Gaining advantage from Complexity in Defence: a new DST research initiative

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Outline

- Complexity – Good or Bad (for us)?
- DST Modelling Complex Warfighting SRI
- Models of Warfighting
- What’s missing?
- Decisions & Attrition: a ‘Kuramoto-Lanchester’ model
- Complexity advantage
- What are we looking for?
- Conclusions
Complexity – Beauty ...

Motor and Somatosensory Cortex

The Internet
Complexity – and The Beast?
Complexity ‘phenomena’

Can these properties be exploited to advantage in national defence?
The problem

- How can ‘emergence’/’criticality’/’self-synchronisation’/’self-organisation’ be exploited by a Force
  - To make it robust against shocks – resilience
  - To give it advantage against a near-peer adversary

- We know some of the answers in abstracto – now is the time to see it for things that look like national defence.
Creating a Future Force: how DST supports Force Design

- What are the parts?
- How do they connect?
- What else should we get?
- How do we gain advantage from them as a system for the future?

Modelling Complex Warfighting SRI*: Revolutionising the analytical approach to force design

*SRI = Strategic Research Investment

**Conquering Uncertainty**
Scientific methods to enable robust Force Design decisions to produce a resilient force through the understanding and management of uncertainty in Defence.

**Innovative simulations**
Novel modelling and simulation techniques to enable exploration of whole-of-force warfighting concepts and force options.

**Knowledge synthesis**
Synthesis of analytical and simulation results to support development of a joint force which is integrated by design.

**Modelling complexity**
Methods to enable understanding of properties of the joint force emerging as a result of nonlinear interactions between the many constituent elements.
Modelling Complex Warfighting SRI: Revolutionising the analytical approach to force design

Conquering Uncertainty
- Modelling unknowns
- Modelling complex human systems

Innovative simulations
- Capability decision evaluation under uncertainty

Knowledge synthesis
- Force design data culture

Modelling complexity
- Concepts for complexity-enabled warfare
  - Simulation-based concept exploration
  - Machine discovered behaviour

Current Force

Future Force

Time Horizon (years)

Project Leader:
A. Kalloniatis

Current Force

Future Force

Time Horizon (years)
Mathematical Models of Warfighting

- Lanchester 1916 – ("Directed") Force-on-Force Attrition
  \[
  \dot{B}(t) = \alpha_s B(t) - \alpha_{RDA} R(t) \quad \text{B= Size of Blue Force}
  \]
  \[
  \dot{R}(t) = \beta_s R(t) - \beta_{BDA} B(t) \quad \text{R = Size of red Force}
  \]

- Protopopescu et al 1989 – Diffusion, Advection, Inhomogeneity

- Hughes 1995 – Missile Salvos, Staying Power

- McLemore et al 2016 – Manoeuvre, Dispersion, Swarming, Swarming
What’s missing – in one or another – or all?

- Logistics
- Deception/Reconnaissance
- Manoeuvre
- States of Readiness/Damage
- **Command and Control (ie organisational decision-making) – hierarchical or networked**
A model for C2 – the Kuramoto Model (1984)

\[ \beta_i = \omega_i + \sigma \sum_j A_{ij} \sin(\beta_j - \beta_i) \]

Rate of progress through decision cycle

Measure of synchronisation:

\[ r(t) = \frac{1}{N} \left| \sum_j e^{i\beta_j(t)} \right| \]

Low \( \sigma \): “Loosely Coupled”

High \( \sigma \): “Tightly Coupled”

Socio/technical applications:
- Rhythmic applause (Neda et al 2000);
- Opinion dynamics (Pluchino et al 2006);
- Pedestrian crowds (Strogatz 2005);
- Decision making in animal groups (Leonard et al 2012);
- Planar vehicle coordination (Paley et al 2007);
- Control systems (Jadbabie et al 2004);
- Consensus protocol (Sarlette & Sepulchre 2009).

Organisational Interactions

Tightness of Organisational Coupling

Frequency of decision-making when left to self.

\( \omega_i \)

\( \beta \) = state in continuous decision cycle

Spontaneous synchronisation through network interactions
External C2 driven resupply and symmetric direct attrition

- Kuramoto

\[ \dot{\beta}_i(t) = \omega_i + \sigma \sum_{j} B_{ij} \sin(\beta_j(t) - \beta_i(t)) \]  

Blue C2 system

\[ \dot{\beta}_j(t) = \nu_j + \sigma \sum_{i} R_{ij} \sin(\rho_j(t) - \rho_i(t)) \]  

Red C2 system

- Order parameter

\[ r_B(t) = \frac{1}{N} \sum_{j} e^{i\beta_j(t)} \]

\[ r_R(t) = \frac{1}{N} \sum_{j} e^{i\rho_j(t)} \]

- Lanchester

\[ \dot{B}(t) = r_B(t)B(t) - r_R(t)R(t) \]

\[ \dot{R}(t) = r_R(t)R(t) - r_B(t)B(t) \]

C2 capability *sits outside* the combat force

Good C2 \(\Rightarrow\) Good resupply of own and good firepower on adversary
Resupply and internal C2-direct attrition

Attrition undermines ability to couple on the network

- Kuramoto

\[ \beta_i(t) = \omega_i + \frac{B(t)}{B(0)} \sum_j B_{ij} \sin(\beta_j(t) - \beta_i(t)) \]  
Blue C2 system

\[ \rho_i(t) = \nu_i + \frac{R(t)}{R(0)} \sum_j R_{ij} \sin(\rho_j(t) - \rho_i(t)) \]  
Red C2 system

- Order parameter

\[ r_B(t) = \frac{1}{N} \left| \sum_j e^{i\beta_j(t)} \right| \]

\[ r_R(t) = \frac{1}{N} \left| \sum_j e^{i\rho_j(t)} \right| \]

- Lanchester

\[ B(t) = r_B(t)B(t) - r_R(t)R(t) \]  
C2 capability resides in the combat force

\[ R(t) = r_R(t)R(t) - r_B(t)B(t) \]  
Good C2 \( \Rightarrow \) Good resupply of own and good firepower on adversary

\[ \text{Resupply} \quad \text{Attrition} \]
Resupply and internal C2-direct attrition

- **Kuramoto**
  \[ \dot{\beta}_i(t) = \omega_i + \sigma_B \frac{\chi_B}{N_B} \sum_j B_{ij}(t) \sin(\beta_j(t) - \beta_i(t)) \]  
  \[ \dot{\rho}_i(t) = \omega_i + \sigma_R \frac{\chi_R}{N_R} \sum_j R_{ij}(t) \sin(\rho_j(t) - \rho_i(t)) \]

- **“Order” parameter**
  \[ \chi_B(t) = \left| \sum_j e^{i\beta_j(t)} \right| \]
  \[ \chi_R(t) = \left| \sum_j e^{i\rho_j(t)} \right| \]

- **Lanchester**
  \[ \dot{\mathcal{B}}(t) = -\kappa_R \chi_R(t) + \eta_B \chi_B(t) \]
  \[ \dot{\mathcal{R}}(t) = -\kappa_B \chi_B(t) + \eta_R \chi_R(t) \]

Attrition undermines coupling on and links of the network

C2 capability is identical to the combat force

- Blue C2 system
- Red C2 system

O O O
A D A D
Detecting criticality

- Kuramoto order parameter
  \[ r_B(t) = \frac{1}{N} \left| \sum_j e^{i \beta_j(t)} \right| \]

- Fisher information
  \[ \mathcal{F} \equiv E \left( \frac{\partial}{\partial \sigma} \log P(X; \sigma) \right)^2 | \sigma \]
  \[ = \prod_i \int dX_i P(X_i; \sigma) \left( \frac{\partial}{\partial \sigma} \log P(X_i; \sigma) \right)^2 \]
  \[ \mathcal{F}_n \equiv \frac{\mathcal{F}}{\mathcal{F}_{\text{max}}} \leq 1 \]

- Minimum description length
  Proxy: in numerical solution, the minimum number of points required to describe time-series for a given value of coupling.

Time-averaged synchronisation measure, \( r \)
Normalised Fisher information

[Kalloniatis, Zuparic, Prokopenko – PRE subm.]

(Kuramoto, 1984)

\[ K_c = \frac{2}{\pi g(\omega)} \]
\[ \Rightarrow \sigma_c = 0.000398 \]
A Scenario

Blue – pseudo-hierarchical – headquarters entity covering two task groups of complete networks

Red – pseudo random network

Strategy:
1. Solve ordinary Kuramoto dynamics for criticality indicators as function of coupling
2. Solve Kuramoto-Lanchester dynamics with static network
3. Solve Kuramoto-Lanchester with attrition of network

Does (1) give insight into (2) and (3)?
Complexity Advantage?

Three measures of Criticality

Crit. Ind. Blue

Crit. Ind. Red

1. Boost in synchronisation at low coupling for Blue – due to complete graph connectivity – gives it early advantage that is maintained

2. 

B, R

Coupling

$\dot{r}_B, r_R$

$t$

DST Group Science and Technology for Safeguarding Australia
3. Attrition of networks - HQ ‘protected’
3. **Attrition of networks**
   - HQ ‘protected’
3. Attrition of networks - HQ ‘unprotected’
3. Attrition of networks - HQ ‘unprotected’
Attrition of networks - HQ ‘unprotected’

This is the more typical behaviour – unless the ‘full connectedness’ of the Task Groups is preserved, the ‘boost in r’ for Blue is lost.
Conservation of criticality?
Early days ...

- Approach to statistical limit – convergence of criticality indicators?
- Criticality indicators for dynamical network scenarios?
- Stochasticity, Advection – Gaussian and non-Gaussian
- Generalisation to more sophisticated representations of modern combat?

- *Is concentration of mass/increase of number of actors the only way to achieve complexity/criticality?*
What are we looking for?

- Collaboration

- Just completed – initial Expression of Interest (EoI) process for start-up collaborations.

  Modelling Complex Warfighting Symposium
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  Victoria Division of Engineers Australia,
  Bourke Place, 600 Bourke Street, Melbourne

- First of many ...
Conclusion

- Complexity – feared but exploitable
- Marrying complex systems dynamical models with mathematical combat models enables generation of new warfighting concepts.
- New DST Strategic Research Initiative “Modelling Complex Warfighting” to pursue this.
- Opportunities for peer-to-peer collaboration with academic partners in ranges of areas:
  - Statistical physics
  - Network Theory
  - High Performance Computing
- Watch this space – or contact me ...