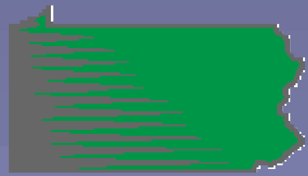
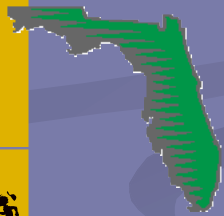
Understanding Teaching and Learning of Mathematical Problem Solving: A Tale of Two Sites





Asha Jitendra

Lehigh University

E-mail: akj2@lehigh.edu

Presentation Overview

- Introduction
- How to use the schema-based representational strategy to solve word problems?
- Findings of design studies using the schema-based problem solving curriculum in third grade classrooms
- Summary

Students with Learning Difficulties

- True math deficits are specific to mathematical concepts and problem types (Zentall & Ferkis, 1993).
- Learning difficulties may not be related to IQ, motivation or other factors that influence learning (Geary, 1996).

Students with Learning Difficulties

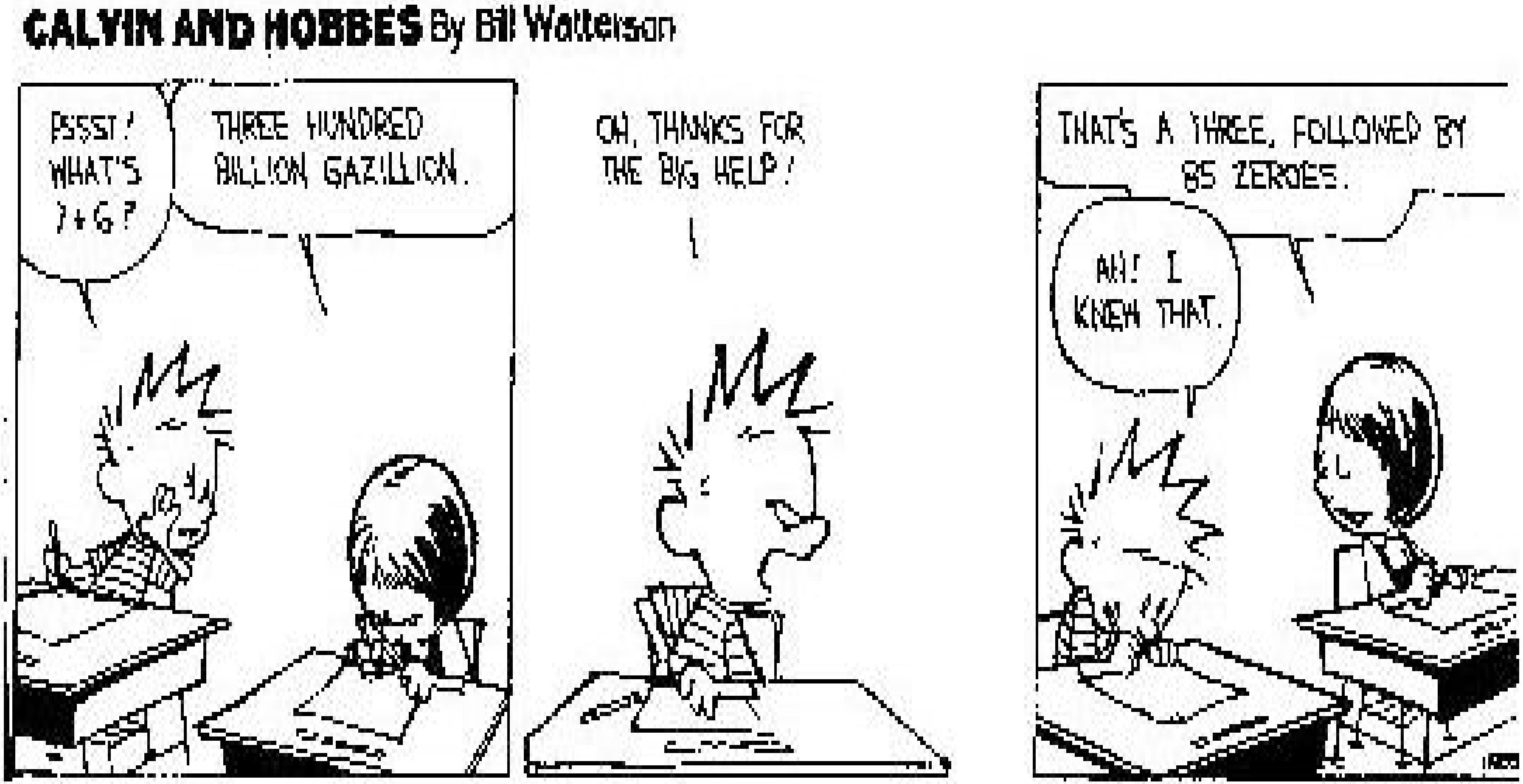
Difficulties in learning math may be related to a variety of learner characteristics. For example, students with learning difficulties

–Cannot remember basic math facts

–Use immature (and time-consuming) problem-solving procedures to solve simple math problems

–Also have problems learning to read and write (Geary, 1996)

Evidence cognitive disadvantage in attention, organization skills, and working memory (e.g., Gonzales & Espinel, 1999, Zentall & Ferkis, 1993).


CALVIN AND HOBBES By Bill Watterson
PSSST! WHAT'S 7+6?
THREE HUNDRED BILLION GAZILLION.
OH, THANKS FOR THE BIG HELP!
THAT'S A THREE, FOLLOWED BY 85 ZEROES.
AH! I KNEW THAT.

Assumptions about Learning

1. Learning involves the recognition of patterns, which become bits of knowledge that are then organized into larger and more meaningful units.
2. Learning for some children is more difficult than for others because of visual, auditory, or motor deficits that interfere with the ready recognition of patterns.
3. Some children have difficulty with the organization of parts into a whole due to a developmental lag or a disability (i.e., with integration, sequencing, memory, or spatial relationships).

Watson

The Patterns in Mathematics

0 x 9 + 8 = 8

9 x 9 + 7 = 88

98 x 9 + 6 = 888

987 x 9 + 5 = 8,888

9,876 x 9 + 4 = 88,888

98,765 x 9 + 3 = 888,888

987,654 x 9 + 2 = 8,888,888

9,876,543 x 9 + 1 = 88,888,888

98,765,432 x 9 + 0 = 888,888,888

Posamentier, A. (2004). Marvelous math! *Educational Leadership, 61*, 44-47.

Assumptions about Teaching

Communication in teaching should insure that the learning task is made clear to the learner and that the student's problems in learning are clear to the teacher.

Assumptions about Teaching

The student must be provided with the sequence and structure that will enable him/her to recognize the patterns into which he/she must organize larger and more meaningful units. The experiences that develop the sequence and structure must be designed to:

- circumvent or overcome sensory modality deficits or weaknesses,
- circumvent the problems of spatial or temporal relationships and sequencing,
- develop organization and integration, and
- provide the association that will insure memory.

Assumptions about Teaching

The learner needs to internalize each concept learned as the basis for further learning. Learning should not only take you somewhere, but should allow you to move ahead more easily.

Bruner

Principles for the Successful Teaching of Students at Risk

1. Have high expectations for all students, letting them know that you believe in them.
2. Use "instructional scaffolding."
3. Make instruction the focus of each class. (Avoid busy work.)
4. Extend students' thinking and abilities beyond what they already know.
5. Work at gaining in-depth knowledge of your students as well as knowledge of the subject matter.

PROMOTE CONCEPTUAL UNDERSTANDING USING CAREFULLY DESIGNED AND EXPLICIT INSTRUCTION!!

Ladson-Billings (1995)

Mathematical Proficiency

1. *Conceptual understanding* – comprehension of mathematical concepts, operations, and relations
2. *Procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. *Strategic competence* – ability to formulate, represent, and solve mathematical problems
4. *Adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
5. *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

NRC (2001, p. 5)

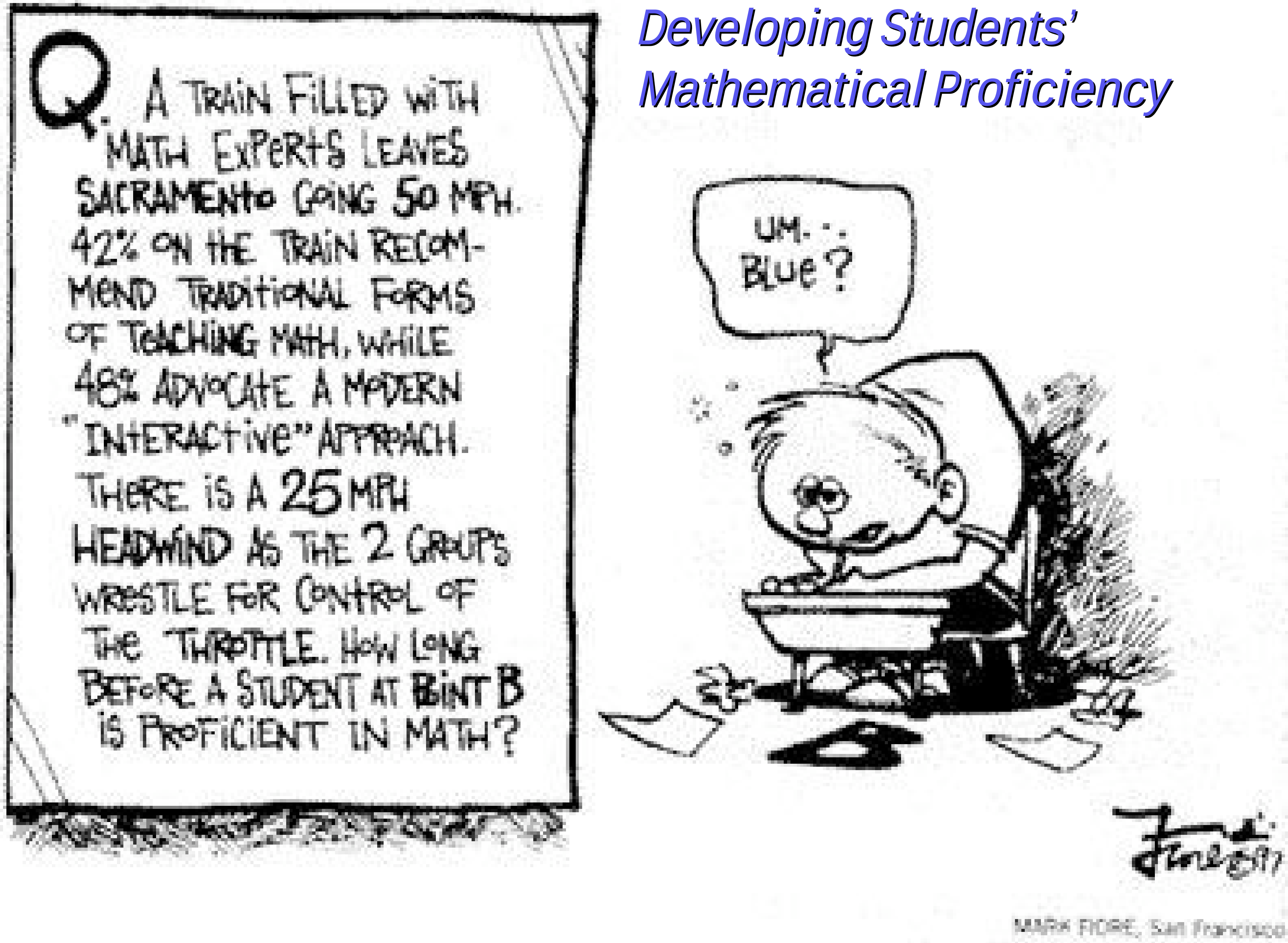
Mathematical Problem Solving

Problem solving involves:

•detecting steps or processes “between the posing of the task and the answer” (Goldin, 1982, p. 97).

•a task that: (a) "the individual or group confronting it wants or needs to find a solution"; (b) "there is not a readily accessible procedure that guarantees or completely determines the solution"; and (c) "the individual or group must make an attempt to find a solution” (Charles & Lester, Jr. , 1983; p. 232).

Developing Students' Mathematical Proficiency



Traditional Models of Problem Solving

- Four-stage model of problem solving (Polya, 1957):
  - Understand the problem,
  - Develop a plan,
  - Carry out the plan,
  - Look back to check if the solution makes sense.
- Key word strategy
  - Jill gave away 6 cookies in the morning. She gave away 2 cookies in the afternoon. How many cookies did she <u>give away</u> that day?" (Kelly & Carnine, 1996, p.5)

Alternative to Traditional Instruction

- Make explicit the key aspects of domain knowledge.
  - Domain knowledge encompasses both conceptual and procedural knowledge.
  - Pattern recognition and knowledge organization are key aspects of conceptual knowledge.
- Need strategies "in the middle range" (Prawat, 1989, p. 33) to meet the needs of students with learning disabilities.

Students with Learning Difficulties and Problem Solving

- Emerging research indicates that students with learning difficulties can be taught to be effective problem solvers (e.g., Jitendra & Xin, 1997; Xin & Jitendra, 1999).
- These students benefit from visual representational techniques and metacognitive procedures.

Mediated Schema Knowledge Instruction as an Alternative

- Skilled problem solving is dependent on schema acquisition (Sweller, Chandler, Tierney, & Cooper, 1990; Willis & Fuson, 1988).
- A distinctive feature of schema knowledge is that when one piece of information is retrieved from memory during problem solving, other connected pieces of information will be activated.

What is the Schema-based Representational Strategy?

- The schema-based instructional approach emphasizes an analysis of the semantic structure of problems and uses schemata diagrams to organize and represent the critical information described in the text, thus mediating effective problem solution.

Problem Solving Instruction: Schema-based Representational Strategy

Essential elements of the schema-based instruction are:

- Schema Identification (i.e., recognize the problem pattern)
- Schema Representation (i.e., translate a problem from words into a meaningful representation)
- Working Schema/Plan (i.e., select appropriate mathematical operations)
- Executive Knowledge/Solution (i.e., execute selected mathematical operations)

Marshall (1990); Riley, Greeno, & Heller (1983)

Problem Solving Instruction:
Schema-based Representational Strategy

Address the "big ideas," or "schemata," by focusing on carefully chosen problems (e.g., *change, group, compare, multiplicative comparison, vary or proportion;* Van de Walle, 2001).

A Schema-Based Representational Strategy: Application to Addition and Subtraction Word Problems

Word Problem Solving Curriculum: A Schema-Based Representational Strategy

Features

- Word Problems selected from commonly adopted US mathematics textbooks and modified to meet the needs of students with diverse experiential backgrounds.
- Word problems are varied and formatted as verbal text, graphs, tables, and pictographs.
- Teaches “big ideas” or salient problem schemata (change, group, compare, vary, multiplicative comparison).
- Instruction focuses on both conceptual and procedural understanding.

Word Problem Solving Curriculum: A Schema-Based Representational Strategy

- Instruction uses a model-lead-independent practice paradigm.
- Includes appropriate scaffolding of instruction.

  - Teacher-led instruction followed by paired learning and independent learning activities.
  - Tasks begins with story situations followed by word problems with unknown information.
  - Visual diagrams and checklists are used until students learn to apply the strategy independently.

Word Problem Solving Curriculum: A Schema-Based Representational Strategy

- Incorporates adequate practice and mixed review of problem types.
- Instruction is aligned with state assessment in terms of communicating, reasoning, and representing word problems.
- Employs frequent measures of student word problem solving performance to monitor student progress.

Schemata Identification and Representation Instruction

- Provide students with story situations of each problem type (e.g., change, group, compare) that do not contain any unknown information.
- Introduce problem schema analysis (i.e., discerning the key features of the problem) using modeling with several examples of story situations.
- During guided practice, use frequent student exchanges to facilitate the identification of critical elements of the story.

Problem Schemata Identification and Representation Instruction

- Use a checklist to help students identify and map key information onto the diagram.
- Have students underline (e.g., words, sentences) and circle (e.g., numbers) key information before mapping the information onto the schema diagram.

CHANGE STORY CHECKLIST

Step 1. Find the problem type.

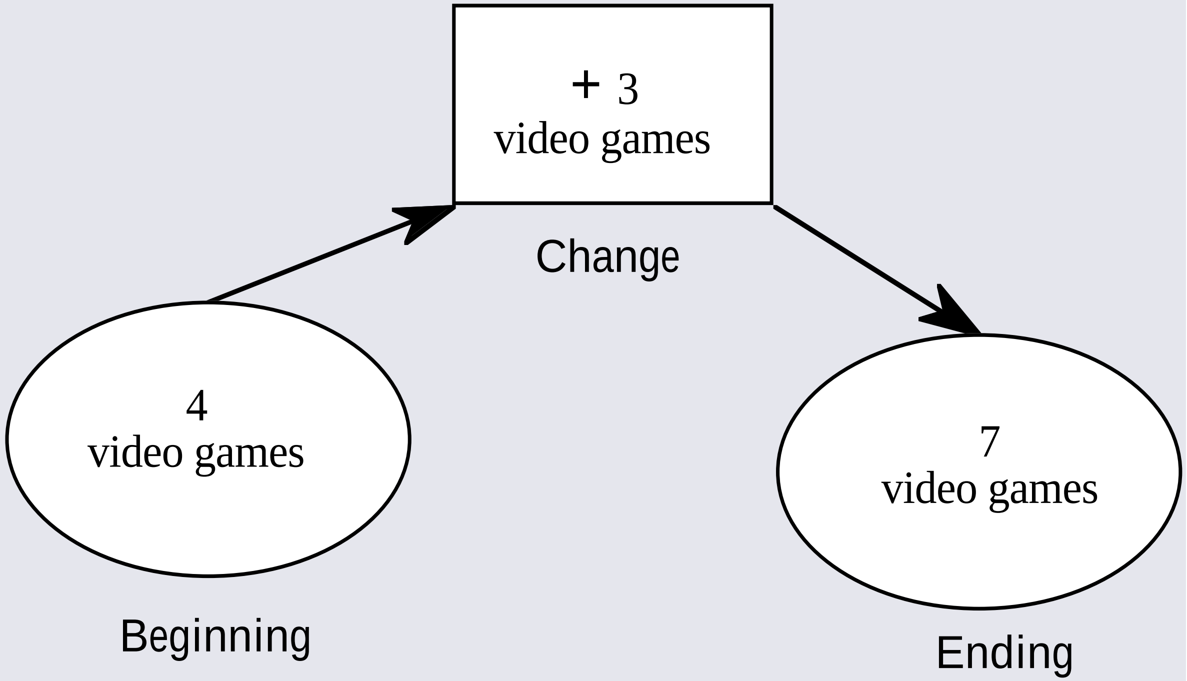
- [ ] Did I read and retell the story?
- [ ] Did I ask if it is a change problem? (Did I look for the beginning, change, and ending? Do they all describe the same thing?)

Step 2. Organize the information using the change diagram.

- [ ] Did I underline the label that describes the beginning, change, and ending and write in label in the diagram?
- [ ] Did I underline important information, circle numbers, and write in numbers in the diagram?

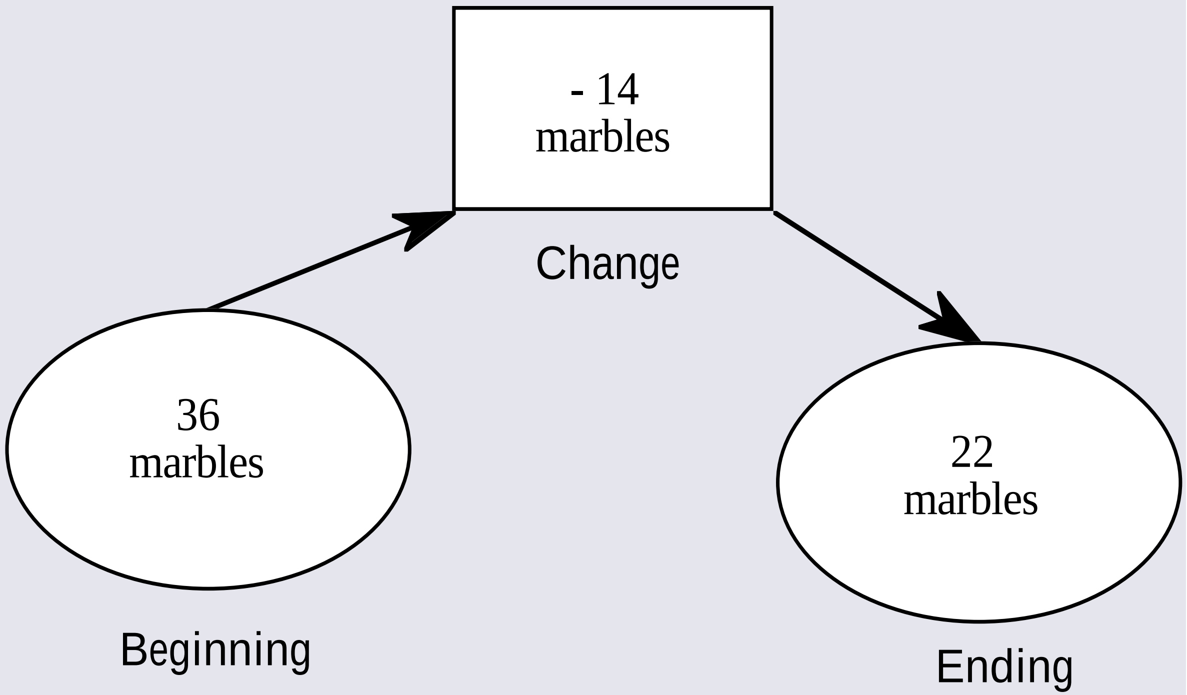
Example: Change Story Situation

Jane had 4 video games. Then her mother gave her 3 more video games for her birthday. Jane now has 7 video games.



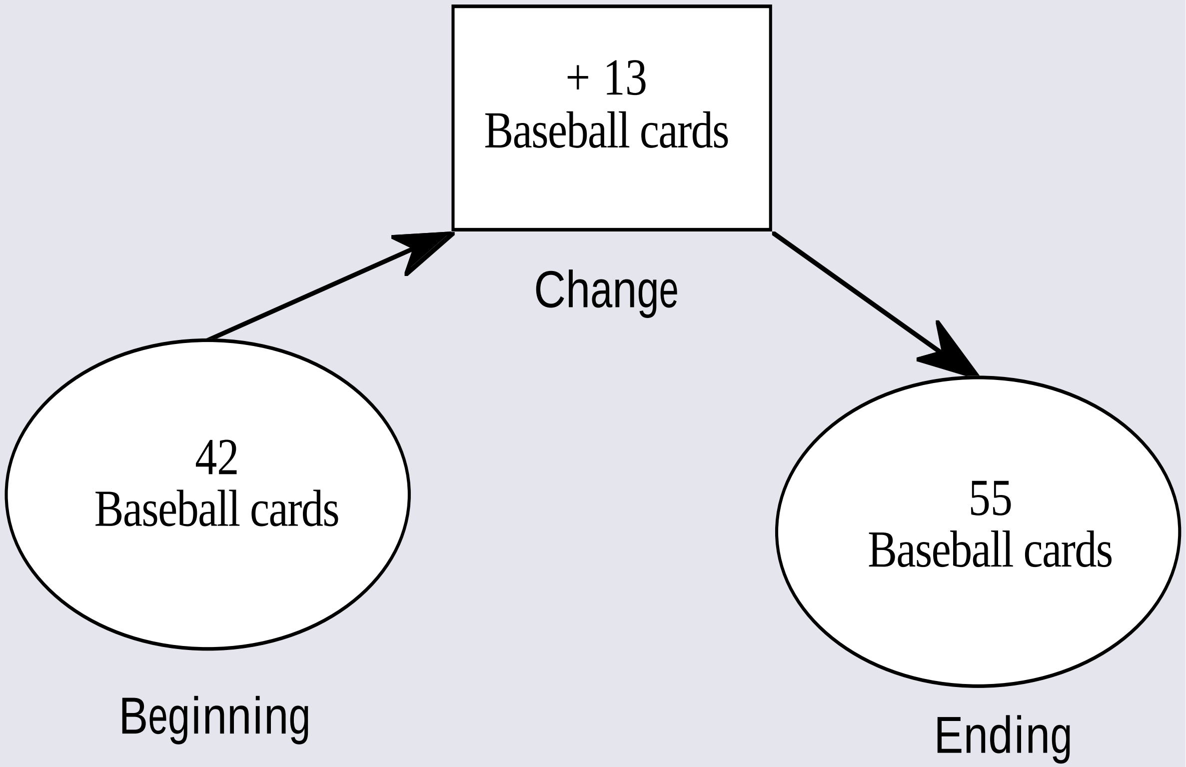
Example: Change Story Situation

Before he gave away 14 marbles, James had 36 marbles. Now he has 22 marbles.



Example: Change Story Situation

Tom had 42 baseball cards. He now has 55 baseball cards after he bought 13 more cards.



ADDITION AND SUBTRACTION ONE-STEP PROBLEMS: CHANGE

Problem Type	Example
Change	
Unknown ending amount (Addition)	Gail had 43 music albums in her collection. Then, she bought 11 albums at a garage sale. How many albums does Gail have now?
Unknown change amount (Subtraction)	Roger had 36 comic books. Then his father bought him some more for his birthday. Roger now has 52 comic books. How many comic books did he receive from his father?
Unknown beginning amount (Subtraction)	There were some Halloween masks in the fifth grade classroom. Then the class made 15 more masks. Now they have 42 masks. How many masks were in the classroom?

Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

Planning and Solution Instruction

- Solve all addition and subtraction problems using the 4-step (FOPS) procedure.
- Encourage metacognition by having students use the problem checklist as they solve *change*, *group*, and *compare* problems.
- Fade the diagrams once students become independent in correctly mapping and solving the problem using the schematic diagrams.

WORD PROBLEM SOLVING STEPS (FOPS)

Step 1. Find the problem type.

Step 2. Organize the information in the problem using the diagram (change, group, or compare).

Step 3. Plan to solve the problem.

Step 4. Solve the problem.

CHANGE PROBLEM CHECKLIST

Step 1. **F**ind the problem type.

- ☐ Did I read and retell the problem?
- ☐ Did I ask if it is a change problem? (Did I look for the beginning, change, and ending? Do they all describe the same thing?)

Step 2. **O**rganize the information using the change diagram.

- ☐ Did I underline the label that describes the beginning, change, and ending and write in label in the diagram?
- ☐ Did I underline important information, circle numbers, and write in numbers in the diagram?
- ☐ Did I write a "?" for what must be solved? (Did I find the question sentence?)

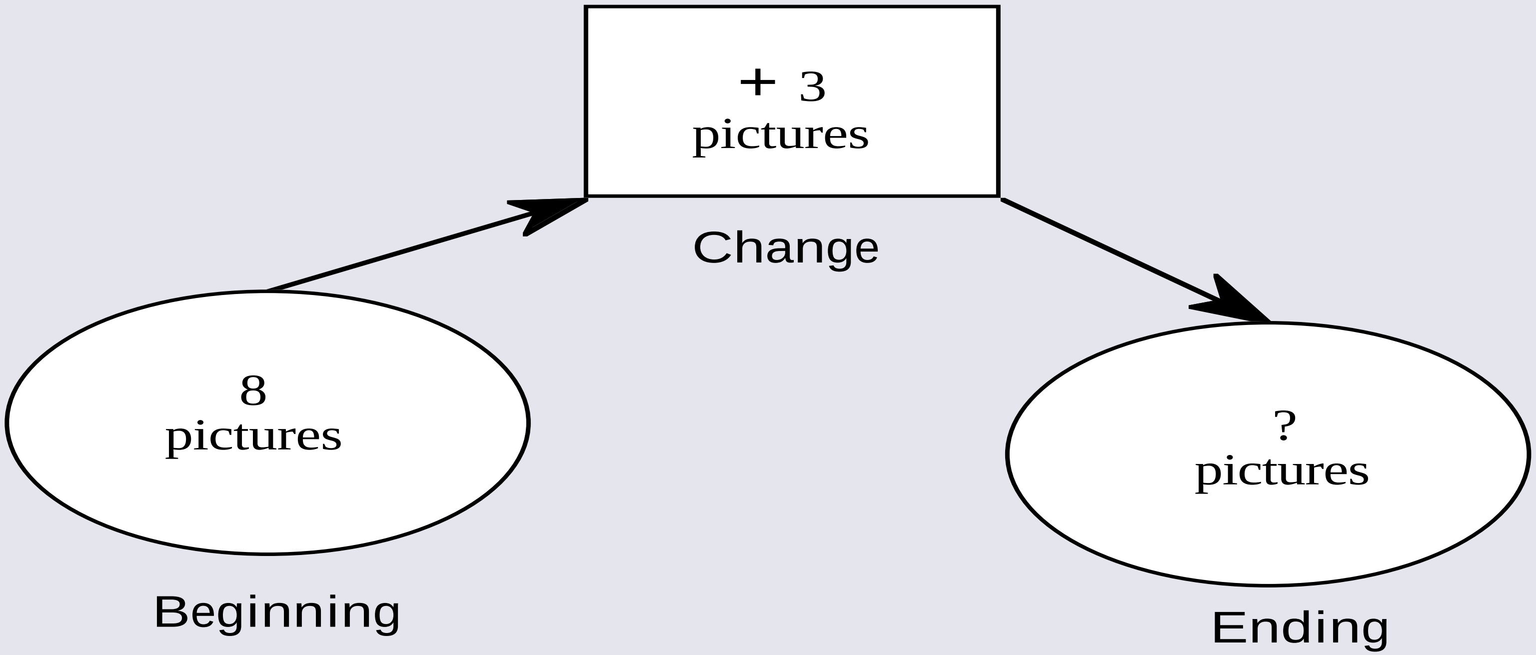
Step 3. **P**lan to solve the problem.

- ☐ Do I add or subtract? (If the "BIG" number is given, subtract. If the "BIG" number is not given, add)
- ☐ Did I write the math sentence?

Step 4. **S**olve the problem.

- ☐ Did I solve the math sentence?
- ☐ Did I write the complete answer?
- ☐ Did I check if the answer makes sense?

Change Problem: Tammy likes to paint pictures. She has painted 8 pictures so far. If she paints 3 more pictures, how many will she have?



Math Sentence: 8

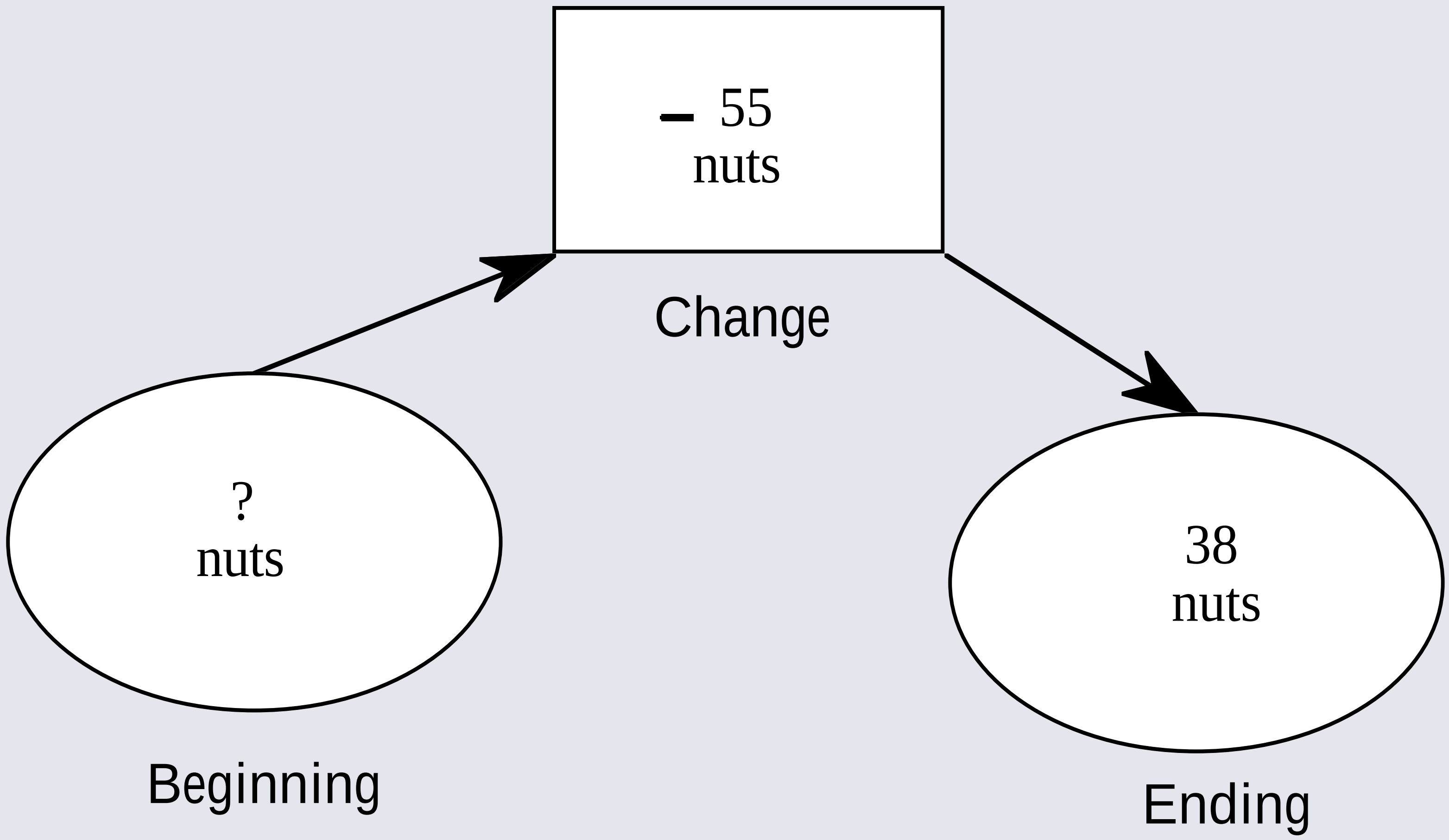
+3

___

___

Change Answer Sheet: Paired Learning

The gray squirrel made a pile of nuts. It carried away 55 nuts up to its nest. Now there are 38 nuts left in the pile. How many nuts were in the pile at the beginning?



Change Answer Sheet (continued)

Work	Explanation
$$\begin{array}{r} 55 \\ +38 \\ \hline 93 \\ \hline \end{array}$$ **nuts**	**First**, I figured out that this is a change problem, because it has a beginning, a change, and an ending. **Next**, I organized the information on the change diagram. **Then,** I decided to add 55 and 38 to figure out the number of nuts in the pile at the beginning, because there were more nuts in the pile at the beginning than at the end. **Finally,** I wrote my math sentence and solved it. I also wrote the complete answer with the number and label.

Answer: **93 nuts**

GROUP STORY CHECKLIST

Step 1. **F**ind the problem type.

- [ ] Did I read and retell the story?
- [ ] Did I ask if it is a group problem? (Did I look to see if two or more small groups combine to make up a large group?)

Step 2. **O**rganize the information using the group diagram.

- [ ] Did I underline the large group and small groups and write in group names in the diagram?
- [ ] Did I circle numbers for the groups and write in numbers for groups in the diagram?

Example: Group Story Situation

68 students at Hillcrest Elementary took part in the school play. There were 22 third graders, 19 fourth graders, and 27 fifth graders in the school play.

3rd graders		4th graders		5th graders		All students (3rd, 4th, & 5th graders)
22	+	19	+	27	=	68

Small Groups or Parts

Large Group or Whole (Sum)

ADDITION AND SUBTRACTION ONE-STEP PROBLEMS: GROUP

Problem Type	Example
Group	
Unknown larger group amount (Addition)	Meg saw 13 bear cubs running and 16 bear cubs walking at the zoo. How many bear cubs did Meg see at the zoo?
Unknown smaller group amount (Subtraction)	In an apple picking contest, the third and fourth graders picked 84 apples. If the third graders picked 41 apples, how many apples did the fourth graders pick?

Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

GROUP PROBLEM CHECKLIST

Step 1. Find the problem type.

- [ ] Did I read and retell the problem?
- [ ] Did I ask if it is a group problem? (Did I look to see if two or more small groups combine to make up a large group?)

Step 2. Organize the information using the group diagram.

- [ ] Did I underline the large group and small groups and write in group names in the diagram?
- [ ] Did I circle numbers for the groups and write in numbers for groups in the diagram?
- [ ] Did I write a "?" for what must be solved? (Did I find the question sentence?)

Step 3. Plan to solve the problem.

- [ ] Do I add or subtract? (If the "BIG" number is given, subtract. If the "BIG" number is not given, add)
- [ ] Did I write the math sentence?

Step 4. Solve the problem.

- [ ] Did I solve the math sentence?
- [ ] Did I write the complete answer?
- [ ] Did I check if the answer makes sense?

Group Problem

In a survey, 98 people were asked what their favorite flower is and 45 named tulips. How many named another kind of flower?

Named tulip	+	Named another kind of flower	=	Named tulip and another kind of flower
45 people		? people		98 people
Small Groups or Parts				Large Group or Whole (Sum)

Math Sentence: 98

$$\begin{array}{r} 98 \\ -45 \\ \hline 53 \end{array}$$

Group Answer Sheet: Paired Learning

Three buses took students on a field trip. One bus carried 35 students, another bus carried 28 students, and the third bus carried 27 students. How many students went on the trip?

First bus		Second bus		Third bus		All three buses
35 students	+	28 students	+	27 students	=	? students

Small Groups or Parts

Large Group or Whole (Sum)

Group Answer Sheet (continued)

Work	Explanation
35 **28** **+27** **90 students**	**First**, I figured out that this is a group problem, because it has three small groups that combine to make a large group (all students). **Next**, I organized the information on the group diagram. **Then,** I decided to add 35, 28, and 27 to figure out all the students who went on the field trip on the three buses, because the large group is the sum of the small groups. **Finally,** I wrote my math sentence and solved it. I also wrote the complete answer with the number and label.

Answer: **90 students went on the trip**

COMPARE STORY CHECKLIST

Step 1. **F**ind the problem type.

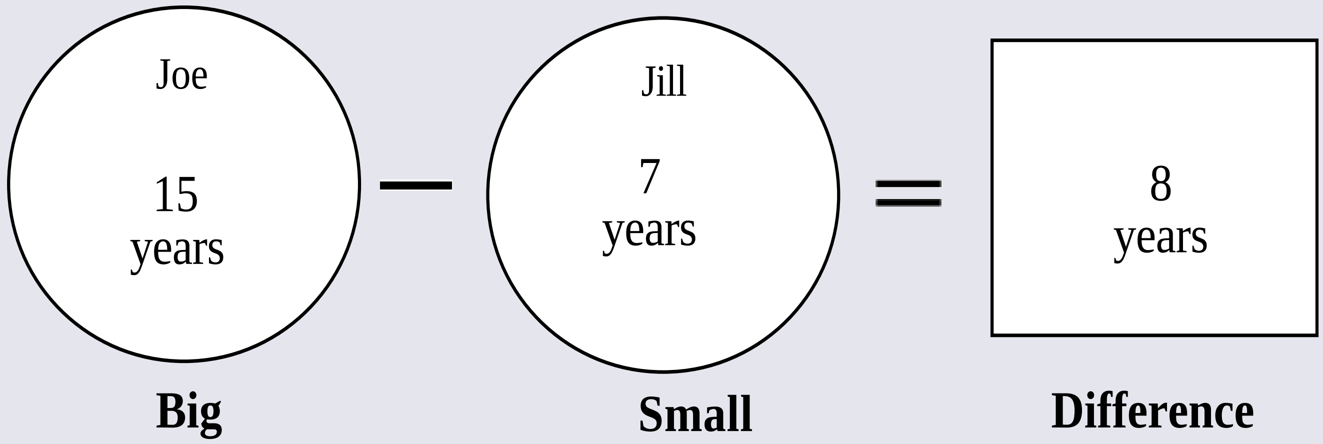
- [ ] Did I read and retell the story?
- [ ] Did I ask if it is a compare problem? (Did I look for compare words – taller than, shorter than, more than, less than?)

Step 2. **O**rganize the information using the compare diagram.

- [ ] Did I underline the comparison sentence and circle the two things compared?
- [ ] Did I reread the comparison sentence to ask, "Which is the "BIG" amount and the "SMALL" amount?" and write in names of things compared in the diagram?
- [ ] Did I underline important information, circle numbers and labels and write in numbers and labels in the diagram?

Example: Compare Story Situation

Joe is 15 years old. He is 8 years older than Jill. Jill is 7 years old.



ADDITION AND SUBTRACTION ONE-STEP PROBLEMS: COMPARE

Problem Type	Example
Compare	
Unknown difference amount (Subtraction)	The pet store is having a sale of 21 hamsters and 32 kittens. How many more kittens are on sale than hamsters?
Unknown compared amount (Addition)	72 people came to the school play on Monday. 26 more people attended it on Tuesday than Monday. How many people went to the school play on Tuesday?
Unknown referent amount (Subtraction)	Janice is 85 centimeters tall. She is 16 centimeters taller than Melinda. How tall is Melinda?

Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

COMPARE PROBLEM CHECKLIST

Step 1. **F**ind the problem type.

- [ ] Did I read and retell the problem?
- [ ] Did I ask if it is a compare problem? (Did I look for compare words – taller than, shorter than, more than, less than?)

Step 2. **O**rganize the information using the compare diagram.

- [ ] Did I underline the comparison sentence or question and circle the two things compared?
- [ ] Did I reread the comparison sentence or question to ask, "Which is the "BIG" amount and the "SMALL" amount?" and write in names of things compared in the diagram?
- [ ] Did I underline important information, circle numbers and labels and write in numbers and labels in the diagram?
- [ ] Did I write a "?" for what must be solved? (Did I find the question sentence?)

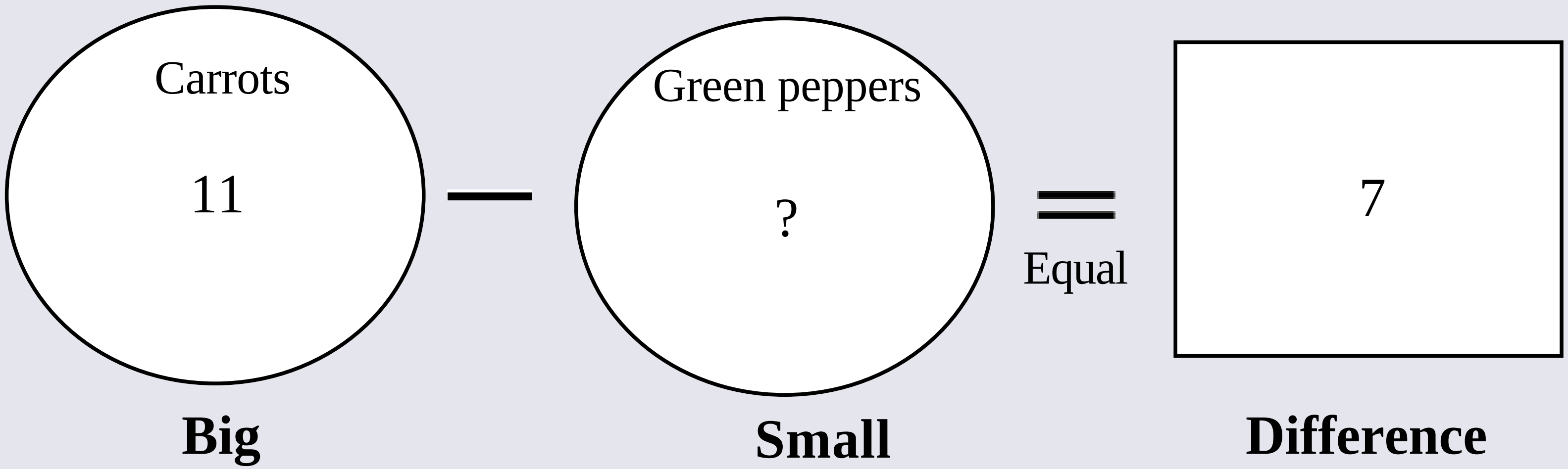
Step 3. **P**lan to solve the problem.

- [ ] Do I add or subtract? (If the "BIG" number is given, subtract. If the "BIG" number is not given, add)
- [ ] Did I write the math sentence?

Step 4. **S**olve the problem.

- [ ] Did I solve the math sentence?
- [ ] Did I write the complete answer?
- [ ] Did I check if the answer makes sense?

Compare Problem: Steve picked 11 carrots. He picked 7 fewer green peppers than carrots. How many green peppers did Steve pick?



Math Sentence: 11
- 7

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Mixed Review:One-Step Problems

1. In one week Samuel read 35 pages. He read 16 fewer pages than Wes. How many pages did Wes read?
2. Ted collected some pictures of butterflies in his scrapbook. This week, he added 25 more pictures. Now he has 90 pictures of butterflies in his scrapbook. How many pictures did he have in the beginning?
3. At top speed, a giraffe can run 32 miles an hour. This speed is 3 miles an hour faster than that of an antelope. How many miles an hour does the antelope run?
4. Your teacher made some snacks for the class. There were 8 left after the students ate 14 snacks. How many snacks did the teacher make for the class?
5. You have a collection of 12 marbles. If 5 of the marbles in your collection are large, how many marbles are small?
6. Karen had 16 of her friends come to her birthday party. 6 of her friends left the party early. How many are still at Karen's birthday party?
7. Olivia has two puzzles. A balloon picture puzzle has 25 pieces. A boat picture puzzle has 5 fewer pieces than the balloon picture puzzle. How many pieces does the boat picture puzzle have?

Design Studies

Purposes

- To provide third grade students (including students with learning difficulties) with math problem solving instruction that includes the use of schematic diagrams to represent the information in addition and subtraction problems
- To conduct an in-depth understanding of teaching and learning using the new word problem solving curriculum prior to conducting the formal experimental study in Year 3 of the project

Student Participants
in Pennsylvania

- 38 third graders from two general education classrooms and a learning support classroom in a suburban school district.
  - 9 were students with learning disabilities
  - 9 were classified as low achievers based on their performance on the Terra Nova Mathematics Subtests (cutoff score was the 33rd percentile)
- 20 were boys and 18 were girls.
- The average age of the students was 102.60 months (range = 91 to 119 months; SD = 5.54).
- 28 (74%) were Caucasian, 3 (8%) were African American, 6 (16%) were Hispanic, and one student was a middle easterner (3%)

Teacher Participants

in Pennsylvania

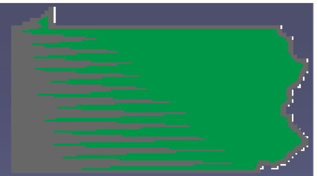
- Two general education classroom teachers and one special education teacher participated in the study.
- All teachers were female and held a master's degree in education.
- The two general education teachers had more than 25 years of teaching experience, whereas the special education teacher had 19 years of teaching experience.
- The three teachers were exposed to an hour of inservice training on implementing the intervention and were provided with on-going support throughout the study.

School Demographics

in Pennsylvania

- The school served 472 students in grades K-5.
- Approximately 15% of the student population was ethnically diverse (i.e., African American, Hispanic, Asian).
- About 17% of the student population was economically disadvantaged
- 5.36% were ELL students.

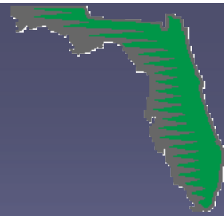
Classroom Description



in Pennsylvania

- Students in this school were grouped according to ability levels
- Teachers in two of the low ability third grade classrooms as well as the learning support teacher at this school participated in the study in order to learn more about innovative approaches that would help their students improve their mathematical problem solving behavior.
- At the time of the study, both classrooms were using the “Heath Mathematics Connections” textbook (Manfre, Moser, Lobato, & Morrow, 1994).

Student Participants
in Florida



- 56 third graders from two general education classrooms in a parochial school in a small southeastern town.
- 9 students were identified as having either a learning disability, attention deficit disorder, or were classified as low achievers based on their performance on the Iowa Test of Basic Skills (cutoff score was the 34th percentile); middle achievers had ITBS scores between the 35th and 66th percentiles; and high achievers between 67th and 99th percentiles.
- The total sample included 29 boys and 27 girls.
- The average age of the students was 108 months, or 9 years old (range = 96 to 113 months).
- 44 (78%) were Caucasian, 5 (8%) were African American, 6 (10%) were Hispanic, and one student was Asian American (2%).

Teacher Participants

in Florida

- Two general education classroom teachers participated in the study.
- Both teachers were female, one held a master’s degree and the other a bachelor’s degree in education.
- The teacher with a master’s degree had been teaching 10 years; the teacher holding a bachelor’s degree had 30 years of teaching experience.
- The teachers were exposed to an hour of inservice training on implementing the intervention and were provided with some support throughout the study.

School Demographics

in Florida

- The school served 570 students in grades PreK-8.
- Approximately 20% of the student population was racially / ethnically diverse (i.e., African American, Hispanic, Asian).

Classroom Description

in Florida

- Students were heterogeneously grouped in the two classrooms with math scores on the ITBS ranging from the 4th percentile in problem solving and data interpretation and 10th percentile in computation to the 99th percentile in problem solving and data interpretation and the 99th percentile in computation.
- At the time of the study, both classrooms were using the textbook, "Mathematics - The Path to Math Success" Silver Burdett and Ginn (Altieri, Krulik, et al., 1999).

Measures

- Word Problem Solving Criterion Referenced Tests (CRT).
- Word Problem Solving Probes.
- Basic Math Computation Probes (Fuchs, Hamlett, & Fuchs, 1998).
- Terra Nova Mathematics Subtests (CTB / McGraw-Hill, 1997).
- Iowa Test of Basic Skills (ITBS) (Houghton Mifflin, 1999)
- B.O.S.S. (Shapiro, 1996)
- Satisfaction Questionnaires

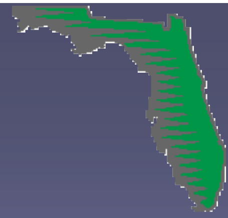
Data Analysis

- Scores from the CRT, word problem solving probes, and computation measures were analyzed using repeated measures ANOVA with time as the repeated measure and teacher or group as the independent variable.
- For the word problem solving probes, the average of the first two probes and the average of the last two probes were the dependent measures.

Results

Pretest (All students: Pennsylvania site)

- No significant differences between teachers on CRT-total, CRT-TOMA, CRT-WPS pretest scores as well as on word problem solving probes-total, one-step, and two-step pretest scores and the pretest computation scores.



Results

Pretests (*All students: Florida site)*

- No significant differences between teachers on CRT-total and CRT-TOMA pretest scores as well as on word problem solving probes-total, one-step, two-step pretest scores, and the pretest computation scores.

- Significant differences between teachers on CRT-WPS pretest scores, $F(1, 54) = 5.75$, $p = .02$.
  - The mean scores for students in Teacher 4's ($M = 21.55$; $SD = 8.44$) classroom were higher than that of Teacher 5's ($M = 17.00$; $SD = 5.28$).

Pretests		Teacher 1 (n=16)	Teacher 2 (n=16)	Teacher 3 (n=6)	Teacher 4 (n=29)	Teacher 5 (n=27)
CRT-Total (25/50)	M	21.38	16.69	16.92	31.16	27.07
	SD	7.62	7.17	5.18	11.25	7.24
CRT-TOMA (9/18)	M	9.06	7.34	6.08	9.60	10.07
	SD	3.02	2.29	3.02	3.74	2.94
CRT-WPS (16/32)	M	12.31	9.38	10.83	21.55*	17.00*
	SD	5.26	5.73	3.01	8.44	5.28
WPS Probes (8/16)	M	7.09	7.55	7.25	9.90	9.90
	SD	3.11	2.18	2.95	4.24	3.2
One-step (6/12)	M	6.58	6.59	6.58	8.07	8.04
	SD	2.73	1.64	1.99	3.04	2.07
Two-step (2/4)	M	0.52	0.95	0.67	1.88	1.84
	SD	0.63	0.85	1.03	1.46	1.29
Computation (25/25)	M	2.00	1.44	1.83	17.35	19.00
	SD	1.67	1.32	1.33	3.68	4.75

Results

Pretests (LD and LA sample: Pennsylvania Site)

- No significant differences between groups on CRT-total, and CRT-WPS pretest scores as well as on word problem solving probes-total, one-step, and two-step pretest scores.
- Significant differences between groups on CRT-TOMA, F (1, 16) = 6.11, p = .025 and computation probe pretest scores, F (1, 16) = 15.09, p = .001.
  - The mean CRT-TOMA scores for students with LD (M = 6.11; SD = 2.42) were lower than that for LA students (M = 9.06; SD = 2.63).
  - The mean computation scores for students with LD (M = 10.44; SD = 4.67) were higher than that for LA students (M = 4.00; SD = 1.73).

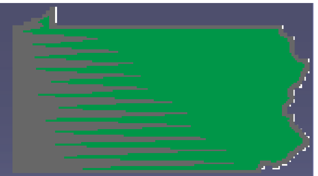
Results

Pretests *(Low, Middle, and High Achievers: Florida site)*

- Significant differences between groups on all pretests (i.e., CRT-total, CRT-TOMA, CRT-WPS, word problem solving probes-total, one-step, and two-step, and the computation probe).
- For every pretest, mean scores for students in the high achieving group were higher than students in the middle and low achieving groups. Scores for students in the middle group were higher than those for students in the low achieving group. Low achievers had the lowest scores on all pretests.

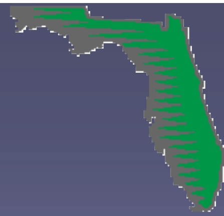
Pretests		LD (n=9)	LA (n=9)	Low* (n=9)	Middle* (n=23)	High* (n=24)
CRT-Total (25/50)	M	14.83	18.22	21.28	26.91	34.33
	SD	5.40	8.34	12.28	8.13	7.10
CRT-TOMA (9/18)	M	6.11*	9.06*	7.00	9.33	11.38
	SD	2.42	2.63	5.07	2.79	2.15
CRT-WPS (16/32)	M	8.78	9.17	14.28	17.59	22.96
	SD	4.18	6.25	8.07	6.69	6.21
WPS Probes (8/16)	M	6.64	6.97	6.04	8.70	12.45
	SD	2.67	2.6	2.33	3.21	2.75
One-step (6/12)	M	6.13	6.25	5.49	7.25	9.79
	SD	1.88	2.17	1.89	2.38	1.76
Two-step (2/4)	M	0.50	0.72	0.57	1.47	2.72
	SD	0.87	0.68	0.83	1.14	1.19
Computation (25/25)	M	1.56	0.78	14.78	18.04	19.50
	SD	1.24	0.83	3.03	4.57	3.75

Results



Posttests *(All Students: Pennsylvania site)*

- Results indicated a main effect for time for CRT-Total, $F(1, 35) = 40.90$, $p < .000$; CRT-TOMA, $F(1, 35) = 7.39$, $p < .010$; CRT-WPS, $F(1, 35) = 208.44$, $p < .000$; computation probe, $F(1, 35) = 244.07$, $p < .000$; word problem solving probes -Total, $F(1, 35) = 13.18$, $p < .001$; and word problem solving probes-one-step, $F(1, 35) = 19.27$, $p < .000$.
- In addition, results revealed a main effect for teacher, $F(2, 35) = 12.92$, $p < .000$ and an interaction effect for time by teacher on the computation probe scores only, $F(2, 35) = 13.19$, $p < .000$.
  - The mean scores for students in Teacher 1's ($M = 10.37$) classroom were higher than that of Teacher 2's ($M = 6.63$) and Teacher 3's ($M = 6.667$).
  - The mean gain score from pretest to posttest for students in Teacher 1's classroom ($M = 16.75$) was greater than that in Teacher 2's ($M = 10.37$) and Teacher 3's ($M = 9.67$) classrooms.



Results

Posttests *(All Students: Florida site)*

- Results indicated a main effect for time for CRT-Total, F (1, 54) = 10.82, p = .002; CRT-WPS, F (1, 54) = 12.99, p = .001; computation probe, F (1, 54) = 23.47, p = .000; word problem solving probes -Total, F (1, 54) = 11.03, p = .002; word problem solving probes-one-step, F (1, 54) = 3.93, p = .052; word problem solving probes-two-step, F(1,54) = 19.33, p < .000
- No significant main effect for time on the CRT-TOMA, F (1, 54) = 3.35, p = .073
- No significant interaction effects for time by teacher on any of the posttests.

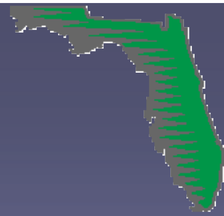
Posttests		Teacher 1 (n=16)	Teacher 2 (n=16)	Teacher 3 (n=6)	Teacher 4 (n=29)	Teacher 5 (n=27)
CRT-Total (25/50)	M	29.03	25.47	24.83	27.33	23.85
	SD	7.02	7.66	5.03	12.17	8.87
CRT-TOMA (9/18)	M	9.13	9.47	8.5	8.98	8.93
	SD	2.43	1.98	2.07	4.72	3.83
CRT-WPS (16/32)	M	19.91	16.34	16.33	18.34	14.93
	SD	5.47	6	4.64	7.96	6.06
WPS Probes (8/16)	M	9.45	9.08	8.63	11.32	10.61
	SD	2.5	3.01	2.1	4.25	1.99
One-step (6/12)	M	8.38	7.89	7.5	8.86	8.18
	SD	1.76	2.29	1.72	3.08	1.49
Two-step (2/4)	M	1.08	1.19	1.13	2.47	2.46
	SD	0.94	1.06	0.89	1.35	0.75
Computation (25/25)	M	18.75*	11.81*	11.50*	18.03	20.37
	SD	2.82	3.87	5.68	3.82	4.46

Results

Posttests *(LD and LA sample: Pennsylvania Site)*

- Results indicated a main effect for time for CRT-Total, F (1,16) = 26.94, p =.000; CRT-WPS, F (1,16) = 28.17, p = .000; computation probe, F (1,16) = 64.61, p = .000; word problem solving probes -Total, F (1,16) = 8.56, p = .010; word problem solving probes-one-step, F (1,16) = 8.52, p = .010; and word problem solving probes-two-step, F (1,16) = 4.55, p = .049.
- A time by group interaction was found only on the CRT-TOMA, F (1,16) = 4.65, p = 0.047
  - The mean difference scores from pretest to posttest was higher for students with LD (M = 2.67) than that for LA students (M = 0.06).

Results



Posttests *(Low, Middle, and High Achievers: Florida site)*

- Results indicated a main effect for time for CRT-Total, $F(1,53) = 26.94$, $p = .000$; CRT-WPS, $F(1,53) = 12.27$, $p = .001$; computation probe, $F(1,53) = 18.08$, $p = .000$; word problem solving probes - Total, $F(1,16) = 8.56$, $p = .010$; word problem solving probes-one-step, $F(1,53) = 8.54$, $p = .005$; and word problem solving probes-two-step, $F(1,53) = 22.86$, $p = .000$.
- No significant main effect for time for the CRT-TOMA, $F(1,53) = 3.04$, $p = .087$
- A time by group interaction was found only on the WPS probes-Total scores, $F(2,53) = 17.49$, $p = .049$.
  - The mean difference scores from pretest to posttest were higher for low achieving students ($M = 2.82$) than for middle achievers ($M = 1.078$) or for high achievers ($M = .479$).

Posttests		LD (n=9)	LA (n=9)	Low (n=9)	Middle (n=23)	High (n=24)
CRT-Total (25/50)	M	22.61	25.83	16.44	23.59	31.08
	SD	5.61	7.64	12.5	9.40	8.34
CRT-TOMA (9/18)	M	8.78	9.00	5.83	8.20	10.85
	SD	1.77	2.14	5.20	3.40	3.92
CRT-WPS (16/32)	M	13.83	16.83	10.61	15.39	20.23
	SD	5.44	6.96	7.86	6.55	5.84
WPS Probes (8/16)	M	8.31	8.94	8.86	9.78	12.93
	SD	1.86	3.21	4.45	3.05	1.95
One-step (6/12)	M	7.22	7.86	7.22	7.60	9.91
	SD	1.51	2.68	3.07	2.36	1.49
Two-step (2/4)	M	1.08	1.08	1.64	2.19	3.04
	SD	0.78	0.86	1.53	0.95	0.69
Computation (25/25)	M	11.22	13.44	15.78	19.35	20.25
	SD	4.97	5.94	3.46	4.37	3.92

Results

Strategy Satisfaction (All Students: Pennsylvania Site)

- Results of a MANOVA indicated no significant differences between teachers (Wilk's lambda = .69; approximate F (2, 35) = 1.29, p = 0.25).

Strategy Questionnaire		Teacher1 (n = 16)	Teacher 2 (n =16)	Teacher 3 (n =6)	Total	ES
Enjoyed (5/5)	M	3.94	3.31	4.67	3.79	T1 vs. T2: +0.62
	SD	0.68	1.35	0.52	1.09	T1 vs. T3: -1.22
						T2 vs. T3: -1.45
Diagram (5/5)	M	3.81	3.87	4.67	3.97	T1 vs. T2: -0.06
	SD	0.83	1.2	0.82	1.03	T1 vs. T3: -1.04
						T2 vs. T3: -0.79
Help solve (5/5)	M	4.2	3.67	5	4.16	T1 vs. T2: +0.51
	SD	1.03	1.03	0	1.03	T1 vs. T3: -1.55
						T2 vs. T3: -2.58
Recommend (5/5)	M	3.56	3.88	4.83	3.89	T1 vs. T2: -0.24
	SD	1.32	1.31	0.41	1.27	T1 vs. T3: -1.47
						T2 vs. T3: -1.10
Continue (5/5)	M	3.81	3.69	4.33	3.84	T1 vs. T2: +0.10
	SD	1.47	0.95	0.82	1.17	T1 vs. T3: -0.45
						T2 vs. T3: -0.72
Total (5/5)	M	19.13	18.75	23.5	19.66	T1 vs. T2: +0.44
	SD	0.87	0.87	1.41	3.77	T1 vs. T3: -3.83
						T2 vs. T3: -4.17

Results

Strategy Satisfaction (LD and LA sample: Pennsylvania Site)

- Results of a MANOVA indicated a main effect for group (Wilk's lambda = .43; approximate F (1, 16) = 3.16, p = 0.047).
  - Results of separate univariate ANOVA's indicated significant differences between groups with regard to:
    - Total satisfaction scores, F (1, 16) = 13.02, p = .002,
    - Enjoyed the strategy, F (1, 16) = 6.15, p = .025,
    - Usefulness of the strategy in solving word problems, F (1, 16) = 16.49, p = .001, and
    - Continue to use the strategy in solving word problems, F (1, 16) = 8.24, p = .011
  - Specifically, LD students rated the items higher than LA students.

Strategy Questionnaire		LD (n = 9)	LA (n = 9)	Total	ES
Enjoyed (5/5)	M	4.56	3.44	4.00*	+1.20
	SD	0.73	1.13	1.08	
Diagram (5/5)	M	4.44	4	4.22	+0.44
	SD	1.13	0.87	1	
Help solve (5/5)	M	4.89	3.44	4.17***	+2.16
	SD	0.33	1.01	1.04	
Recommend (5/5)	M	4.56	3.89	4.22	+0.65
	SD	1.01	1.05	1.06	
Continue (5/5)	M	4.44	3	3.72**	+1.40
	SD	0.73	1.32	1.27	
Total (5/5)	M	22.89	17.78	20.33**	+1.71
	SD	3.26	2.73	3.93	

B.O.S.S. Results: *Means (%) and Standard deviations for Classroom Observations of Target Students vs. Peer Comparisons* (Pennsylvania Site)

Measure	Target Students (n = 4)	Comparison Peer (n = 4)	ES
AET			
Pre	48.81 (15.27)	64.56 (11.27)	-1.19
Post	43.35 (23.47)	46.27 (18.68)	-0.14
PET			
Pre	36.13 (11.02)	25.61 (7.47)	+1.14
Post	24.81 (17.45)	21.36 (16.47)	+0.20
OFT-M			
Pre	30.11 (8.68)	7.35 (9.88)	+2.45
Post	21.31 (30.21)	5.78 (7.12)	+0.83
OFT-V			
Pre	12.81 (5.96)	0.53 (1.05)	+3.50
Post	2.71 (2.37)	3.10 (3.54)	-0.13
OFT-P			
Pre	11.66 (4.67)	5.71 (6.41)	+1.07
Post	9.93 (6.17)	12.66 (9.83)	-0.34

Lessons Learned from Design Studies

- TEACHERS
  - Teachers need ongoing support during the initial implementation of a newly developed intervention
  - Teacher input helped us to:
    - Modify the curriculum (problem schema instruction, checklists)
    - Reevaluate and design the “compare” problem instruction
    - Modify the word problems to meet diverse student needs
    - Align instruction with state wide testing (e.g., problem types, use of explanation format)

Lessons Learned from Design Studies

- STUDENTS
  - Strategy instruction that incorporates explicit modeling and explanations using several examples enhances some students' problem solving skills; particularly lower performers
  - Student use of the strategy steps during paired or independent learning activities requires direct monitoring by the teacher
  - Scaffolded instruction (modeling, guiding, presenting schemata diagrams and checklists) is important as students learn to <u>apply</u> the strategy

Lessons Learned from Design Studies

- STUDENTS
  - Students may have been tired of the testing by the end of the school year; CRT is a long test - 25 items.
  - Consider the type of student groupings that might benefit most from the schema-based instruction designed for this study (i.e., homogeneous vs. heterogeneous).
  - Use of strategy fading, changes in the “compare” problem instruction, and extensive support to teachers may have helped students perform better in the Pennsylvania site.

Fading Change Diagrams: You had 32 French fries. You got 15 more French fries from your sister. How many French fries do you have now?

+15
French fries

C

32
French fries

B

?
French fries

E

Math Sentence: 32
+15
47

Fading Group Diagrams: A new bike costs $80. A new helmet costs $20. How much would it cost to buy the bike and the helmet?

New bike		New helmet		New bike and helmet
$80		$20		$?
____	+	____	=	____
SG		SG		LG

Math Sentence: $80
+$20
$100

Fading Compare Diagrams: Lin is 5 years older than his cousin. If Lin is 11 years old, how old is his cousin?

Lin		Lin's cousin		
11 years	−	? years	=	5 years
B		S		D

Math Sentence: 11

$$\begin{array}{r} - 5 \\ \hline 6 \\ \hline \end{array}$$

Compare Story Situation

Mitch has 43 music CD's and Anne has 70 music CD's. Anne has 27 more music CD's than Mitch.

From Schemas in Problem Solving (p. 135) by S. P. Marshall, 1995, New York: Cambridge University Press.

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