January 29, 2007

# The Effect of Inflation on Market Participation and Search Intensity

Xiuhua Huangfu[*]
University of Toronto

---

[*] This paper is part of my doctoral dissertation at the University of Toronto. I am grateful to my thesis supervisor Miquel Faig for an enormous guidance and inspiration. I thank Shouyong Shi, Randall Wright and seminar participants at the University of Toronto and the Summer Workshop on Money, Banking and Payments at the Federal Reserve Bank of Cleveland for very helpful comments. All remaining errors are my own.

## Abstract

Inflation changes individuals' trading behavior and, as clearly pointed out by Lucas (2000), one of the reasons is that inflation induces individuals to spend extra efforts to get rid of their money holdings. The two alternative margins that have been used to model this effect are the extensive margin (e.g., frequency of shopping per period) and the intensive margin (e.g., average shopping time per trip). This paper investigates the effect of inflation on these two margins. It deviates from the previous literature by allowing individuals to vary both margins simultaneously. I conclude that if the returns to search are strongly decreasing, individuals carry money and participate in the market every day throughout a period. Therefore, inflation can affect only the intensive margin. If the returns to search are close to constant, individuals find it optimal to exert all their efforts on shopping on each trip, but not to go shopping every day. Hence, inflation varies only the extensive margin. There exists an intermediate region of returns to search where inflation affects both margins.

# 1 Introduction

Inflation changes individuals' trading behavior and, as clearly pointed out by Lucas (2000), one of the reasons is that inflation induces individuals to make an extra effort to get rid of their money holdings. The two alternative margins that have been used to model this effect are the *extensive margin* (frequency of shopping per period) and the *intensive margin* (average shopping time per trip). While the *extensive margin* reveals the frequency that individuals participate in the market, the *intensive margin* measures the effort with which market participants search for trade opportunities. This paper investigates the effect of inflation on these two margins. It deviates from the previous literature by allowing individuals to vary both margins simultaneously. The analysis generates interesting implications for the impact of inflation on the velocity of the circulation of money, and social welfare.

The study of individuals' trading behavior and its implication has generated a considerable amount of research. However, individuals' behavior in the previous literature has been modelled in such a way that only one margin can be adjusted.[1] This restriction raises several related questions that provide the motivation for this paper. Are individuals willing to vary only one margin? If so, which margin is relevant? What would be the effect of inflation if individuals were allowed to vary both margins? Would the outcome be substantially different from the outcomes of models in which individuals vary only one margin?

This paper shows that the impact of inflation on these two margins is determined by the interaction between the opportunity cost of holding money (which depends on the level of inflation) and the direct costs of searching (which depends on the properties of the returns to search). Specifically, if the returns to search are strongly decreasing, individuals carry

---

[1] For example, in both Berentsen, Rocheteau and Shi (2006), and Lagos and Rocheteau (2005), it is assumed that individuals always go shopping. As a result, only the intensive margin could possibly be affected by inflation. In contrast, Lagos and Wright (2005) assume that individuals either do not participate in the market or shop full time. Therefore, inflation can only change the extensive margin.

money and participate in the market every day throughout a period. Therefore, inflation can change only the intensive margin. If the returns to search are close to constant, individuals find it optimal to exert all their efforts on shopping on each trip, but not to go searching every day. There exists an intermediate region of the returns to search where inflation affects both margins.

The basic intuition is as follows: When the nominal interest rate is positive,[2] there is a complementarity between the effort with which an individual searches for trade opportunities and the money balances she carries. As the individual increases her search effort, the probability of finding a trading partner rises. As a result, it is optimal to increase the money balances to extract maximum benefit from the trade. Likewise, due to the opportunity cost of carrying money, a large amount of money balances increases the incentive to search more intensively. This positive complementarity determines that when making decisions about their trading behavior, individuals should take into account both the opportunity cost of carrying money, and the direct costs of searching. In some cases, this complementarity leads to extreme optimal choices. That is, individuals may spend all their endowed efforts on searching or may spend no effort at all.

To see how the direct costs of searching interact with the opportunity cost of carrying money, consider an individual who gets paid at the beginning of each month; she faces two options in regard to her purchasing behavior: during that month, she could carry certain money balances and spend a fraction of her endowed effort on shopping every day. In this case, since the individual goes shopping every day, the extensive margin is 1. The intensive margin is strictly positive but less than 1, as on each trip the individual exerts only part of her effort on shopping. Alternatively, the individual could go shopping a few days during that month, but spends all her effort on shopping on each trip. In that case, the intensive margin is 1, but the extensive drops below 1.

The pros and cons associated with each option are the following: Under option 1, the individual searches less intensively on each trip (than under option 2), which leads to a

---

[2] The complementarity diminishes at a zero nominal interest-rate level as there is no opportunity cost of carrying money. Consequently, the individual's money balance is undetermined.

lower probability of finding a trading partner. As a result, the individual is exposed to a higher opportunity cost of carrying money. However, depending on the returns to search, the individual might face higher direct costs of searching under option 2. Specifically, if the returns to search are decreasing strongly, it is not optimal to search "too much" in a given day. Therefore, the individual is better off spreading the purchasing activities evenly throughout the month, regardless of the inflation rate. Instead, if the returns of search are close to constant, then the individual is primarily concerned with the opportunity cost of carrying money. When this is the case, specializing in shopping for a few days during the month allows the individual to avoid carrying money every day, and it certainly dominates option 1. To summarize, when deciding her trading behavior, the individual compares the benefits and the costs of the two alternatives. The outcome is determined by the relative importance of the opportunity cost of carrying money and the direct costs of searching.

The contribution of this paper is to develop a model in which individuals can vary both the intensive and the extensive margins simultaneously. The model predicts that the effect of inflation on these two margins is determined by the interaction between the properties of the returns to search and the level of inflation.

The rest of the paper is organized as follows. Section 2 describes the model and Section 3 studies individuals' optimal behavior. Section 4 analyzes equilibrium conditions and comparative statics. Quantitative analysis is conducted in Section 5 to examine the effects of inflation on the velocity of money, and social welfare. Finally, Section 6 concludes.

# 2 The Model

## 2.1 The Environment

The environment is based on the competitive pricing framework introduced in Rocheteau and Wright (2005). However, the quasi-linear preference assumption they adopt typically gives rise to equilibria in which individuals only want to vary one margin. Therefore, I replace that assumption by the village structure as introduced in Faig (2005).[3]

The economy is composed of a continuum of measure one of symmetric villages. Inside each village, there is a continuum of measure 2 of individuals equally divided into two types that differ in terms of their preferences to be specified below. It is convenient to call them *buyers* and *sellers*. Inside their villages, individuals know each other. Outside their villages, individuals can easily hide their true identity to be anonymous.

The goods are non-durable. Individuals from different villages produce different goods, and each village is characterized by one specific good. While everyone is able to produce the good specific to his/her own village, buyers want to consume every day the good produced in their own village as well as one specific good from another village (but which specific good to consume differs randomly across time and across buyers). Sellers only consume the good produced in their village of origin.

Time is discrete, and the horizon is infinite. Each period is divided into two subperiods: morning and afternoon. In the morning, individuals from different villages trade goods specific to each village. As the good is traded inside the village where it is produced, it takes a seller no effort to find a trading partner. However, a buyer has to search for the appropriate market place of the good she wants to consume.

Each individual is endowed with 1 unit of labor every day. Since there is no disutility associated with production or trading activities, everyone inelastically supplies all his/her endowed labor. In the morning of date $t$, a buyer spends a fraction $s_t$ of her effort on searching for the market trading the good she would like to consume. Following the same terminology adopted by previous literature, I name $s_t$ *search intensity*, and $s_t \in [0, 1]$. With probability $h(s_t)$ the buyer successfully finds her targeted market, and with the complementary probability she fails to do so. The matching function $h$ is increasing and concave, with $h(0) = 0$ and $h(1) \leq 1$. I define $n_t$ as the fraction of buyers who search with positive intensities and carry positive money balances.[4] $n_t$ is named the *participation rate*,

---

[3] An alternative would be the large-households framework proposed by Shi (1997).

[4] As will become clear later, due to the opportunity cost of holding money, if the buyer decides not to search in the morning, it is not optimal to bring any money.

and $n_t \in [0, 1]$.

A buyer who successfully reaches the targeted market trades competitively with other market participants at a price of $p_t$. As a result of the trade, the seller produces $q_t^s$, the buyer consumes $q_t^b$, and money changes hands from the buyer to the seller. For simplicity, a linear production technology is assumed here. The seller produces the good at a time cost of $l_t$ and $l_t = q_t^s/w$, where $w$ is the output produced per unit of time. The competitive market closes when the morning ends.

During the afternoon of date $t$, both buyers and sellers want to consume the good produced in their own village, and individuals (buyers and sellers) trade with their fellow villagers inside the village in a competitive good market. The market is centralized and frictionless. The price of the good at each date is normalized to 1, and the relative price of money is denoted $\phi_t$.

The effort spent on production in the afternoon differs across individuals. For a seller, his available effort is $1 - l_t$, and, therefore, the real production revenue is $(1 - l_t)w$. Instead, a buyer's real production revenue is $(1 - s_t)w$. The competitive goods market closes when the afternoon ends.

Certainly, there is an opportunity cost for the buyer to search in the morning market: the production revenue that has to be sacrificed. As the buyer increases her search intensity, less time is allocated to production. This opportunity cost plays the same role as the disutility of searching in some other papers on similar topics (e.g., Lagos and Rocheteau (2005)). It captures that in our daily life the time we spend on shopping cannot be spent on producing goods.

Despite the competitive nature of the morning and afternoon markets, there is a role for money because in the morning individuals are anonymous in the markets and there is a lack of double coincidence of wants. Money is an intrinsically useless, perfectly divisible, and storable asset. The money supply grows at a constant factor $\gamma$; that is, $M_{t+1} = \gamma M_t$, where $M_t$ is the quantity of money per buyer at date $t$. New money is injected via lump-sum transfers to buyers at the beginning of each afternoon.

As in Faig (2005), individuals are not risk neutral, and they are allowed to buy insurance in their village against the risks affecting their trading opportunities in the morning market. Specifically, during the afternoon of date $t$,[5] buyers can purchase a contract for the delivery of $I_{t+1}$ dollars the next afternoon contingent upon reaching the targeted market tomorrow morning.[6] The fair premium of acquiring the insurance contract is $I_{t+1}h(s_{t+1})$. For notational ease, the premium is also paid the next afternoon.

In addition to insurance contracts, credit is also available every afternoon inside the village.[7] Individuals can issue one-period risk-free nominal bonds that promise $(1 + i_t)$ units of money the next afternoon, where $i_t$ is the nominal interest rate. These bonds are traded in competitive markets inside the village in the afternoon at a price of 1. Outside the village, these personal bonds are not valued as individuals from other villages do not know the issuer of these bonds. Initial bond holdings are assumed to be zero.

To summarize, in the afternoon of date $t$, a buyer chooses the quantity to be consumed of the good from her own village $(x_t^b)$ and the demand for bonds $(b_t^b)$. At the same time, since there is no new information arriving during the night, the buyer also chooses the intensity in which she would search for the targeted market $(s_{t+1})$ the next morning, and the demand for money $(m_{t+1})$. Finally, the buyer decides the quantity of a specific good to be consumed $(q_{t+1}^b)$ and purchases an insurance contract which delivers $I_{t+1}$ dollars the next afternoon if her search is successful. New money is also injected to buyers in the afternoon. Instead, a seller chooses the demand for bonds $(b_t^s)$ and the good from his own village $(x_t^s)$. In addition, the seller decides the next morning's output $(q_{t+1}^s)$.

The one-period utility of a buyer and a seller at date $t$ are, respectively, $U_t^b = v(x_t^b) + u(q_t^b)$ and $U_t^s = v(x_t^s)$. The functions $u$ and $v$ are bounded, continuously differentiable, increasing and concave. Moreover; $v(0) = u(0) = 0$. The objective of individuals is to maximize their expected lifetime utility: $E_0 \sum_{t=0}^{\infty} \beta^t U_t^j$, where $j = b, s$. The discount factor $\beta \in (0, 1)$, and

---

[5] These insurance contracts are not feasible in the morning as agents are anonymous outside their villages.

[6] A linear production technology and a competitive market imply that the seller's trading surplus is zero in the morning. Therefore, sellers face no risks in the morning market.

[7] The combination of credit and insurance enables individuals to rebalance their portfolio every period, and this leads to a tractable model of divisible money in a search environment.

the money growth factor $\gamma$ is greater than $\beta$.

The paper focusses on stationary, monetary competitive equilibria. In a competitive equilibrium, individuals maximize utility taking the sequence of prices of the goods $(p_t)$, prices of the money $(\phi_t)$, the nominal interest rate $(i_t)$ and the insurance premia $(I_t)$ as given. Buyers also take as given lump-sum transfers from the government. The equilibrium is monetary if money is valued. The equilibrium is stationary if the quantity of goods consumed in the morning $(q^b)$ and the quantity of goods traded in the afternoon $(x^b, x^s)$ are constant over time. The time subscript $t$ is omitted and $t+1$ is shortened to $+1$, etc., in what follows.

In a stationary equilibrium, the following two results ought to hold: First, the nominal prices of the goods traded in the morning and the afternoon markets must grow at the same rate as the money supply. That is, $p_{+1}/p = \phi/\phi_{+1} = \gamma$. Second, the real interest rate on bonds $(r)$ must be equal to the subjective discount rate: $r = 1/\beta - 1$; otherwise, consumption would grow or decline over time. Consequently, $i = (1+r)\gamma - 1 = (\gamma - \beta)/\beta$.

## 2.2 The Behavior of Sellers

A representative seller who starts the afternoon at date $t$ with real wealth $a_0^s$ chooses $\left\{x^s, b^s, q_{+1}^s, l_{+1}\right\}$ to solve the following maximization problem:

$$V^s(a_0^s) = \max \ v(x^s) + \beta V^s(a_1^s), \tag{1}$$

subject to the budget constraint $a_0^s = b^s\phi + x^s$ and the production possibility constraint $q_{+1}^s = l_{+1}w$. The term $a_1^s$ is the seller's next afternoon's real wealth, and

$$a_1^s = b^s(1+i)\phi_{+1} + w. \tag{2}$$

Due to the linear production technology, the arbitrage condition determines that in equilibrium $p_{+1} = 1/\phi_{+1}$. Hence, the seller is indifferent between producing in the morning or in the afternoon, and $q_{+1}^s$ are undetermined.

The seller's demand for the good produced in his own village is equal to the sum of the interest payment of his initial wealth and the production revenue $w$. For simplicity, I assume the seller's initial wealth is 0. This implies the seller consumes all of his output; that is, $x^s = w$.[8]

## 2.3 The Behavior of Buyers

A representative buyer who starts the afternoon at date $t$ with real wealth $a_0^b$ faces the following constraints:

$$a_0^b = (m_{+1} + b^b)\phi + x^b, \text{ and} \tag{3}$$

$$p_{+1} q_{+1}^b \leq m_{+1}. \tag{4}$$

Constraint (3) is the buyer's budget constraint,[9] and (4) simply says the buyer cannot pay more money than she has got.

The buyer's optimal choice of $\left\{x^b, b^b, s_{+1}, m_{+1}, q_{+1}^b, I_{+1}\right\}$ solves the following maximization problem:

$$\begin{aligned} V^b(a_0^b) &= \max\ v(x^b) + \beta h(s_{+1})[u(q_{+1}^b) + V^b(a^{b1})] \\ &\quad +\beta[1 - h(s_{+1})]V^b(a^{b0}), \end{aligned} \tag{5}$$

subject to (3) and (4). The terms $a^{b1}$ and $a^{b0}$ denote, respectively, the real wealth next afternoon contingent on the success of reaching the targeted market (superscript $b1$) or not (superscript $b0$). Therefore,

$$a^{b1} = [m_{+1} + b^b(1 + i) + I_{+1} + \tau_{+1} - p_{+1} q_{+1}^b - I_{+1} h(s_{+1})]\phi_{+1} + (1 - s_{+1})w, \text{ and} \tag{6}$$

$$a^{b0} = a^{b1} + (p_{+1} q_{+1}^b - I_{+1})\phi_{+1}, \tag{7}$$

---

[8] To understand the result, suppose $x^s < w$, then the seller will be strictly better off by consuming more of his own output. If instead $x^s > w$, this implies the seller has borrowed at an interest rate of $i$, this would reduce the seller's next period consumption of $x$.

[9] Indeed, $a_0^b = \tilde{a}_0^b + (1 - s)w$. $\tilde{a}_0^b$ is the buyer's physical capital, which includes an unspent money balance, bonds, insurance payment and government transfer. $(1 - s)w$ is the buyer's human capital. Since $\tilde{a}_0^b$ and $s$ are both state variables, we use $a_0^b$ to simplify notations.

where $\tau_{+1}$ is the lump-sum transfer from the government, growing at a constant rate $\gamma$.

In an environment as described above, two results are immediate: First, the gross real rates of return on bonds and money are $\frac{1}{\beta}$ and $\frac{1}{\gamma}$, respectively. Therefore, the assumption that $\gamma > \beta$ implies that bonds earn a higher return than money. As a result, a buyer who searches with zero intensity should also not carry any money balances. Second, the buyer must fully insure against the risk on trading opportunities. Hence, it is optimal for the buyer to purchase contracts that satisfy: $I_{+1} = p_{+1}q^b_{+1}$. With full insurance, the buyer's next-afternoon real wealth is independent of the random success of finding her targeted market, and $a^{b1} = a^{b0} \equiv a^b_1$.

Based on all these results, the buyer's maximization problem can be simplified into:

$$V^b(a^b_0) = \max_{\left\{x^b, s_{+1}, q^b_{+1}\right\}} v(x^b) + \beta[h(s_{+1})u(q^b_{+1}) + V^b(a^b_1)], \tag{8}$$

subject to

$$a^b_1 = \frac{a^b_0 - x^b}{\beta} + \frac{\tau_{+1}}{p_{+1}} - q^b_{+1}i - h(s_{+1})q^b_{+1} + (1 - s_{+1})w. \tag{9}$$

To solve (8), denoting $\mu$ as the Lagrangian multiplier associated with the budget constraint, I obtain:

$$\begin{aligned} V^b(a^b_0) &= \max_{\left\{x^b\right\}} v(x^b) - \mu(a^b_1 - \frac{a^b_0 - x^b}{\beta} - \frac{\tau_{+1}}{p_{+1}}) + \beta V^b(a^b_1) \\ &\quad + \max_{\left\{q^b_{+1}, s_{+1}\right\}} h(s_{+1})[\beta u(q^b_{+1}) - \mu q^b_{+1}] - \mu q^b_{+1}i + \mu(1 - s_{+1})w. \end{aligned} \tag{10}$$

The buyer's optimal choice of $x^b$ must equate the marginal utility of consumption to the marginal cost:

$$\beta v'(x^b) = \mu. \tag{11}$$

Program (10) implies that the buyer's optimal plan of $\left\{x^b\right\}$ and $\left\{q^b_{+1}, s_{+1}\right\}$ are separable. As this paper focuses on buyers' choices of search intensities, I study the following equivalent maximization problem: (in a stationary equilibrium, $q^b = q^b_{+1}$ and $s = s_{+1}$; therefore, all time subscripts are dropped hereafter. I also omit the superscript $b$ when no confusion could result.)

$$\max_{\{q,s\}} h(s)[\beta u(q) - \mu q] - \mu q i + \mu(1-s)w, \tag{12}$$

subject to the constraints $q \geq 0$ and $s \in [0,1]$.

Program (12) provides a simpler way of interpreting the buyer's optimization problem. In the afternoon of date $t$, the expected payoff for a buyer who chooses to search with intensity $s$ during the next morning is her expected trading surplus (the first term) plus the discounted production revenue (the last term).[10] The expected trading surplus equals the probability of finding the targeted market $h(s)$ multiplied by the buyer's surplus discounted back to date $t$. In addition, there is an opportunity cost of carrying money (the second term). As the buyer exhausts her money balance, the opportunity cost is determined by the discounted value of payment $\mu q$ and the interest rate $i$.

# 3 Solving Buyers' Maximization Problem

In this section, I show that the solution to program (12) is determined by the interaction between the shape of the matching function $h(s)$ and the inflation rate $\gamma$.

**Lemma 1**. *When $i$ is strictly positive, program (12) is not concave.*

Due to the nonconcavity of objective function (12), both an interior solution (i.e., $\{q > 0, 0 < s < 1\}$) and corner solutions (i.e., $\{q = 0, s = 0\}$ or $\{q > 0, s = 1\}$) are possible. However, the difficulty in solving the buyer's maximization problem is to keep track of the many possibilities caused by the nonconcavity. To facilitate this task, I specialize the matching function $h(s)$ and the utility function $u(q)$ to be isoelastic. This functional form is widely used in economics.

**Proposition 1.** *Suppose $h(s) = A\frac{s^{1-\alpha}}{1-\alpha}$, $u(q) = \frac{q^{1-\theta}}{1-\theta}$, where $\alpha \in (0,1)$ and $\theta \in (0,1)$. For a given inflation rate $\gamma$, the solution to the buyer's maximization problem is determined by*

---

[10] $\mu$ is the discounted marginal utility of wealth as $\mu = \beta V^{b\prime}(a_1^b)$.

*the value of* $\alpha$ *and* $\theta$*.* (a) *If* $\alpha+\theta>1$*, there exists at most one interior solution of* $(q,s)$*. (b) If* $\alpha+\theta<1$*, there exists a critical value of* $\mu$*, called* $\mu^*$*, such that*

$$(q,s)=\begin{cases}(0,0) & \text{if } \mu>\mu^* \\ (q_1,s_1) & \text{if } \mu<\mu^* \\ (0,0) \textit{ or } (q_2,s_2) & \text{if } \mu=\mu^*\end{cases}, \textit{where } q_1>0,\ q_2>0,\ s_1\in(0,1),\ s_2\in(0,1].^{11}$$

As indicated by Proposition 1, if $\alpha+\theta>1$, program (12) is strictly concave so that there is a unique solution, which is interior. That is, every buyer searches with positive intensity (but less than 1). As a result, the participation rate is one, and only the search intensity could be adjusted as the inflation rate fluctuates. This interior solution is commonly assumed (and therefore studied) by most of the search literature.[12] Intuitively, for a given $\alpha$, the bigger the curvature parameter of the utility function $\theta$ is, the more risk averse the buyer is. Therefore, she prefers a more smoothed consumption profile and would like to participate in the search market every day. As $\alpha$ increases, the marginal returns to search decrease more rapidly. This provides another incentive for the buyer to spread the purchasing activities evenly throughout a period.

In the model of this paper, the nonconcavity of the objective function gives rise to a new possible solution: if $\alpha+\theta<1$, both the search intensity and the participation rate could be adjusted. I will focus on that case in what follows. Apparently, the marginal utility of wealth $\mu$ is the key variable in determining the solution to the buyer's maximization problem. As will be seen, $\mu$ serves several functions in the model. For each buyer, $\mu$ is such that her intertemporal budget constraint is binding. In equilibrium, $\mu$ is determined by the good market-clearing condition. Different values of $\mu$ result in different choices of $\{q,s\}$.

---

[11] In the limiting case where $\alpha=0$, there exists a critical value of $\mu$, called $\hat{\mu}^*$, such that $(q,s)=\begin{cases}(0,0) & \text{if } \mu>\hat{\mu}^* \\ (0,0) \text{ or } (\hat{q},1) & \text{if } \mu=\hat{\mu}^* \\ (\hat{q},1) & \text{if } \mu<\hat{\mu}^*\end{cases}$, where $\hat{q}>0$.

[12] This concavity also appears in Rocheteau and Wright (2005). However, the quasi-linear preference in their model rules out a generic equilibrium in which buyers are indifferent between $(0,0)$ and $(q^*,s^*)$. That is, the only monetary equilibrium is that every buyer searches with strictly positive intensity (but less than 1), and that corresponds to the case of $\alpha+\theta>1$ in this paper.

Intuitively, with a high marginal utility of wealth, staying at home producing certainly dominates the benefit of going searching. Therefore, no buyer wants to go shopping. In contrast, a low value of $\mu$ makes searching for a good more attractive than producing. As a result, every buyer searches with positive intensity. For each inflation rate level, there exists one $\mu^*$ such that when $\mu$ equals $\mu^*$, a buyer is indifferent between going searching or staying at home producing. In the following subsections, I will characterize the buyer's optimal choices of $(q, s)$ in detail.

## 3.1 If $\mu < \mu^*$: Intensive Search

For any given inflation rate $\gamma$, if $\mu$ is less than $\mu^*$, there exists a unique interior solution of $(q, s)$. When this is the case, I say that buyers search intensively as their search intensities are strictly positive.

When buyers search intensively, the optimal plan $(q_1, s_1)$ that solves maximization program (12) is characterized by the following set of first-order conditions:

$$\frac{\beta u'(q_1)}{\mu} = 1 + \frac{i}{h(s_1)}, \text{ and} \tag{13}$$

$$h'(s_1)[\beta u(q_1) - \mu q_1] = \mu w. \tag{14}$$

Condition (13) equates the marginal benefit and the marginal cost of acquiring money. Condition (14) states that the optimal search intensity should balance the marginal benefit and the marginal cost of searching. The benefit (LHS) is the higher probability of a trade meeting, and the cost (RHS) is the production revenue that has to be sacrificed.

## 3.2 If $\mu = \mu^*$: Extensive/Extreme Search

If $\mu$ equals $\mu^*$, choosing $(q_2, s_2)$ or $(0, 0)$ yields the same utility to the buyer. I study the cases in which $s_2 \in (0, 1)$ and $s_2 = 1$ separately.

When $s_2 \in (0, 1)$, I say that buyers search extensively. The following conditions that characterize $(q_2, s_2, \mu)$ should be satisfied:

$$\frac{\beta u'(q_2)}{\mu} = 1 + \frac{i}{h(s_2)}, \tag{15}$$

$$h'(s_2)[\beta u(q_2) - \mu q_2] = \mu w, \text{ and} \tag{16}$$

$$h(s_2)[\beta u(q_2) - \mu q_2] - \mu q_2 i + (1 - s_2)\mu w = \mu w. \tag{17}$$

Conditions (15) and (16) are identical to conditions (13) and (14). In addition, condition (17) requires the benefit of going searching (LHS) equal to the benefit of staying at home producing (RHS).

When $s_2 = 1$, the buyer is indifferent between searching with full intensity or staying at home producing; and, I call this an extreme search. Now the following conditions characterize the buyer's optimal behavior:

$$\frac{\beta u'(q_2)}{\mu} = 1 + \frac{i}{h(1)}, \text{ and} \tag{18}$$

$$h(1)[\beta u(q_2) - \mu q_2] - \mu q_2 i = \mu w. \tag{19}$$

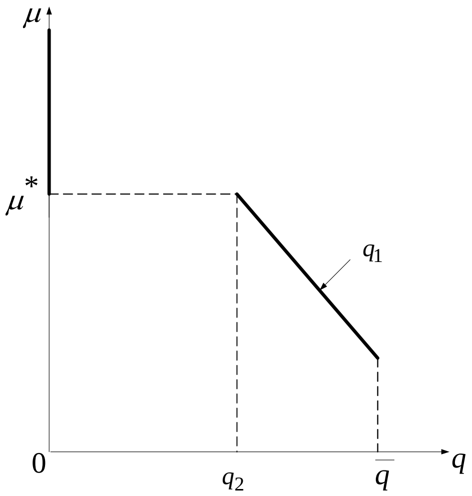
Conditions (18) and (19) are analogous to (15) and (17) replacing 1 for $s_2$.

For a given $\gamma$, if the marginal utility of wealth $\mu$ is greater than $\mu^*$, the benefit of staying at home producing certainly outweighs the expected payoff associated with search. If this were to happen for all buyers, the search market breaks down.

To summarize, Figure 1 shows how buyers' optimal choice of $q$ is determined. In the figure, $\bar{q}$ is the point where $\beta u'(\bar{q}) = \mu$. I restrict my attention to $q \in [0, \bar{q}]$, since buyers' marginal benefit of consumption would be negative for any $q$ greater than $\bar{q}$. As Proposition 1 points out, for each inflation rate $\gamma$ there exists one $\mu^*$ that makes buyers indifferent between $\{q_2, s_2\}$ and $\{0, 0\}$. If $\mu$ is greater than $\mu^*$, buyers are better off not searching; and if $\mu$ falls below $\mu^*$, it is optimal to choose $\{q_1, s_1\}$. Moreover, $q_1$ decreases with $\mu$ for a given $\gamma$.

Figure 1

Buyer's Optimal Choice of $q$



# 4 Equilibrium

In a competitive equilibrium, all buyers share the same value of $\mu$, which clears the goods market:

$$x^b + x^s + nh(s)q = (2 - ns)w.$$

By Walras's law, the equilibrium value of $\mu$ is also the $\mu$ that makes the buyers' budget constraints binding. Also, the money market must clear, and this determines the price of the good $p$ (which in turn determines the price of money $\phi$).

$$M = npq.$$

Finally, the lump-sum transfer $\tau$ must satisfy the government budget constraint:

$$\tau = (1 - \gamma^{-1})npq.$$

**Definition:** *A competitive equilibrium consists of:*

*(a) The buyer's optimal plan* $\{x^b, q, s\}$*, a marginal value of wealth* $\mu$*, and a market participation rate* $n$*.*

*(b) The seller's optimal plan* $\{x^s\}$*.*

*(c) The price of the goods in the search market* $p$*, the price of money* $\phi$*, and the nominal interest rate* $i$*.*

*(d) The government lump-sum transfer* $\tau$*,*

*such that,*

*(1) given* $\{\mu, p, i\}$*, the buyer's optimal plan solves the maximization problem (*$10$*).*

*(2) The seller consumes all his output,* $x^s = w$*.*

*(3)* $\{\mu, p, i\}$ *clears the goods market, the money market and the bond market.*

*(4)* $\tau$ *satisfies the government budget constraint.*

The existence of the monetary equilibrium requires that $\beta \leq \gamma \leq \bar{\gamma}$. The condition $\gamma \leq \bar{\gamma}$ ensures that the buyer's payoff from participating in the market $h(s)[\beta u(q) - \mu q] - \mu q i + (1 - s)\mu w$ is greater than or equal to the payoff from staying at home producing $\mu w$. As claimed in the previous section, the condition $\gamma \geq \beta$ ensures that the return on money does not dominate the return on bonds.

The analysis of the preceding sections shows that for any given inflation rate, three types of search (intensive, extensive and extreme search) are possible, depending on the value of $\mu$. Based on that result, the following proposition states that three different equilibria arise at different inflation levels, each corresponding to one type of search mentioned above. Buyers' trading behaviors are not identical across these three regimes.

**Proposition 2.** *In a stationary monetary equilibrium where* $\gamma > \beta$*, if the matching function satisfies* $\alpha + \theta < 1$ *and* $\alpha > 0$*, then there exist two critical levels of inflation, called* $\gamma_1$ *and* $\gamma_2$*, such that the following three types of equilibria are possible, depending on the level of* $\gamma$*. (a) If* $\gamma < \gamma_1$*,* $n = 1$*,* $(q, s) = (q_1, s_1)$*, where* $q_1 > 0$*,* $s_1 \in (0, 1)$*. (b) If* $\gamma_1 \leq \gamma < \gamma_2$*,* $n \in (0, 1]$*,* $(q, s) = (0, 0)$ *or* $(q_2, s_2)$*, where* $q_2 > 0$*,* $s_2 \in (0, 1)$*. (c) If* $\gamma \geq \gamma_2$*,* $n \in (0, 1)$*,* $(q, s) = (0, 0)$ *or* $(q_3, 1)$*, where* $q_3 > 0$*.*

This is a very intuitive result: as I mentioned earlier, due to the complementarity between the direct costs of searching (which depends on the shape of the matching function) and the opportunity cost of holding money (which depends on the level of inflation), the buyer takes both costs into account when deciding the trading behavior. With a weakly concave matching function, the opportunity cost of carrying money is negligible if the inflation rate is low ($\gamma < \gamma_1$). In that case, the concavity of the matching function determines that every buyer spends a fraction of her effort on searching, regardless of the inflation rates. I call this an *intensive-search equilibrium.* In contrast, as the inflation rate reaches a certain level ($\gamma \geq \gamma_2$), the opportunity cost of carrying money becomes the key determinant. Instead of participating in the search market every day, the buyer now either spends no time or spends all the endowed effort on searching. I call this an *extreme-search equilibrium.* If the inflation rate falls between $\gamma_1$ and $\gamma_2$, then the concavity of the matching technology and the opportunity cost of carrying money jointly determine that it is neither optimal for the buyer to carry money and participate in the search market every day, nor is it optimal to exert all the effort on searching on each shopping trip. Therefore, in equilibrium $s = 0$ or $s \in (0, 1)$, and I call this an *extensive-search equilibrium.*

Figure 2

Equilibrium Output

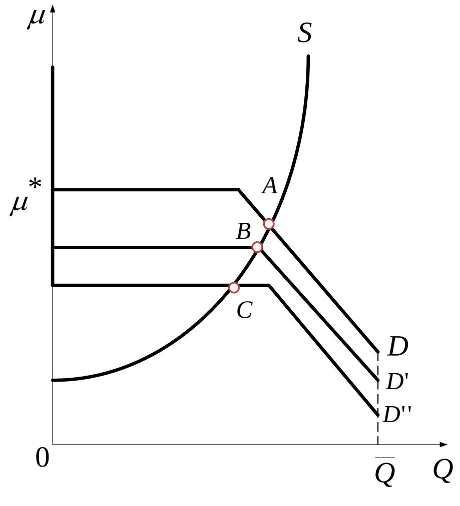


Figure 2 combines the buyer's optimal decision with the goods market-clearing condition.

It illustrates that three different types of equilibria arise at different inflation levels.

In the figure, the upward-sloping curve represents $S = 2w - x^b - x^s$, which can be interpreted as the aggregate supply of output in the morning search market. Curve $D$ represents the aggregate demand in the morning market at each marginal value $\mu$, and $D = n[h(s)q + sw]$. The horizontal part of the curve represents market demand when buyers search extensively (or extremely). At $\mu^*$, buyers are indifferent between going searching or not; therefore, some of them go searching in the morning market, while others spend all their endowed effort on producing. Hence, the aggregate demand at $\mu^*$ is perfectly elastic. The downward-sloping part of curve $D$ represents the market demand when buyers search intensively, and the vertical section corresponds to a non-monetary equilibrium in which $q = 0$. Curve $D$ shifts downwards as the inflation rate increases.

The intersections of $D$ and $S$ defines an equilibrium. In the figure, points $A$, $B$ and $C$ represent three alternative equilibria, which are intensive search, extensive search, and extreme search equilibrium, respectively. At point $A$, the equilibrium value of $\mu$ is lower than the $\mu^*$ that makes buyers indifferent between searching or not. In that case, buyers are strictly better off by spending a fraction of their endowed effort on searching. At both points $B$ and $C$, $\mu^*$ equals the market-clearing value of $\mu$; buyers are now indifferent between going searching or not. The participation rate is 1 at point $B$, and is less than 1 at point $C$.

The following subsections characterize the equilibrium allocations $(x^b, q, s, \mu, n)$ for each type of equilibrium.

## 4.1 Intensive Search Equilibrium

If $\gamma < \gamma_1$, an intensive-search equilibrium arises. The following equations characterize the equilibrium:

$$\frac{\beta u'(q_1)}{\mu_1} = 1 + \frac{i}{h(s_1)}, \tag{20}$$

$$h'(s_1)[\beta u(q_1) - \mu_1 q_1] = \mu_1 w, \tag{21}$$

$$\beta v'(x_1^b) = \mu_1, \tag{22}$$

$$x_1^b + h(s_1)q_1 = (1 - s_1)w, \text{ and} \tag{23}$$

$$n_1 = 1. \tag{24}$$

The system implies the following comparative statics:

$$\frac{\partial q_1}{\partial \gamma} < 0, \frac{\partial s_1}{\partial \gamma} \text{ ambiguous}, \frac{\partial n_1}{\partial \gamma} = 0, \frac{\partial x_1^b}{\partial \gamma} > 0, \ \frac{\partial \mu_1}{\partial \gamma} < 0.$$

Intuitively, an increase in $\gamma$ represents an inflation tax imposed on buyers who carry money and participate in the search market. Therefore, inflation reduces the demand for the specific good. Buyers switch the consumption from $q$ to $x^b$ as there is no inflation tax imposed on $x^b$, which leads to a decrease in $\mu_1$ (which is the discounted marginal utility of consuming $x^b$).

However, $\frac{\partial s_1}{\partial \gamma}$ cannot be unambiguously determined. As shown in the Appendix, $\frac{\partial s_1}{\partial \gamma}$ is positive (negative) at relatively low (high) inflation rates. The key to the effect of inflation on $s$ lies in the relationship between the benefit $(\beta u(q_1) - \mu_1 q_1)$ and the cost $(\mu_1 w)$ of searching defined by (21). Although both terms are decreasing with inflation, the decrease in the cost dominates at relatively low inflation rates. Therefore, individuals find it is still optimal to increase their search intensity, and vice versa.

## 4.2 Extensive Search Equilibrium

If $\gamma_1 \leq \gamma < \gamma_2$, the equilibrium values of $(x_2^b, q_2, s_2, \mu_2, n_2)$ are determined by the following set of conditions:

$$\frac{\beta u'(q_2)}{\mu_2} = 1 + \frac{i}{h(s_2)}, \tag{25}$$

$$h'(s_2)[\beta u(q_2) - \mu_2 q_2] = \mu_2 w, \tag{26}$$

$$h(s_2)[\beta u(q_2) - \mu_2 q_2] - \mu_2 q_2 i + (1 - s_2)\mu_2 w = \mu_2 w, \tag{27}$$

$$\beta v'(x_2^b) = \mu_2, \text{ and} \tag{28}$$

$$x_2^b + n_2 h(s_2) q_2 = (1 - n_2 s_2) w. \tag{29}$$

The system implies that the comparative statics are as follows:

$$\frac{\partial q_2}{\partial \gamma} > 0, \ \frac{\partial s_2}{\partial \gamma} > 0, \ \frac{dn_2}{d\gamma} < 0, \frac{\partial x_2^b}{\partial \gamma} > 0, \ \frac{\partial \mu_2}{\partial \gamma} < 0.$$

In an extensive search equilibrium, due to the higher opportunity cost of carrying money associated with higher inflation, the buyer's search intensity increases with inflation.

It is not commonly found in the literature that inflation has a positive effect on the amount of goods traded. The intuition for my result is as follows: The marginal benefit of a search drops as the buyer increases her search intensity. As a result, the amount of goods traded in the search market has to rise so that the buyer is indifferent between participating in the morning market and staying at home producing. Since $q$, $s$ and $x$ are all increasing with inflation, the participation rate $n$ has to drop to clear the market.

## 4.3 Extreme Search Equilibrium

An extreme search equilibrium arises if $\gamma \geq \gamma_2$. Combining first-order conditions (15), and (17) with the marketing-clearing condition, I obtain:

$$\frac{\beta u'(q_3)}{\mu_3} = 1 + \frac{i}{h(1)}, \tag{30}$$

$$h(1)[\beta u(q_3) - \mu_3 q_3] - \mu_3 q_3 i = \mu_3 w, \tag{31}$$

$$\beta v'(x_3^b) = \mu_3, \tag{32}$$

$$x_3^b + n_3 h(1) q_3 = (1 - n_3) w, \text{ and} \tag{33}$$

$$s_3 = 1. \tag{34}$$

The system implies the following comparative statics:

$$\frac{\partial q_3}{\partial \gamma} < 0, \ \frac{\partial s_3}{\partial \gamma} = 0, \ \frac{dn_3}{d\gamma} \text{ ambiguous}, \ \frac{\partial x_3^b}{\partial \gamma} > 0, \ \frac{\partial \mu_3}{\partial \gamma} < 0.$$

$\frac{dn_3}{d\gamma}$ is positive (negative) at relatively low (high) inflation rates. Intuitively, the effect of inflation on $n_3$ depends on the change of $q_3$ and $x_3^b$. When inflation is relatively low, the decrease of $q$ dominates; therefore, the participation rate has to rise to clear the market, and vice versa.

# 5 Quantitative Analysis

This section studies a quantified version of the model. I estimate the model using annual observations on the velocity of the circulation of money and the short-term commercial paper rate in the United States from 1892 to 2005. My objective is to study the effects of inflation on the velocity of money, and welfare.

To parametrize the model, I adopt the following functional forms:

$$u(q) = \frac{q^{1-\theta}}{1-\theta},$$

$$v(x) = B \ln(1 + x), \text{ and}$$

$$h(s) = As^{1-\alpha}.$$

To estimate the five parameters of the model $(\alpha, \theta, B, A, \beta)$, I use annual observations on the velocity of the circulation of money and the short-term commercial-paper rate in the United States from 1892 to 2005. The velocity of the circulation of money is $GDP$ over $M1^*$, where $M1^*$ is $M1$ in circulation inside the United States. The model's counterpart of the average annual velocity of the circulation of money is $[x^b + x^s + nh(s)q]/(nq)$.

The time series of the velocity of money displays a noticeable upward trend since 1946, and the velocity has tripled at the end of the sample period compared to the beginning. This upward trend cannot be explained well by variations in the interest rate because the interest rates at the end of the sample are quite similar to those at the beginning. According to the existing literature, many factors account for the upward trend in the velocity. For example, innovations in payment system allows individuals to use alternative payment methods, such as credit cards, and this dramatically reduces the demand for money. However, since this paper abstracts away from those factors, I detrend both the velocity of the circulation of money and the commercial-paper rate with a fourth-power polynomial of time.[13]

I now describe the method that I use to identify those parameters. The basic idea is, as all equilibrium values of $(q, s, x^b, \mu, n)$ are determined uniquely by the nominal interest rate $i$, I searched for the vector of the parameter values that gives the best fit of the model predictions to the observed relationship between the short-term commercial paper rate and the velocity of money.

Some of the parameters cannot be precisely identified. Therefore, I choose the value of the discount factor $\beta$ to match the average real rate of interest rate in the U.S. data, which is 3 per cent. In addition to that, the coefficient $A$ is normalized to be 1, which implies that buyers reach their targeted market with probability one if they exert all their efforts on searching.

The remaining three parameters $(a, \theta, B)$ are estimated to minimize the sum of squared residuals (the difference between actual and predicted velocities), subject to the following constraint: as reported in Robinson, Andreyenkov, and Patrushev (1989, p.76), in 1986, individuals spent about 11 per cent of their total time on shopping in the U.S.[14] This gives me a condition which requires $n * s = 0.11$ at the average commercial-paper rate of

---

[13] I normalize the time variable to go from $-1$ in 1892 to 1 in 2005.

[14] This is the most recent estimate of the average shopping time that I could find. According to their survey, on average an individual spends 3.4 hours/week on shopping and 27.9 hours/week on regular work. This implies that the search intensity is 0.11 ($\simeq 3.4/(3.4 + 27.9)$).

7.26%. Finally, the length of the period is set to be one year. Table 1 reports the estimated parameter values.

Table 1

Estimation of the Model

| Parameter | $\alpha$ | $\theta$ | $B$ | $\beta$ | $A$ |
|---|---|---|---|---|---|
| Values | 0.073 | 0.228 | 1.096 | 1/1.03 | 1 |

Using the estimated parameters, Figure 3 examines the impact on the velocity of the circulation of money when inflation increases from the Friedman rule ($\gamma = \beta = 1/1.03$) to 10 per cent ($\gamma = 1.1$). The model predicts that there exist two critical levels of inflation rate $\gamma_1 = 0.974$ and $\gamma_2 = 0.994$ that divide the entire range into three regimes, which correspond to the intensive, extensive and extreme-search equilibria, respectively. It is apparent from the figure that the velocity of money is much more interest elastic in the second regime where both the search intensity and the participation rate are affected by inflation than if only one margin were relevant.

Figure 3

The Effect of Inflation on the Velocity of Money

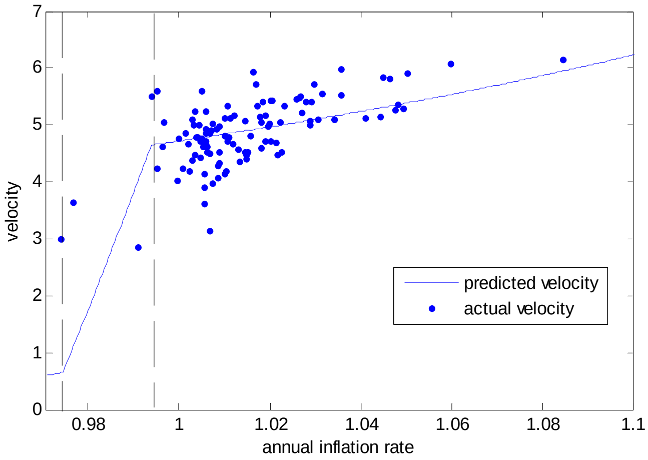
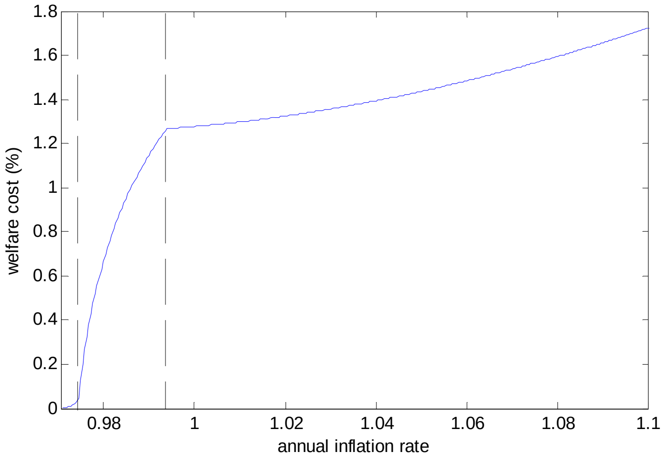


Figure 3 illustrates another finding of interest: most observations fall into the third regime, which is the extreme-search equilibrium. It is because the two endogenous threshold values of inflation rate $\gamma_1$ and $\gamma_2$ are both smaller than 1, and central banks rarely implement a deflationary policy. To better understand the result, I conduct two contrast experiments in which individuals are restricted to vary only one margin: either the intensive or the extensive margin. The overall fit of the model is about the same when only the extensive margin is adjusted. However, when only the intensive margin is considered, the model cannot fit the data well. This result suggests that the frequency of shopping is more relevant than the search intensity in modelling individuals' purchasing behavior.

Figure 4 plots the welfare cost of inflation. It is a widely-held view that inflation reduces social welfare as inflation induces individuals to make extra efforts to get rid of their money holdings, and that the extra efforts made are a social cost.

Figure 4

Welfare Cost of Inflation



Here, steady-state welfare is defined as $W = v(x^b) + v(x^s) + nh(s)u(q)$. The welfare cost of inflation measures the percentual increase in endowment $w$ that compensates for the welfare loss of individuals at $\gamma$ inflation (subscript $\gamma$) compared with the Friedman rule (subscript $F$). That is, the portion $k$ solves the following equation:

$$W_F(w) = W_\gamma(w(1+k)).$$

As indicated by Figure 4, the welfare cost of a 10-percent inflation rate is equivalent to a 1.7% change of endowment. It is also noted that the welfare cost increases more rapidly when individual search extensively because inflation creates inefficiencies in both margins.

# 6 Conclusion

This paper investigates the effect of inflation on individuals' trading behavior, namely search intensity and participation rate. Instead of restricting individuals to vary only one margin, it allows for both and characterizes the conditions under which each margin is relevant. I show that due to the interaction between the direct costs of searching and the opportunity cost of carrying money, different inflation rates and/or different matching technologies affect individuals' purchasing behavior in different ways.

Specifically, if the matching technology is strongly concave, every individual carries money and searches with positive intensity (but less than 1). Therefore, the participation rate is 1. In contrast, if the matching technology is linear, individuals are concerned mainly with the opportunity cost of carrying money. As a result, they do not participate in the market every day. In equilibrium, individuals are indifferent between going searching or not. Some of them stay at home producing, and others go to the market and search with full intensity. In that case, the participation rate drops below 1. Lastly, if the matching technology is weakly concave, then three different types of equilibria are possible, depending on the level of inflation.

This study provides interesting insights into the effects of inflation on the velocity of money, welfare, and individuals' trading behavior. For example, I show that when individuals vary both margins simultaneously, the velocity of money is much more interest elastic than if only one margin is adjusted. In addition to that, the numerical analysis discovers that the shopping frequency is the more relevant margin in modelling individuals' purchasing behavior.

## References

[1] Berentsen, Aleksander, Gabriele Camera and Christopher Waller (2006), “Money, Credit and Banking,” *Journal of Economic Theory*, forthcoming.

[2] Faig, Miquel (2004), “Money, Credit and Banking,” manuscript.

[3] Faig, Miquel (2005), “Divisible Money in an Economy with Villages,” manuscript.

[4] Faig, Miquel and Belen Jerez (2005), “A Theory of Commerce,” *Journal of Economic Theory* 122, 60-99.

[5] Faig, Miquel and Belen Jerez (2006), “Precautionary Balances and the Velocity of Circulation of Money,” *Journal of Money, Credit and Banking*, forthcoming.

[6] Lagos, Ricardo and Randall Wright (2005), “A Unified Framework for Monetary Theory and Policy Analysis,” *Journal of Political Economy* 113, 463-484.

[7] Lagos, Ricardo and Guillaume Rocheteau (2005), “Inflation, Output, and Welfare,” *International Economic Review* 46, 495-522.

[8] Lucas, Robert E., Jr. (2000), “Inflation and Welfare,” *Econometrica* 68, 247-274.

[9] Robinson, John P., Vladimir G. Andreyenkov and Vasily D. Patrushev (1989), *The Rhythm of Everyday Life: How Soviet and American Citizens Use Time,* Westview Press: Boulder Colorado.

[10] Rocheteau, Guillaume and Randall Wright (2005), “Money in Search Equilibrium, in Competitive Equilibrium, in Competitive Search Equilibrium,” *Econometrica* 73, 175-202.

[11] Rodriguez Mendizabal, Hugo (2006), “The Behavior of Money Velocity in High and Low Inflation Countries,” *Journal of Money, Credit and Banking* 38, 91-118.

[12] Shi, Shouyong (1997), “A Divisible Search Model of Fiat Money,” *Econometrica* 65, 75-102.

## Appendix

**1. Proof of Lemma 1.** The Hessian of the objective function (12) is:

$$\begin{bmatrix} \beta h(s)u''(q) & h'(s)[\beta u'(q)-\mu] \\ h'(s)[\beta u'(q)-\mu] & h''(s)[\beta u(q)-\mu q] \end{bmatrix}.$$

Clearly, the determinant of the Hessian $\triangle = \beta h(s)u''(q)h''(s)[\beta u(q)-\mu q]-\{h'(s)[\beta u'(q)-\mu]\}^2$ could be negative. Therefore, (12) is not concave.

### 2. Proof of Proposition 1.

$$F(q,s) = A\frac{s^{1-\alpha}}{1-\alpha}(\beta\frac{q^{1-\theta}}{1-\theta}-\mu q)-\mu qi+(1-s)\mu w;$$

where $\alpha \in [0,1)$ and $\theta \in (0,1)$.

### 2.1 Some Useful Facts.

1. Define $\bar{q}$ as $\beta\bar{q}^{-\theta} = \mu$. I focus on $q \in [0,\bar{q}]$ since when $q$ is greater than $\bar{q}$, a buyer's marginal benefit of offering $(q,d)$ would be negative. 2. $F_s = 0 \Rightarrow s = (A\frac{\beta\frac{q^{1-\theta}}{1-\theta}-\mu q}{w\mu})^{\frac{1}{\alpha}}$, define this function as $\tilde{s}(q)$, and it determines the optimal $s$ for a given $q$. 3. $F_q = 0 \Rightarrow q = [\frac{\mu}{\beta}(1+\frac{i(1-\alpha)}{s^{1-\alpha}A})]^{-\frac{1}{\theta}}$, define this as $\tilde{q}(s)$, and it determines the optimal $q$ for a given $s$. 4. To calculate the inverse function of $\tilde{q}(s)$, I get $s = [\frac{\mu i(1-\alpha)}{\beta q^{-\theta}-\mu}\frac{1}{A}]^{\frac{1}{1-\alpha}}$; define this function as $\tilde{q}^{-1}(s)$. 5. It is a simple matter to show that $\tilde{s}(0) = 0$ and $\tilde{q}(0) = 0$. 6. (1) $\lim_{q\to 0}\tilde{s}(q) = 0 = \tilde{s}(0)$, so $\tilde{s}(q)$ is continuous at 0. (2) $\lim_{s\to 0}\tilde{q}(s) = 0 = \tilde{q}(0)$, so $\tilde{q}(s)$ is continuous at 0. 7. The slope of $\tilde{s}(q) = \frac{ds}{dq} = -\frac{F_{sq}}{F_{ss}} = \frac{1}{\alpha}(\frac{A}{w\mu})^{\frac{1}{\alpha}}(\frac{\beta q^{1-\theta}}{1-\theta}-\mu q)^{\frac{1-\alpha}{\alpha}}(\beta q^{-\theta}-\mu) > 0$. As $q \to 0$, (1) if $\alpha+\theta > 1$, slope of $\tilde{s}(q)$ goes to $+\infty$; (2) if $\alpha+\theta < 1$, the slope of $\tilde{s}(q)$ goes to 0. 8. The slope of $\tilde{q}^{-1}(s) = \frac{ds}{dq} = -\frac{F_{qq}}{F_{sq}} = \frac{\beta\theta A}{\mu i(1-\alpha)^2}(\frac{\mu i(1-\alpha)}{\beta q^{-\theta}-\mu}\frac{1}{A})^{\frac{2-\alpha}{1-\alpha}}q^{-\theta-1} > 0$. As $q \to 0$, (1) if $\alpha+\theta > 1$, the slope of $\tilde{q}^{-1}(s)$ goes to 0; (2) if $\alpha+\theta < 1$, the slope of $\tilde{q}^{-1}(s)$ goes to $+\infty$. 9. At interior max, $\triangle = F_{ss}F_{qq}-(F_{sq})^2 > 0$. That is, the slope of $\tilde{s}(q) <$ the slope of $\tilde{q}^{-1}(s)$. 10. At interior max, $F_s = 0$ and $F_q = 0$ should both be satisfied. When this is the case, the slope of $\tilde{s}(q) = \frac{1-\alpha}{\alpha}\frac{i}{w}$, and the slope of $\tilde{q}^{-1}(s) = \frac{\theta q^{-\theta-1}(\frac{\beta q^{1-\theta}}{1-\theta}-\mu q)}{(\beta q^{-\theta}-\mu)^2}\frac{i}{w}$.

11. The slope of $\tilde{q}^{-1}(s)$ is strictly increasing with $q$ at interior max (where $F_s = 0$ and $F_q = 0$ are both satisfied) when $q \in [0, \bar{q}]$ and $\mu$ is positive. It is because $\frac{\partial(\frac{\theta q^{-\theta-1}(\frac{q^{1-\theta}}{1-\theta}-\mu q)}{(q^{-\theta}-\mu)^2}\frac{i}{w})}{\partial q} = \frac{\theta q^{-\theta-1}(q^{-\theta}-\mu)^3+2\theta^2 q^{-2\theta-2}(\frac{q^{1-\theta}}{1-\theta}-\mu q)(q^{-\theta}-\mu)-\theta(\theta+1)q^{-\theta-2}(\frac{q^{1-\theta}}{1-\theta}-\mu q)(q^{-\theta}-\mu)^2}{(q^{-\theta}-\mu)^4}\frac{i}{w}$. The numerator is positive when $q \in [0, \bar{q}]$ and $\mu$ is positive. To show this, $q^{-\theta} > \mu$ implies that $\frac{1+\theta}{1-\theta}q^{-\theta} > \mu$; therefore, $\mu^2 - 2(1+\theta)\mu q^{-\theta} > \mu^2(1+\theta) - \frac{(1+\theta)(2-\theta)}{1-\theta}\mu q^{-\theta}$. As a result, $\theta q^{-\theta-1}(q^{-\theta} - \mu)^3 + 2\theta^2 q^{-2\theta-2}(\frac{q^{1-\theta}}{1-\theta} - \mu q)(q^{-\theta} - \mu) > \theta(\theta+1)q^{-\theta-2}(\frac{q^{1-\theta}}{1-\theta} - \mu q)(q^{-\theta} - \mu)^2$. 12. Obviously, the slope of $\tilde{s}(q)$ at interior max is independent of $q$. 13. Combining facts 5 to 12, it must be true that, (1) If $\alpha + \theta > 1$, there will be at most one interior max. (2) If $\alpha + \theta < 1$, there will be one corner solution $(0, 0)$, one interior min, and at most one interior max $(q^*, s^*)$, where $q^* \geq 0$ and $1 \geq s^* > 0$.

### 2.2 Case One: $\alpha + \theta > 1$.

At most one interior max of $(q, s)$.

### 2.3 Case Two: $\alpha + \theta < 1$.

#### 2.3.1 $\alpha = 0$.

The solutions to the buyers' max problem are

$$(q, s) = \begin{cases} (0, 0) \text{ if } \mu \text{ is high, i.e. } \mu > \beta(\frac{\theta A}{(1-\theta)w})^{\theta}(1 + \frac{i}{A})^{\theta-1} \\ ((\frac{\mu}{\beta}(1 + \frac{i}{A}))^{\frac{-1}{\theta}}, 1) \text{ if } \mu \text{ is low, i.e. } \mu < \beta(\frac{\theta A}{(1-\theta)w})^{\theta}(1 + \frac{i}{A})^{\theta-1}. \end{cases}$$

There will be one $\hat{\mu}^*$ such that $\mu = \beta(\frac{\theta A}{(1-\theta)w})^{\theta}(1 + \frac{i}{A})^{\theta-1}$ is satisfied; buyers are indifferent between $(0, 0)$ and $((\frac{\mu}{\beta}(1 + \frac{i}{A}))^{\frac{-1}{\theta}}, 1)$. There is no interior max when $\alpha = 0$.

#### 2.3.2 $\alpha > 0$.

The solutions to the buyers' max problem are

$$(q,s)=\begin{cases}(0,0) \text{ if } \mu \text{ is high, i.e. } \mu > \beta(sw)^{-\theta}[\frac{\theta}{1-\theta}(i+\frac{s^{1-\alpha}A}{1-\alpha})]^{\theta}[1+\frac{i(1-\alpha)}{s^{1-\alpha}A}]^{-1}\\ (q^{*},s^{*}) \text{ if } \mu \text{ is low, i.e. } \mu < \beta(sw)^{-\theta}[\frac{\theta}{1-\theta}(i+\frac{s^{1-\alpha}A}{1-\alpha})]^{\theta}[1+\frac{i(1-\alpha)}{s^{1-\alpha}A}]^{-1}.\end{cases}$$

There will be one $\mu^{*}$ such that $\mu = \beta(sw)^{-\theta}[\frac{\theta}{1-\theta}(i+\frac{s^{1-\alpha}A}{1-\alpha})]^{\theta}[1+\frac{i(1-\alpha)}{s^{1-\alpha}A}]^{-1}$ is satisfied, and buyers are indifferent between $(0,0)$ and $(q^{*},s^{*})$, where $q^{*} \geq 0$ and $1 \geq s^{*} > 0$. There will be one interior min in this case, which I can ignore.

**3. Buyers' Optimal Choice.**

$$F(q,s) = h(s)(\beta u(q) - \mu q) - \mu q i + (1-s)\mu w.$$

$F_s = h'(s)[\beta u(q) - \mu q] - w\mu$, $F_q = h(s)[\beta u'(q) - \mu] - ui$, $F_{ss} = h''(s)[\beta u(q) - \mu q]$, $F_{sq} = h'(s)[\beta u'(q) - \mu]$, $F_{qq} = h(s)\beta u''(q)$. Therefore,

$$\begin{bmatrix} F_{qq} & F_{sq} \\ F_{sq} & F_{ss} \end{bmatrix}\begin{bmatrix} dq \\ ds \end{bmatrix} = \begin{bmatrix} \mu di + [h(s)+i]d\mu \\ \mu dw + [h'(s)q + w]d\mu \end{bmatrix}. \tag{35}$$

Solving (35), I obtain $\frac{dq}{d\mu} = \frac{[h(s)+i]F_{ss} - [h'(s)q+w]F_{sq}}{F_{qq}F_{ss} - (F_{sq})^2} < 0$.

**4. Proof of Proposition 2.** The existence of an equilibrium requires $h(s)[\beta u(q) - \mu q] - \mu q i + (1-s)\mu w \geq \mu w$. Therefore, there are only two possibilities; either $h(s)[\beta u(q) - \mu q] - \mu q i + (1-s)\mu w > \mu w$ or $h(s)[\beta u(q) - \mu q] - \mu q i + (1-s)\mu w = \mu w$. Suppose $h(s)[\beta u(q) - \mu q] - \mu q i + (1-s)\mu w > \mu w$, and this is the intensive search equilibrium. Using equilibrium conditions (20) to (24) to simplify, $(1-\alpha-\theta)q^{-\theta}\beta < (1-\alpha)(1-\theta)\mu$. As inflation increases, $q$ and $\mu$ both decrease. Therefore, there must exist one critical level of inflation $\gamma_1$ such that when $\gamma$ equals to $\gamma_1$, $(1-\alpha-\theta)q^{-\theta}\beta = (1-\alpha)(1-\theta)\mu$. This implies that $h(s)[\beta u(q) - \mu q] - \mu q i + (1-s)\mu w = \mu w$. This proves that an increase in inflation switches the equilibrium from an intensive search to an extensive search one.

At $\gamma = \gamma_1$, $h(s)[\beta u(q) - \mu q] - \mu q i + (1-s)\mu w = \mu w$. This implies that $\mu$ coincides with $\mu^{*}$. If $\gamma > \gamma_1$, $\mu$ always equals to $\mu^{*}$. That is to say, there exists an extensive search equilibrium if and only if $\gamma_1 \leq \gamma < \gamma_2$. In an extensive search equilibrium, since $s$ is increasing with inflation, there must exist another critical level of inflation $\gamma_2$ such that an extreme search equilibrium in which $s = 1$ arises if $\gamma \geq \gamma_2$.

**5. Comparative Statics of an Intensive Search Equilibrium.** Substituting conditions (22) and (23) into conditions (20) and (21), I obtain

$$\frac{u'(q)}{v'[(1-s)w - h(s)q]} = 1 + \frac{i}{h(s)}, \text{ and} \tag{36}$$

$$h'(s)\left\{\beta u(q) - \beta v'[(1-s)w - h(s)q]q\right\} = \beta v'[(1-s)w - h(s)q]w. \tag{37}$$

The comparative statics of the system (36) and (37) are described by

$$\begin{bmatrix} h(s)[u'' + v''\frac{h(s)u'}{v'}] & v''[h(s)+i]\frac{h'(s)u}{v'} + v'\frac{ih'(s)}{h(s)} \\ h'(s)v'\frac{i}{h(s)} + v''h(s)\frac{h'(s)u}{v'} & h''(s)\frac{v'w}{h'(s)} + v''[\frac{h'(s)u}{\beta v'}]^2 \end{bmatrix} \begin{bmatrix} dq \\ ds \end{bmatrix} = \begin{bmatrix} v'di \\ 0di \end{bmatrix}. \tag{38}$$

Using (36) and (37) to simplify, the determinant of the $2 \times 2$ matrix in (38) is:

$$\begin{aligned} \triangle \;=\; & h(s)\frac{h''(s)}{h'(s)}w[u''v' + h(s)v''u'] + [\frac{h'(s)u}{v'}]^2 h(s)u''v'' \\ & -v''[h'(s)]^2 ui(\frac{u'}{v'} + 1) - [v'\frac{ih'(s)}{h(s)}]^2. \end{aligned} \tag{39}$$

Using the fact that at interior max where the second-order conditions are satisfied, it must be true that $F_{ss}F_{qq} - (F_{sq})^2 = h(s)u''(q)h''(s)[\beta u(q) - v'q] - [h'(s)(\beta u'(q) - \beta v')]^2 > 0$. As $[h'(s)(u' - v')]^2 = [v'\beta\frac{ih'(s)}{h(s)}]^2$, which is just the last term in (39), I obtain

$$\begin{aligned} \triangle \;>\; & h(s)\frac{h''(s)}{h'(s)}w[u''v' + h(s)v''u'] + [\frac{h'(s)u}{v'}]^2 h(s)u''v'' \\ & -v''[h'(s)]^2 ui(\frac{u'}{v'} + 1) - h(s)u''(q)h''(s)(u - v'q) \end{aligned}$$

$$= h(s)\frac{h''(s)}{h'(s)}wh(s)v''u' + [\frac{h'(s)u}{v'}]^2 h(s)u''v'' - v''[h'(s)]^2 ui(\frac{u'}{v'} + 1).$$

Therefore, the determinant $\triangle$ is positive. Solving (38), it implies

$$\frac{dq}{di} = \frac{h''(s)\frac{v'w}{h'(s)} + v''[\frac{h'(s)u}{v'}]^2}{\triangle} v' < 0, \text{ and}$$

$$\frac{ds}{di} = -\frac{[\frac{v'i}{h(s)} + v''h(s)\frac{u}{v'}]}{\triangle} v'h'(s). \tag{40}$$

The numerator in (40) is negative when $i$ is low; therefore, $\frac{ds}{di} > 0$ ($< 0$) when $i$ is low (high).

To show $\frac{dx}{di} > 0$, substituting conditions (22) and (23) into conditions (20) and (21), I obtain

$$\frac{u'[\frac{(1-s)w-x}{h(s)}]}{v'(x)} = 1 + \frac{i}{h(s)}, \text{ and} \tag{41}$$

$$h'(s)\left\{\beta u[\frac{(1-s)w-x}{h(s)}] - \beta v'(x)[\frac{(1-s)w-x}{h(s)}]\right\} = \beta v'(x)w. \tag{42}$$

The comparative statics of systems (41) and (42) are described by

$$\begin{bmatrix} -u'' - v''\frac{h(s)u'}{v'} & -u''\frac{h'(s)u}{v'} + v'\frac{ih'(s)}{h(s)} \\ -\frac{h'(s)}{h(s)}v'\frac{i}{h(s)} - v''\frac{h'(s)u}{v'} & h''(s)\frac{v'w}{h'(s)} - [\frac{h'(s)}{h(s)}]^2 iu \end{bmatrix} \begin{bmatrix} dx \\ ds \end{bmatrix} = \begin{bmatrix} v'di \\ 0di \end{bmatrix}. \tag{43}$$

Using (41) and (42) to simplify, the determinant of the $2 \times 2$ matrix in (43) is:

$$\begin{aligned} \triangle &= -h''(s)\frac{v'w}{h'(s)}[u'' + h(s)v''\frac{u'}{v'}] + [\frac{h'(s)}{h(s)}]^2 iuv''h(s)\frac{u'}{v'} \\ &\quad + \frac{[h'(s)]^2 iu}{h(s)}v'' - [\frac{h'(s)u}{v'}]^2 u''v'' + [v'\frac{ih'(s)}{h(s)}]^2\frac{1}{h(s)}. \end{aligned} \tag{44}$$

Following the same procedure, I can show that $\triangle$ is negative. Therefore,

$$\frac{dx}{di} = \frac{h''(s)\frac{v'w}{h'(s)} - [\frac{h'(s)}{h(s)}]^2 iu}{\triangle} > 0, \text{ and}$$

$$\frac{d\mu}{di} < 0.$$

**6. Comparative Statics of an Extensive Search Equilibrium.** Substituting conditions (27), (28) and (29) into (25) and (26), I obtain

$$\beta u' - \frac{\beta h(s)u}{h(s)q + qi + sw}(1 + \frac{i}{h(s)}) = 0, \text{ and} \tag{45}$$

$$h'(s)\beta u - \frac{\beta h(s)u}{h(s)q + qi + sw}[h'(s)q + w] = 0. \tag{46}$$

The comparative statics of systems (45) and (46) are described by

$$\begin{bmatrix} u''[h(s)q + qi + sw] & ui\frac{h'(s)}{h(s)} \\ h'(s)ui & h''(s)u(qi + sw) \end{bmatrix} \begin{bmatrix} dq \\ ds \end{bmatrix} = \begin{bmatrix} (u - u'q)di \\ -qh'(s)udi \end{bmatrix}. \tag{47}$$

Using (45) and (46) to simplify, the determinant of the 2 × 2 matrix in (47) is:

$$\begin{aligned} \triangle &= u''[h(s)q + qi + sw]h''(s)u(qi + sw) \\ &\quad - \frac{1}{h(s)}[uih'(s)]^2. \end{aligned} \tag{48}$$

Using the fact that at interior max where the second-order conditions are satisfied, it must be true that $F_{ss}F_{qq} - (F_{sq})^2 = h(s)u''(q)h''(s)(u - v'q) - [h'(s)(u' - v')]^2 > 0$. Since $\frac{1}{h(s)}[h'(s)(u' - v')]^2[h(s)q + qi + sw]^2 = \frac{1}{h(s)}[uih'(s)]^2$, which is just the last term in (48), I obtain

$$\begin{aligned} \triangle &> u''[h(s)q + qi + sw]h''(s)u(qi + sw) \\ &\quad - u''(q)h''(s)(u - v'q)[h(s)q + qi + sw]^2. \end{aligned}$$

Since $u''(q)h''(s)(u - v'q)[h(s)q + qi + sw]^2 = u''(q)h''(s)\frac{wh(s)u}{h'(s)}[h(s)q + qi + sw]$, it must be true that

$$\begin{aligned} \triangle &> u''[h(s)q + qi + sw]h''(s)u(qi + sw) \\ &\quad - u''(q)h''(s)\frac{wh(s)u}{h'(s)}[h(s)q + qi + sw] \end{aligned}$$

$$= u''h''(s)u[h(s)q + qi + sw][qi + sw - \frac{wh(s)}{h'(s)}].$$

Next, since $qi + sw = h(s)[\frac{u}{v'} - q] = \frac{wh(s)}{h'(s)}$, the determinant $\triangle$ is positive. Solving (47), I obtain

$$\frac{dq}{di} = \frac{s^{-2\alpha}(\frac{q^{1-\theta}}{1-\theta})^2 \left\{[\alpha\theta(1-\theta) - (1-\alpha)(1-\theta)]q\mu - [\alpha\theta - (1-\alpha)(1-\theta)]q^{1-\theta}\right\}}{\triangle}, \text{ and}$$

$$\frac{ds}{di} = -\frac{\theta(\frac{q^{1-\theta}}{1-\theta})^2 s^{-\alpha} i(\frac{q^{-\theta}}{q^{-\theta}-\mu} - 1)}{\triangle} > 0.$$

Using the fact that $(1-\alpha-\theta)q^{-\theta} = (1-\alpha-\theta+a\theta)\mu/\beta$ in an extensive search equilibrium, I can show $\frac{dq}{di} > 0$. Following the same procedure, I can show that $\frac{dx}{di} > 0$ and $\frac{d\mu}{di} < 0$. To solve $\frac{dn}{di}$, from (29) I obtain $n = \frac{w-x}{h(s)q+sw}$. As $\frac{dq}{di} > 0$, $\frac{ds}{di} > 0$ and $\frac{dx}{di} > 0$, it must be true that $\frac{dn}{di} < 0$.

**7. Comparative Statics of an Extreme Search Equilibrium.** Substituting equation (32) into (30) and (31), I obtain

$$u'(q) - v'(x)(1 + \frac{i}{h(1)}) = 0, \text{ and} \tag{49}$$

$$h(1)[\beta u(q) - \beta v'(x)q] - \beta v'(x)qi = \beta v'(x)w. \tag{50}$$

The comparative statics of systems (49) and (50) are described by

$$\begin{bmatrix} h(1)u'' & -v''(x)\,h(1)\,\frac{u'}{v'} \\ 0 & -v''(x)\frac{h(1)u(q)}{v'(x)} \end{bmatrix} \begin{bmatrix} dq \\ dx \end{bmatrix} = \begin{bmatrix} v'(x)di \\ v'(x)qdi \end{bmatrix}. \tag{51}$$

Using (49) and (50) to simplify, the determinant of the $2 \times 2$ matrix in (51) is:

$$\triangle = -h(1)u''(q)\beta v''(x)\frac{h(1)u(q)}{v'}.$$

Obviously, $\triangle$ is negative. Solving (51), I obtain

$$\frac{dq}{di} = \frac{v''(q)h(1)[u'(q)q - u(q)]}{\triangle} < 0, \text{ and}$$

$$\frac{dx}{di} = \frac{v'(x)qh(1)u''(q)}{\triangle} > 0.$$

Similarly, I can show $\frac{d\mu}{di} < 0$. Since $n = \frac{w-x}{h(1)q+w}$, it is hard to tell $\frac{dn}{di}$, which depends on the relative changes of $q$ and $x$. If the change of $q$ dominates the change of $x$, then $\frac{dn}{di} > 0$, and vice versa.

**8. Data Sources.** The interest rate is the short-term commercial paper rate. For 1892-1971, it is taken from Friedman and Schwartz (1982), Table 4.8, Column 6. For 1972-2004, it is taken from the DRI series FYCP90 (averaged).

Money is $M1^* = M1$- currency outside the country. M1 is the stock at the end of June of each year. For 1892-1928, the source of M1 is the United States Bureau of the Census (1960), Series X267. For 1929-1958, it is the series constructed by the St. Louis FED that extends modern M1back in time: http://reserarch.stlouisfed.org/aggreg. For 1959-2004, it is the DRI series FZM1. Currency in circulation abroad is from the FED *Flow of Funds Table* L-204 in the file ltab204d.prn downloaded from http://www.federalreserve.gov/releases/z1/current/data.htm.

For 1892-1928, GDP is calculated from the real GDP series in Kendrick (1961) and the implicit price deflator in Friedman and Schwartz (1982), Table 4.8, Column 4. For 1929-2004, it is taken from BEA NIPA Table 1.1.5 downloaded from www.bea.doc.gov/bea/dn/nipaweb.