0 \\
\hat{\sigma}_{AE} & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

Appendix E Interdependence between Art and Equity

Given the true model is

\[y_t = \alpha y_{t-1} + v_t\]

\[v_t \sim (0, \sigma^2)\]

With respect to the true model,

\[y_t - \hat{\alpha}y_{t-1} = \hat{v}_t\]

\[\text{plim} \sum_{t=1}^{T} \frac{\hat{v}_t^2}{T} = \sigma^2\]

Suppose we assume $y_t$ to follow an iid process incorrectly such that

\[y_t = v_t\]

\[\text{Var}(v_t) = \sigma^2\]

\[\text{plim} \sum_{t=1}^{T} \frac{(y_t - \bar{y})^2}{T - 1} = \frac{\sigma^2}{1 - \alpha} > \sigma^2\]
Table 0.1: Ratio of Art to Equity Investment under the Portfolio Construction Approach

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \sigma_E^2(\mu_A + p) - \sigma_AE\mu_E )</th>
<th>( \sigma_A^2\mu_E - \sigma_AE(\mu_A + p) )</th>
<th>( \frac{\omega_A}{\omega_E} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.076</td>
<td>0.923</td>
<td>0.082</td>
</tr>
<tr>
<td>0.015</td>
<td>0.095</td>
<td>0.904</td>
<td>0.105</td>
</tr>
<tr>
<td>0.02</td>
<td>0.113</td>
<td>0.886</td>
<td>0.127</td>
</tr>
<tr>
<td>0.03</td>
<td>0.147</td>
<td>0.852</td>
<td>0.173</td>
</tr>
<tr>
<td>0.04</td>
<td>0.179</td>
<td>0.820</td>
<td>0.219</td>
</tr>
<tr>
<td>0.05</td>
<td>0.210</td>
<td>0.789</td>
<td>0.266</td>
</tr>
<tr>
<td>0.1</td>
<td>0.340</td>
<td>0.659</td>
<td>0.516</td>
</tr>
<tr>
<td>0.15</td>
<td>0.442</td>
<td>0.557</td>
<td>0.794</td>
</tr>
<tr>
<td>0.2</td>
<td>0.524</td>
<td>0.475</td>
<td>1.102</td>
</tr>
<tr>
<td>0.25</td>
<td>0.591</td>
<td>0.408</td>
<td>1.448</td>
</tr>
<tr>
<td>0.3</td>
<td>0.647707</td>
<td>0.352</td>
<td>1.838</td>
</tr>
</tbody>
</table>

Table 0.2: Ratio of Art to Equity Investment under the Equilibrium Approach

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \sigma_E^2d_1 - \sigma_AE d_2 )</th>
<th>( \sigma_A^2d_2 - \sigma_AE d_1 )</th>
<th>( \frac{\omega_A}{\omega_E} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.028</td>
<td>0.735</td>
<td>0.039</td>
</tr>
<tr>
<td>0.005</td>
<td>0.041</td>
<td>0.732</td>
<td>0.056</td>
</tr>
<tr>
<td>0.01</td>
<td>0.052</td>
<td>0.729</td>
<td>0.071</td>
</tr>
<tr>
<td>0.015</td>
<td>0.064</td>
<td>0.727</td>
<td>0.088</td>
</tr>
<tr>
<td>0.02</td>
<td>0.076</td>
<td>0.724</td>
<td>0.105</td>
</tr>
<tr>
<td>0.1</td>
<td>0.265</td>
<td>0.683</td>
<td>0.388</td>
</tr>
<tr>
<td>0.2</td>
<td>0.502</td>
<td>0.633</td>
<td>0.794</td>
</tr>
<tr>
<td>0.3</td>
<td>0.739</td>
<td>0.582</td>
<td>1.270</td>
</tr>
</tbody>
</table>

Table 0.3: Estimation of VAR(2)

<table>
<thead>
<tr>
<th></th>
<th>( r_{A,t} )</th>
<th>( r_{E,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{A,t-1} )</td>
<td>-0.521 (0.096)</td>
<td>0.007 (0.046)</td>
</tr>
<tr>
<td>( r_{A,t-2} )</td>
<td>-0.266 (0.095)</td>
<td>0.018 (0.045)</td>
</tr>
<tr>
<td>( r_{E,t-1} )</td>
<td>0.422 (0.207)</td>
<td>0.047 (0.099)</td>
</tr>
<tr>
<td>( r_{E,t-2} )</td>
<td>0.272 (0.214)</td>
<td>-0.110 (0.103)</td>
</tr>
<tr>
<td>( a )</td>
<td>-0.004 (0.041)</td>
<td>0.044 (0.019)</td>
</tr>
<tr>
<td>R squared</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

Bibliography