NUMERICAL SIMULATION OF COLD-FORMED CHANNEL SECTIONS WITH INTERMEDIATE WEB STIFFENERS UNDERGOING PURE SHEAR

SONG HONG PHAM
CAO HUNG PHAM
GREGORY J. HANCOCK

RESEARCH REPORT R930
JULY 2012
ISSN 1833-2781

SCHOOL OF CIVIL ENGINEERING
NUMERICAL SIMULATION OF COLD-FORMED CHANNEL SECTIONS WITH INTERMEDIATE WEB STIFFENERS UNDERGOING PURE SHEAR

RESEARCH REPORT R930

SONG HONG PHAM
CAO HUNG PHAM
GREGORY J. HANCOCK

July 2012

ISSN 1833-2781
Copyright Notice

School of Civil Engineering, Research Report R930
Numerical Simulation of Cold-formed Channel Sections with Intermediate Web Stiffeners Undergoing Pure Shear.

Song Hong Pham
Cao Hung Pham
Gregory J. Hancock

July 2012

ISSN 1833-2781

This publication may be redistributed freely in its entirety and in its original form without the consent of the copyright owner.

Use of material contained in this publication in any other published works must be appropriately referenced, and, if necessary, permission sought from the author.

Published by:
School of Civil Engineering
The University of Sydney
Sydney NSW 2006
Australia

This report and other Research Reports published by the School of Civil Engineering are available at http://sydney.edu.au/civil
ABSTRACT

This report presents a numerical solution to model a simply supported cold-formed steel beam subjected to the pure shear load case. The modelling procedures including the unique concept of pure shear loads in the Finite Element Method (FEM) are discussed thoroughly. The accuracy of the simulation is confirmed by comparison with the available test data. Based on this model, a number of shear strength analyses were conducted on cold-formed channel members with plain webs and various types of web stiffeners. The outcomes are graphically shown in two formats and related discussions are included. Application to the Direct Strength Method (DSM) of design of cold-formed sections is presented.

KEYWORDS

Cold-formed steel; Web stiffener; Shear strength; Pure shear; Complex channel sections; High strength steel; Direct strength method; ABAQUS.
# TABLE OF CONTENTS

ABSTRACT ....................................................................................................................................................... 3  
KEYWORDS .................................................................................................................................................... 3  
TABLE OF CONTENTS .................................................................................................................................... 4  
INTRODUCTION ............................................................................................................................................... 5  
FINITE ELEMENT SIMULATION ...................................................................................................................... 5  
  Boundary Conditions .................................................................................................................................. 5  
  Shear Load Modelling ............................................................................................................................... 6  
  Material Properties ..................................................................................................................................... 7  
  Element Type and Mesh ............................................................................................................................... 8  
  Eigenvale Buckling Prediction .................................................................................................................... 8  
  Post- Buckling Analysis .............................................................................................................................. 8  
  Initial Geometric Imperfection .................................................................................................................. 8  
VALIDATION OF FINITE ELEMENT MODEL .................................................................................................. 8  
  Validation of Buckling Analysis ................................................................................................................ 9  
  Validation of Shear Stress Distribution and Shear Failure Mode ............................................................. 9  
  Validation of Shear Load Application ....................................................................................................... 9  
  Validation of Shear Strength ...................................................................................................................... 10  
  Concluding Remarks on the Validation of FE Modelling ........................................................................ 15  
  Matlab Code for Inputting Purpose ........................................................................................................... 15  
THE EMPLOYMENT OF ELASTIC SHEAR BUCKLING LOAD ...................................................................... 15  
NUMERICAL ANALYSIS FOR SHEAR STRENGTH OF CHANNEL SECTIONS WITH SUBSTANTIAL WEB STIFFENERS UNDER PURE SHEAR .................................................................................. 17  
  Geometry of Lipped Channel Sections with Substantial Web Stiffeners ................................................. 17  
  Shear Strength of Channel Section with One Rectangular Web Stiffener ................................................. 18  
  Shear Strength of Channel Section with Triangular Web Stiffeners ........................................................ 21  
DISCUSSION .................................................................................................................................................. 25  
  The Influence of Initial Geometric Imperfection on Shear Strength of Plain Channel Sections .......... 25  
  Understanding Tension Field Action ......................................................................................................... 25  
  The Effect of Stiffener Geometry .............................................................................................................. 26  
  Discussion on FEM Results ....................................................................................................................... 26  
  The Relations Between the $V_r$ Determined by the SAFSM and the FEM .............................................. 27  
CONCLUSIONS .............................................................................................................................................. 29  
  General ..................................................................................................................................................... 29  
  Finite Element Analysis ............................................................................................................................ 29  
ACKNOWLEDGEMENT .................................................................................................................................. 30  
REFERENCES ................................................................................................................................................ 31  
APPENDIX : SHEAR STRENGTH ANALYSIS RESULTS FOR STIFFENED MEMBERS ............................ 33
INTRODUCTION

Numerical simulation using the Finite Element Method (FEM) of cold-formed steel structures has been accurately calibrated against tests and used to produce a design database for the Direct Strength Method (DSM) of design (Yang and Hancock, 2006, Yu and Schafer, 2006, 2007). The DSM was formally incorporated in the 2007 Edition of the North American Specification (AISI S100-2007) and the Australian/New Zealand Standard for Cold-Formed Steel Structures (AS/NZS 4600:2005) as an alternative to the traditional Effective Width Method (EWM). In these standards and specification, the DSM only applies to axial compression and bending of members. The newly developed Direct Strength Method (DSM) of design for shear has recently been incorporated in 2012 Edition of the North American Specification (NAS S100-2012) by the American Iron and Steel Institute (AISI) Specification Committee. These new rules were based on predominantly shear tests conducted at the University of Missouri Rolla for sections with plain webs and the University of Sydney for sections with plain webs and very small multiple intermediate stiffeners. The purpose of this report is to extend the data numerically using the FEM to include larger intermediate stiffeners in the web.

In order to explicitly understand the shear behaviour of thin-walled sections, it is necessary to investigate a member in a state of pure shear. While an experiment has not been set up for such loading condition, a numerical model of a cold-formed member subjected to shear alone seems to be an appropriate approach. This report presents the procedures to produce such an FEM using the ABAQUS/Standard (2010) package. Various factors are thoroughly studied to determine an appropriate solution. The accuracy of the numerical simulations is validated before a series of FE analyses is performed. Firstly, buckling analyses are conducted by means of the ABAQUS program and the buckling stresses are compared with the outcomes of the well-known program Isoparametric Spline Finite Strip Method (ISFSM) (Eccher, 2007). Further, the predominantly shear test series conducted by Pham and Hancock (2009a, b) are simulated with a few modifications to convert to the pure shear state. The agreement between numerical and experimental results shows that the FE model is reliable to investigate the shear behaviour of cold-formed sections. The analyses are conducted on channel sections with one rectangular, one triangular or two triangular intermediate stiffeners in the web where the stiffener shapes, dimensions and locations are varied. The shear strengths are then compared with the DSM design equations previously proposed (Pham and Hancock, 2012).

FINITE ELEMENT SIMULATION

Boundary Conditions

This report develops an FEM capable of achieving idealized simply supported conditions. This allows the elastic buckling analysis to be verified against the results of the ISFSM program (Eccher, 2007) which assumes simply supported ends. Further, the predominantly shear test series by Pham and Hancock (2009a,b) were also conducted with simply supported end configurations. It is obvious that the idealized simple support as mentioned above rarely exists in practice. However, the study of those boundary conditions is very important to understand theoretically the behaviour of thin-walled members under pure shear. It is worth noting that no experiment on a cold-formed member undergoing shear alone has been set up in practice although conditions close to this can be achieved.

The end boundary conditions for the 3D FEM is shown in Fig. 1 and the restraint conditions are summarised in Table 1. It is noted that one point at the middle of the web at one end section is restrained longitudinally to prevent rigid body motion.

<table>
<thead>
<tr>
<th>Table 1: End Boundary Conditions for 3D Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
<tr>
<td>Lips</td>
</tr>
<tr>
<td>Flanges</td>
</tr>
<tr>
<td>Web</td>
</tr>
</tbody>
</table>

Note: 1 denotes Fixed, 0 denotes Free

Figure 1: End Boundary Conditions for 3D model
Shear Load Modelling
Shear loads may be applied to a structure directly by pure shear forces or shear forces resulting from moment gradient. The former approach may provide an in-depth understanding of pure shear behaviour of cold-formed structures while the latter represents a common load case in practice. The shear flow resulting from a shear force through the section shear centre is shown in Fig. 2.

Pure Shear Load Modelling
Since the cold-formed member studied herein is limited to the ratio of shear span and section depth of 1:1, the effect of the shear flow varying around the cross-section is important. Thus the FE model needs to be able to simulate the variation of the shear flow. The member is meshed into the finite elements as shown in Fig. 3(a) so that at each element the shear stress distribution is assumed to be uniform. The more the cross-section is subdivided, the more accurately the shear stress is represented in order to match the practical shear flow distribution. Fig. 3(b) shows the stress states at element 1 and element 2 extracted from the web. As a result of the shear flow variation, the shear stress $\tau_2$ is larger than $\tau_1$. Consequently, a difference of $(\tau_2 - \tau_1)$ exists at the common edge between two elements.

In practice, the overall equilibrium of the whole structure is maintained since the unbalanced shear stresses are balanced by the moment gradient resulting from the couple of shear forces at two end sections as shown in Fig. 3(c).

Based on the concept described in Figs 3(a) and (b), the pure shear load may be simulated by a series of longitudinal concentrated forces at the common nodal lines between the strips and the two free edges of the lips as shown in Fig. 4(a). The magnitudes of the concentrated forces applied at a particular line are determined as the difference of the shear flows between two adjacent strips. These values are then distributed in each node along the longitudinal line. The values at the centre line of the web are zero since the shear flows at the two adjacent strips are equal. In fact, when the longitudinal forces reach the end sections where there is no restraint along the member length, shear flows will be generated to balance those forces as shown in Fig. 4(b).
Numerical Simulation of Cold-Formed Channel Sections With Intermediate Web Stiffeners under Pure Shear

The University of Sydney
School of Civil Engineering
Research Report R930
Page 7

Shear Force Resulting from Moment Gradient

This load case is simply applied by the line loads at the end section resulting from the moment $M = VL$ as shown in Fig. 5. These loads produce a moment gradient and corresponding uniform shear force.

Material Properties

In this report, both linear and nonlinear analyses are performed. In the linear analysis, only the Young's modulus $E$ and Poisson's ratio $\nu$ for steel are required. These values are taken as 200000 MPa and 0.3, respectively. For non-linear analysis, a stress and strain curve is employed to follow the real behaviour of material. The available data from the tensile coupon testing by Pham and Hancock (2009a,b) provided the stress-strain data for an isotropic material experiencing uniaxial loading.

The Von Mises' yield criterion is adopted. Accordingly, in the case of pure shear stress, at the onset of yielding, the magnitude of the shear stress in pure shear is $\sqrt{3}$ times lower than the tensile stress in the case of simple tension.

$$\tau_y = \frac{f_y}{\sqrt{3}} \quad (Eq.1)$$

where $f_y$ is the tensile yield point for the material.
Element Type and Mesh

The element S4R in the ABAQUS library is employed. This element uses three translation and three rotational degrees of freedom at each node. The element accounts for finite membrane strains and arbitrarily large rotations. Therefore, it is suitable for large-strain analyses and geometrically nonlinear problems. To establish an adequate mesh refinement, the mesh size 10mm x 10mm is employed.

Eigenvalue Buckling Prediction

In ABAQUS/Standard, the *BUCKLE procedure can estimate elastic buckling by eigenvalue extraction. This estimation is typically useful for “stiff” structures, where the pre-buckling response is almost linear. The buckling load estimate is obtained as a multiplier of the pattern of perturbation loads, which are added to a set of base state loads. Eigenvalue buckling analysis results can also be used in the investigation of the imperfection sensitivity of a structure. Mathematically, eigenvalue buckling analyses evaluate the singularities in a linear perturbation of the structure’s stiffness matrix so that the problem:

\[ K\delta = 0 \] (Eq. 2)

has nontrivial solutions. \( K \) is the tangent stiffness matrix when the loads are applied and \( \delta \) are nontrivial displacement solutions. In this study, since the numbers of necessary eigenmodes are not large, the subspace iteration eigensolver is employed to speed the analyses.

Post-Buckling Analysis

For structures for which material nonlinearity and geometric nonlinearity or unstable post-buckling response are of interest, the load-deflection (Riks) analysis must be performed to investigate the problem further. Such analysis can be utilized by *STATIC, RIKS procedure in ABAQUS.

The Riks method uses the load magnitude as an additional unknown to solve simultaneously for loads and displacements. Therefore, another quantity called “arc length” is employed to follow the static equilibrium path in load-displacement space. Riks (1972, 1979) proposed an increment approach to confront the buckling and snapping problems. The Riks method effectively solves the snap-through problem in which the equilibrium path in load-displacement space is smooth and does not branch. However, the exact post-buckling problem cannot be solved directly due to the discontinuous response at the point of buckling. To analyse such problems, it must be turned into a problem with continuous response instead of bifurcation. This effect can be accomplished by introducing an initial imperfection into a “perfect” geometry so that there is some response in the buckling mode before the critical load is reached.

Initial Geometric Imperfection

To analyse the exact post-buckling problem, an initial imperfection should be included into the “perfect” geometry. Unless the precise shape of an imperfection is known, an imperfection consisting of multiple superimposed buckling modes is normally the most appropriate.

An initial analysis is carried out on a perfect mesh using the elastic buckling analysis to generate the possible buckling modes and nodal displacements of these modes. The imperfections are introduced to the perfect mesh by means of linearly superimposing the elastic buckling modes onto the mesh. The lowest buckling modes are usually the critical modes and these are, therefore, used to generate the imperfections. The coordinates of the eigenmodes obtained from the buckling analysis are by default stored in a file with extension *.fil and can subsequently be used as input for the *IMPERFECTION command in the actual simulation with different scaling factors with respect to the thickness of the channel. The imperfection magnitudes for cold-formed steel structures are normally based on two scaling factors of 0.15\( t \) and 0.64\( t \) where \( t \) is the thickness of channel section. These two factors were proposed by Camotim and Silvestre (2004) and Schafer and Pekoz (1998) respectively.

VALIDATION OF FINITE ELEMENT MODEL

A channel section member of 200mm length consisting of a 200mm web depth, 80mm flange width, 20mm lip size and 2mm thickness is utilized. The boundary conditions are as depicted in Fig. 1. A reference pure shear load of 10kN is applied through the shear centre. The incorporation of this shear force follows the concept shown in Fig. 4. The ratio of the shear span to web depth is 1:1.
Validation of Buckling Analysis

The elastic buckling analysis is performed by means of the ABAQUS program and the results are compared with the outcomes obtained from the ISFSM (Eccher, 2007) program. The results are summarized in Fig. 6. The eigenvalue obtained from the ISFSM is 7.2041 which is the same as the value computed by Pham and Hancock (2009c). The eigenvalue extracted from the buckling analysis by ABAQUS is 7.2093 and almost identical to the value from the ISFSM. Also, the buckling mode shapes plotted by these programs are similar. Therefore, it can be concluded that the FE model is appropriate in terms of buckling analysis.

<table>
<thead>
<tr>
<th></th>
<th>ISFSM</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigen Value</td>
<td>7.2041</td>
<td>7.2093</td>
</tr>
<tr>
<td>Buckling Mode Shape</td>
<td><img src="image1" alt="ISFSM Buckling Shape" /></td>
<td><img src="image2" alt="FEM Buckling Shape" /></td>
</tr>
</tbody>
</table>

Figure 6: The Comparison of Buckling Loads and Buckling Mode Shapes.

Validation of Shear Stress Distribution and Shear Failure Mode

The nonlinear analysis is performed for the above member. An initial geometric imperfection of 0.15t (Camotim and Silvestre, 2004) is employed to the shear strength analysis by superimposing the elastic buckling mode.

<table>
<thead>
<tr>
<th>Shear Stress Distribution</th>
<th>Shear Field</th>
<th>Failure mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>End section</td>
<td><img src="image3" alt="Shear Field" /></td>
<td><img src="image4" alt="Failure mode shape" /></td>
</tr>
<tr>
<td>Mid-section</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Shear Stress Distribution  

Figure 7: Shear Strength Analysis Results

Fig. 7(a) shows the shear stress distributions around the cross-sections at the initial step of the analysis which are unchanged along the member length and very close to the classical shear stress distribution as shown in Fig. 2. Fig. 7(b) displays the shear field and failure mode shape propagating from the 45° inclined shear field. This failure mode is commonly observed in the shear test as can be seen in Fig. 9(b).

Validation of Shear Load Application

As mentioned previously, there are two approaches to apply shear loads including the application of direct pure shear loads or indirect shear forces resulting from moment gradient.
The applied shear curves for a plain channel member subjected to pure shear or moment gradient are shown in Fig. 8. The cross-section is chosen consistently with the section mentioned previously. ‘V’ denotes the case where pure shear load is applied and ‘M’ represents the case where shear force is applied indirectly via moment gradient.

It can be observed that the ultimate shear strengths are almost the same for both load cases. This is understandable since the ratio of the span to the section depth (L/b₁) is unity. For such a structure, the loading is predominantly shear and moment only plays a minor role. Further, as will be discussed later, the strength of a plain channel member with supports as in Fig. 1 is significantly dependent on the Tension Field Action (TFA). Distinct failure modes of a member under pure shear and moment gradient may affect the development of the TFA. Therefore, for a relatively short span structure (L/b₁=1), the shear strength for the moment gradient load case is not necessarily always less than that for the pure shear load case. Due to the fact that the effect of moment on shear strength is negligible, the study hereafter only focuses on the pure shear load case.

Validation of Shear Strength

The predominantly shear test series conducted by Pham and Hancock (2009a, b) at the University of Sydney are simulated by means of the ABAQUS program for the pure shear load case. The numerical models are set up as described in Fig. 1 and Fig. 4 where only one shear span is considered. It is worth noting that in the predominantly shear tests, the structure still resists a small moment gradient as shown in Fig. 9(a). However, as shown in Fig. 8, the effect of the moment gradient on the ultimate strength for a member with the ratio of L/b₁ equal to 1:1 is negligible. Therefore, the numerical model subjected for pure shear loads is employed in order to be consistent with the aim of this report.

Further, as can be seen in Fig. 9(a), at the supports and mid span, the vertical rows of M12 high tensile bolts were used to configure the test system. This provides the boundary conditions slightly different from those in the FE model where the ideal simple supports are set up. As a result, the ultimate applied shear loads obtained from FE analyses would be expected to be slightly smaller than those from the predominantly shear tests.
Test Configuration

A diagram of the test set-up is shown in Fig. 9(a) for the predominantly shear (V) tests. The ratio of shear span to depth, the distance between the lines of bolts at the loading and support points, is 1:1. The channel section members were tested in pairs with the flanges facing inwards. For more details for the test configuration, refer to (Pham, C.H, 2010).

Figure 9: Test Configuration and Deformation Mode for Predominantly Shear Test.
**Test Series**

Two test groups with straps across the flanges are selected to be simulated numerically including the shear tests for both the Plain-C purlins and the SupaCee® Section purlins as shown in Fig. 11(a) and (b) respectively. The amount of the straps and their locations are demonstrated in Fig. 10(a). Each group contains six tests in which half are for the 150mm section depth and the rest are for the 200mm section depth. The tests without the straps as shown in Fig. 10(b) were not selected because it is believed that the boundary conditions in Fig. 1 represent the case with the straps.

![Figure 10. Shear Test Series Configuration With and Without Straps](image)

The test specimens were labelled to describe the section, depth and thickness. For instance, the designation ‘C15015’ is defined as follows:

- ‘C’ denotes a channel section (alternatively SC denotes a SupaCee® section).
- ‘150’ indicates the depth of the web (alternatively ‘200’).
- ‘15’ is the thickness times 10 in mm (alternatively ‘19’ and ‘24’).

For the plain-C section group, the nomenclature and mean dimensions of the specimens are summarized in Table 2 and Fig. 11(a).

<table>
<thead>
<tr>
<th>Section</th>
<th>t (mm)</th>
<th>D (mm)</th>
<th>B (mm)</th>
<th>L (mm)</th>
<th>$f_y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15015</td>
<td>1.5</td>
<td>153.30</td>
<td>64.58</td>
<td>15.03</td>
<td>541.13</td>
</tr>
<tr>
<td>C15019</td>
<td>1.9</td>
<td>153.31</td>
<td>64.98</td>
<td>15.73</td>
<td>534.48</td>
</tr>
<tr>
<td>C15024</td>
<td>2.4</td>
<td>153.37</td>
<td>62.94</td>
<td>19.23</td>
<td>485.29</td>
</tr>
<tr>
<td>C20015</td>
<td>1.5</td>
<td>204.64</td>
<td>76.53</td>
<td>15.58</td>
<td>513.40</td>
</tr>
<tr>
<td>C20019</td>
<td>1.9</td>
<td>202.13</td>
<td>78.50</td>
<td>17.41</td>
<td>510.48</td>
</tr>
<tr>
<td>C20024</td>
<td>2.4</td>
<td>203.38</td>
<td>77.38</td>
<td>22.41</td>
<td>483.49</td>
</tr>
</tbody>
</table>

Internal radius $r = 5\text{mm}$

For the SupaCee® section group, the nomenclature and dimensions of six specimens are summarized in Table 3 and Fig. 11(b).

<table>
<thead>
<tr>
<th>Section</th>
<th>t (mm)</th>
<th>D (mm)</th>
<th>B (mm)</th>
<th>l (mm)</th>
<th>lr (mm)</th>
<th>GS (mm)</th>
<th>$\theta_1$ (°)</th>
<th>$\theta_2$ (°)</th>
<th>$f_y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC15012</td>
<td>1.2</td>
<td>153.56</td>
<td>41.87</td>
<td>5.09</td>
<td>6.22</td>
<td>63.85</td>
<td>55.5</td>
<td>84.5</td>
<td>589.71</td>
</tr>
<tr>
<td>SC15015</td>
<td>1.5</td>
<td>152.64</td>
<td>42.49</td>
<td>4.78</td>
<td>5.83</td>
<td>63.48</td>
<td>55.5</td>
<td>81.5</td>
<td>533.88</td>
</tr>
<tr>
<td>SC15024</td>
<td>2.4</td>
<td>153.66</td>
<td>44.49</td>
<td>4.81</td>
<td>4.99</td>
<td>60.84</td>
<td>51.5</td>
<td>81.5</td>
<td>513.68</td>
</tr>
<tr>
<td>SC20012</td>
<td>1.2</td>
<td>205.45</td>
<td>54.61</td>
<td>6.96</td>
<td>6.15</td>
<td>109.43</td>
<td>55.5</td>
<td>84.0</td>
<td>593.30</td>
</tr>
<tr>
<td>SC20015</td>
<td>1.5</td>
<td>203.77</td>
<td>54.19</td>
<td>7.28</td>
<td>6.88</td>
<td>109.46</td>
<td>54.5</td>
<td>87.5</td>
<td>532.03</td>
</tr>
<tr>
<td>SC20024</td>
<td>2.4</td>
<td>204.16</td>
<td>54.67</td>
<td>6.57</td>
<td>8.35</td>
<td>111.48</td>
<td>55.0</td>
<td>85.5</td>
<td>504.99</td>
</tr>
</tbody>
</table>

Internal radius $r = 5\text{mm}$

---

Numerical Simulation of Cold-Formed Channel Sections With Intermediate Web Stiffeners under Pure Shear
Experimental Results

The experimental results are summarized in Table 4 where $P_T$ is the mean applied load of the 3 tests with straps of each section and $V_T$ is the corresponding mean shear load applied to a single member.

Table 4: Mean Experimental Results (Tests with Straps)

<table>
<thead>
<tr>
<th>Section</th>
<th>$P_T$ (kN)</th>
<th>$V_T = P_T/4$ (kN)</th>
<th>Section</th>
<th>$P_T$ (kN)</th>
<th>$V_T = P_T/4$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15015</td>
<td>221.31</td>
<td>55.33</td>
<td>SC15012</td>
<td>168.5</td>
<td>42.13</td>
</tr>
<tr>
<td>C15019</td>
<td>307.04</td>
<td>76.76</td>
<td>SC15015</td>
<td>222.3</td>
<td>55.58</td>
</tr>
<tr>
<td>C15024</td>
<td>381.12</td>
<td>95.28</td>
<td>SC15024</td>
<td>392.0</td>
<td>97.99</td>
</tr>
<tr>
<td>C20015</td>
<td>223.72</td>
<td>55.93</td>
<td>SC20012</td>
<td>185.9</td>
<td>46.48</td>
</tr>
<tr>
<td>C20019</td>
<td>341.27</td>
<td>85.32</td>
<td>SC20015</td>
<td>248.3</td>
<td>62.07</td>
</tr>
<tr>
<td>C20024</td>
<td>455.55</td>
<td>113.89</td>
<td>SC20024</td>
<td>496.8</td>
<td>124.21</td>
</tr>
</tbody>
</table>

Figure 12: DSM Shear Curves and Shear Test Data

$$\lambda_i = \frac{V_y}{V_{cr}}$$
Fig. 12 shows all of the test data and nominal shear capacity curves which include the Tension Field Action (TFA) curve (Basler, 1961), the DSM shear curve without TFA, the elastic buckling curve and the DSM shear curve including TFA. As can be seen, all the test results lie close to the DSM shear curve with TFA. They lie well above the DSM shear curve without TFA presumably because significant TFA was developed in the tests.

**Comparison of Experimental Results and Numerical Results**

Table 5: Finite Element Analysis Results for Plain C and SupaCee® Tests

<table>
<thead>
<tr>
<th>Applied Shear (V_{abq})</th>
<th>V_{abq}/V_{test} (%)</th>
<th>f_y (MPa)</th>
<th>V_{cr} (kN)</th>
<th>V_{y} (kN)</th>
<th>\lambda_v</th>
<th>V_{abq}/V_{y}</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15015</td>
<td>48.01</td>
<td>86.77</td>
<td>541.13</td>
<td>42.58</td>
<td>1.27</td>
<td>0.70</td>
</tr>
<tr>
<td>C15019</td>
<td>70.10</td>
<td>91.32</td>
<td>534.48</td>
<td>86.08</td>
<td>1.00</td>
<td>0.82</td>
</tr>
<tr>
<td>C15024</td>
<td>88.71</td>
<td>93.10</td>
<td>485.29</td>
<td>97.83</td>
<td>0.75</td>
<td>0.91</td>
</tr>
<tr>
<td>C20015</td>
<td>49.38</td>
<td>88.29</td>
<td>513.40</td>
<td>31.29</td>
<td>1.68</td>
<td>0.56</td>
</tr>
<tr>
<td>C20019</td>
<td>72.62</td>
<td>85.12</td>
<td>510.48</td>
<td>63.68</td>
<td>1.32</td>
<td>0.66</td>
</tr>
<tr>
<td>C20024</td>
<td>101.20</td>
<td>88.86</td>
<td>483.49</td>
<td>127.05</td>
<td>1.02</td>
<td>0.77</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Applied Shear (V_{abq})</th>
<th>V_{abq}/V_{test} (%)</th>
<th>f_y (MPa)</th>
<th>V_{cr} (kN)</th>
<th>V_{y} (kN)</th>
<th>\lambda_v</th>
<th>V_{abq}/V_{y}</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC15012</td>
<td>39.59</td>
<td>93.96</td>
<td>589.71</td>
<td>35.89</td>
<td>1.29</td>
<td>0.67</td>
</tr>
<tr>
<td>SC15015</td>
<td>52.17</td>
<td>93.87</td>
<td>533.88</td>
<td>60.14</td>
<td>1.06</td>
<td>0.78</td>
</tr>
<tr>
<td>SC15024</td>
<td>94.25</td>
<td>96.19</td>
<td>513.68</td>
<td>197.26</td>
<td>0.72</td>
<td>0.91</td>
</tr>
<tr>
<td>SC20012</td>
<td>39.73</td>
<td>85.48</td>
<td>593.30</td>
<td>22.07</td>
<td>1.92</td>
<td>0.49</td>
</tr>
<tr>
<td>SC20015</td>
<td>52.28</td>
<td>84.24</td>
<td>532.03</td>
<td>38.78</td>
<td>1.53</td>
<td>0.57</td>
</tr>
<tr>
<td>SC20024</td>
<td>109.20</td>
<td>87.92</td>
<td>504.99</td>
<td>137.51</td>
<td>1.00</td>
<td>0.79</td>
</tr>
</tbody>
</table>

**Figure 13: DSM Shear Curves and Numerical Modelling Data**
Table 5 summarizes the results obtained from the numerical modelling. The non-dimensional slenderness ratio is calculated as \( \lambda_v = \sqrt{\frac{V_y}{V_{cr}}} \). It is noted that the \( V_{cr} \) used in the formula is the buckling load computed by the ABAQUS program. For both plain-C section and SupaCee® section, \( V_f \) is determined by the classical formula \( V_f = 0.6 f_y b_w t_w \) where \( b_w \) is the projected depth of the web and \( t_w \) is the thickness of the web. As can be seen in Table 5, the applied shear loads achieved from numerical simulations are on average about 6% and 13% less than those produced by the experiments for the 150mm and 200mm shear spans respectively. The reason for these differences was discussed previously and is most likely due to the idealised simply supported boundary conditions compared with the tests.

In Fig. 13, the shear strength data resulting from FEM are plotted in comparison with the DSM shear strength curves. As can be seen, the numerical simulation results closely follow the shear strength curve where the Tension Field Action (TFA) is included although they are on average about 7.5% lower.

Concluding Remarks on the Validation of FE Modelling

The relative matching between the numerical and experimental results proves that the FE model where a longitudinal concentrated load pattern is used to simulate the pure shear load case and the boundary conditions as shown in Fig. 1 is appropriate. Therefore, this model can be employed to investigate the shear strength of cold-formed channel members with various types of longitudinal web stiffeners.

Since the FEM results lie close to the shear strength curve with TFA, it is implied that those FE models have created TFA. However, it has not been clear how the TFA affects the ultimate shear strength of a channel member with different types of web stiffeners.

Matlab Code for Inputting Purpose

In order to speed up the simulations and avoid any possible input errors, simple Matlab (2011) code was written to generate an input file for ABAQUS. This input file normally has the extension *.inp and includes all necessary data to create a complete model such as nodes, elements, applied loads, boundary conditions, method of analysis and so on. Once such an input file is available, it can be imported to ABAQUS and no further manual data input is needed.

THE EMPLOYMENT OF ELASTIC SHEAR BUCKLING LOAD

As mentioned above, the DSM nominal shear strength \( (V_v) \) including TFA is determined as

\[
V_v = \left[ 1 - 0.15 \left( \frac{V_{cr}}{V_y} \right)^{0.4} \right] \left( \frac{V_{cr}}{V_v} \right)^{0.4} V_y
\]

(Eq.3)

Accordingly, the shear strength \( (V_v) \) relies on two terms namely the shear strength at yielding \( (V_y) \) and elastic shear buckling load \( (V_{cr}) \). The buckling capacity may be determined by several methods including the Finite Element Method (FEM), the Spline Finite Strip Method (SFSM) (Pham and Hancock, 2009c) and the Semi-Analytical Finite Strip Method (SAFSM) (Hancock and Pham, 2011, 2012).

The SAFSM was originally derived by Cheung (1976) for stress analysis of simply supported isotropic and orthotropic plates in bending. Hancock (1978) applied the SAFSM to beams and firstly identified local, distortional and lateral-torsional modes. The signature curve for a beam being the buckling stress versus the buckling half-wavelength for a single half-wavelength was also identified by Hancock (1978). The buckling loads corresponding to the minimum points in the signature curves for compression and bending have been incorporated in the DSM design equations adopted in AS/NZS 4600:2005 and AISI S100-2007. For shear, Hancock and Pham (2011, 2012) employed the SAFSM to study pure shear buckling of cold-formed channel section with and without web stiffener. The shear signature curve was isolated and provided the buckling load for shear strength determination.

As discussed previously, the buckling analyses conducted by means of the SFSM and the FEM provide similar results. It is noted that both the SFSM and the FEM assume the beam is simply supported at the two
end sections. Conversely, the SAFSM is based on the assumption that both end sections are free to distort. This corresponds to the case of a buckle half-wavelength as part of a much longer length of section. As a result, the buckling load for a specific section produced by the SAFSM is smaller than by the other methods as shown in Table 6.

Table 6: Buckling Loads Determined by SAFSM and FEM

<table>
<thead>
<tr>
<th>Designation</th>
<th>( V_{cr} ) (SAFSM) (kN)</th>
<th>( V_{cr} ) (FEM) (kN)</th>
<th>( \frac{V_{cr(SAFSM)}}{V_{cr(FEM)}} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C20012</td>
<td>10.17</td>
<td>15.96</td>
<td>63.72</td>
</tr>
<tr>
<td>C20015</td>
<td>19.83</td>
<td>31.29</td>
<td>63.36</td>
</tr>
<tr>
<td>C20019</td>
<td>40.17</td>
<td>63.68</td>
<td>63.08</td>
</tr>
<tr>
<td>C20024</td>
<td>80.57</td>
<td>127.1</td>
<td>63.41</td>
</tr>
</tbody>
</table>

Fig. 14 shows the buckling loads determined by means of the SAFSM and the FEM for a plain channel member with various thicknesses as in Table 6. The abscissa represents the thickness whereas the ordinate shows the critical buckling load. As can be seen, the values obtained by the FEM are larger than those achieved by the SAFSM. The difference is approximately 36% irrespective of the thicknesses. Therefore, for a plain channel member, it is possible to use the elastic shear buckling capacity from the analysis using the SAFSM. This possibility is significantly beneficial for a design engineer since the SAFSM is much more practical than the awkward and time consuming FEM in terms of structural design. However, as studied in Pham SH, Pham CH and Hancock (2012a,b), the occurrence of longitudinal stiffeners may eliminate the minimum point in the shear signature curve. Therefore, the application of the SAFSM in shear strength determination has not been available especially in the case of substantial web stiffeners. The analysis hereafter employs the buckling load obtained by the FEM and a discussion of the above issue is continued later in this report. The shear strengths of a plain channel member with three thicknesses are plotted in Fig. 15 where the elastic buckling load is determined by means of the SAFSM. The data shift significantly to the right and well above the DSM curve due to the decrease of \( V_{cr} \), i.e. the increase of the slenderness (\( \lambda_v \)).
Figure 15: Numerical Results in Which $V_{cr}$ Is Determined by the SAFSM

NUMERICAL ANALYSIS FOR SHEAR STRENGTH OF CHANNEL SECTIONS WITH SUBSTANTIAL WEB STIFFENERS UNDER PURE SHEAR

Geometry of Lipped Channel Sections with Substantial Web Stiffeners

The geometries of cross-sections studied in this part are consistent with the sections studied in Pham SH, Pham CH and Hancock (2012a,b). However, the thicknesses herein vary from 1.2mm to 2.4 mm. The geometries of the sections and stiffener shapes are reproduced in Fig. 16. An initial geometric imperfection of $0.15t$ is incorporated in the shear strength analyses.

Figure 16: Lipped Channel Geometry with Intermediate Web Stiffeners
Shear Strength of Channel Section with One Rectangular Web Stiffener

Stiffener Width $b_{s2}=5\text{mm}$, Thickness $t=1.2\text{mm}$

Table 8 shows the ABAQUS simulated strength for a 5mm wide rectangular stiffener in the web ($b_{s2}=5\text{mm}$) and a range of stiffener depths ($b_{s1}$).

### Table 8: Shear Strengths for Members with One Rectangular Stiffener (with $b_{s2}=5\text{mm}$, $t=1.2\text{mm}$)

<table>
<thead>
<tr>
<th>$b_{s1}$ (mm)</th>
<th>Designation</th>
<th>Shear Load ($V_{abq}, \text{kN}$)</th>
<th>$f_y$ (MPa)</th>
<th>$V_{cr}$ (kN)</th>
<th>$V_y$ (kN)</th>
<th>$\lambda_v$</th>
<th>$V_{abq}/V_y$</th>
<th>$V_{abq}/V_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>C20012</td>
<td>37.92</td>
<td>593.30</td>
<td>15.96</td>
<td>82.22</td>
<td>2.27</td>
<td>0.46</td>
<td>2.38</td>
</tr>
<tr>
<td>5</td>
<td>ST20012-RE5x5-V</td>
<td>37.06</td>
<td>593.30</td>
<td>30.96</td>
<td>82.22</td>
<td>1.63</td>
<td>0.45</td>
<td>1.20</td>
</tr>
<tr>
<td>10</td>
<td>ST20012-RE10x5-V</td>
<td>38.34</td>
<td>593.30</td>
<td>37.27</td>
<td>82.22</td>
<td>1.49</td>
<td>0.47</td>
<td>1.03</td>
</tr>
<tr>
<td>20</td>
<td>ST20012-RE20x5-V</td>
<td>40.02</td>
<td>593.30</td>
<td>46.38</td>
<td>82.22</td>
<td>1.33</td>
<td>0.51</td>
<td>0.91</td>
</tr>
<tr>
<td>30</td>
<td>ST20012-RE30x5-V</td>
<td>45.21</td>
<td>593.30</td>
<td>53.82</td>
<td>82.22</td>
<td>1.24</td>
<td>0.55</td>
<td>0.84</td>
</tr>
<tr>
<td>40</td>
<td>ST20012-RE40x5-V</td>
<td>50.37</td>
<td>593.30</td>
<td>62.07</td>
<td>82.22</td>
<td>1.15</td>
<td>0.61</td>
<td>0.81</td>
</tr>
<tr>
<td>50</td>
<td>ST20012-RE50x5-V</td>
<td>54.90</td>
<td>593.30</td>
<td>70.04</td>
<td>82.22</td>
<td>1.08</td>
<td>0.67</td>
<td>0.78</td>
</tr>
<tr>
<td>60</td>
<td>ST20012-RE60x5-V</td>
<td>59.48</td>
<td>593.30</td>
<td>70.51</td>
<td>82.22</td>
<td>1.08</td>
<td>0.72</td>
<td>0.84</td>
</tr>
<tr>
<td>80</td>
<td>ST20012-RE80x5-V</td>
<td>56.52</td>
<td>593.30</td>
<td>51.05</td>
<td>82.22</td>
<td>1.27</td>
<td>0.69</td>
<td>1.11</td>
</tr>
<tr>
<td>100</td>
<td>ST20012-RE100x5-V</td>
<td>48.57</td>
<td>593.30</td>
<td>38.58</td>
<td>82.22</td>
<td>1.46</td>
<td>0.59</td>
<td>1.26</td>
</tr>
<tr>
<td>120</td>
<td>ST20012-RE120x5-V</td>
<td>44.15</td>
<td>593.30</td>
<td>31.41</td>
<td>82.22</td>
<td>1.62</td>
<td>0.54</td>
<td>1.41</td>
</tr>
<tr>
<td>140</td>
<td>ST20012-RE140x5-V</td>
<td>41.15</td>
<td>593.30</td>
<td>26.57</td>
<td>82.22</td>
<td>1.76</td>
<td>0.50</td>
<td>1.55</td>
</tr>
<tr>
<td>160</td>
<td>ST20012-RE160x5-V</td>
<td>40.47</td>
<td>593.30</td>
<td>22.90</td>
<td>82.22</td>
<td>1.89</td>
<td>0.49</td>
<td>1.77</td>
</tr>
<tr>
<td>180</td>
<td>ST20012-RE180x5-V</td>
<td>40.15</td>
<td>593.30</td>
<td>18.99</td>
<td>82.22</td>
<td>2.08</td>
<td>0.49</td>
<td>2.11</td>
</tr>
</tbody>
</table>

In Table 8, the designation ST20012-RE10x5-V, for instance, is defined as follows:
- ‘ST200’ denotes the stiffened channel member with the overall section depth is 200mm.
- ‘12’ is the thickness times 10 in mm.
- ‘RE10x5’ indicates a rectangular stiffener with a depth 10mm and a width 5mm.
- ‘V’ represents the pure shear load case.

![Figure 17: Shear Buckling Loads and Ultimate Shear Strengths for Members with One Rectangular Stiffener (with $b_{s2}=5\text{mm}$, $t=1.2\text{mm}$)](image)

Fig. 17 shows the buckling loads and shear strengths for both plain channel sections and channel sections with rectangular web stiffeners taken from Table 8. The abscissa displays the stiffener depth ($b_{s1}$) which
varies from 5mm to 180mm. The stiffener width is kept unchanged at 5mm. The ordinate depicts the shear capacity. It can be seen that when the stiffener depth increases from 5mm to 60mm, both the buckling capacity and shear strength are improved compared with those for a plain channel member (C20012). The maximum capacities are observed at the stiffener depth of 60mm. It is interesting that the occurrence of a small rectangular stiffener \( (b_{s1}=5-10\text{mm}) \) raises the elastic shear buckling capacity significantly whereas the shear strength is almost the same compared with the strength of a plain channel member. When the stiffener depth increases beyond 60mm, the shear strength gradually decreases from the maximum value and is only slightly larger than that of a plain-C section member when \( b_{s1}=180\text{mm} \). In this range of stiffener depth \( (b_{s1}>70\text{mm}) \), the shear strength is always greater than the buckling capacity demonstrating the post-buckling strength due to TFA.

Fig. 18 shows the shear strengths for the channel members with one rectangular web stiffener as given in Table 8 in another format where the ultimate shear strength is normalized \( \left( \frac{V_{abq}}{V_y} \right) \) and plotted against the section slenderness \( (\lambda_v) \). The strength of a plain channel member with the same overall size is included for comparison. The solid curve is the DSM shear strength curve in which the TFA is not included. The dashed curve is the DSM shear strength curve where the TFA is accounted for as given by Eq.3. It is noted that the shear yielding strength \( (V_y) \) is calculated as the product of the shear stress at yielding \( (\tau = \frac{1}{\sqrt{3}} f_y) \) and the vertically projected web area \( (b_{tw} t_w) \) where \( b_{tw} \) is the vertically projected depth of the web and \( t_w \) is the web thickness. It can be seen that when the stiffener depth increases from 5mm to 60mm, the data unexpectedly lies well below the DSM curve with TFA. The explanation is mainly based on the fact that for these sections, the rate of increase in buckling capacity is significantly larger than the rate of change in shear strength. Therefore, the data shifts rapidly to the left hand side.

As a result, the shear strength data when the stiffener depths are in the range of 5mm to 60mm seem not to follow the DSM curve with TFA. When the stiffener depth increases from 80mm to 180mm, the shear strength points lie close to the DSM curve with TFA as for the plain channel section. Fig. 19 illustrates the failure mechanisms of a stiffened channel member with various values of the stiffener depth where the stiffener width is 5mm.
When the stiffener depth is small ($b_{s1}=5$-$10$mm), only one shear field across the diagonal line of the whole web is observed. The same phenomenon occurs for the section with a deep stiffener ($b_{s1}>80$mm) where the shear zone locates well in the vertical portion of the stiffener. When $b_{s1}$ varies from 20mm to 50mm, the stiffener subdivides the shear field into two inclined parts lying in the two vertical portions of the web. At the stiffener depth of 60mm, three clear diagonal zones are formed which maximize the ultimate shear strength. It is emphasized that the tension field always occurs in the shear failure mechanism irrespective of the existence of the post-buckling strength.

The shear strength analyses are expanded to sections with $b_{s2}=10$mm, $b_{s2}=15$mm and $b_{s2}=30$mm to provide an overview. The results are summarized in Fig. 20.
The solid curves represent the buckling capacity whereas the dashed curves represent the shear strength. Generally, the wider is the stiffener, the higher the buckling capacity and ultimate shear strength obtained. Both capacities reach the maximum values when the stiffener depth is 60mm which approximately subdivides the web into three equal vertical flat portions (except for $b_{s2}=30\text{mm}$ where the ultimate strength is obtained at $b_{s1}=50\text{mm}$). It is interesting that for $b_{s1}<120\text{mm}$, both the buckling load and shear strength are considerably improved when the stiffener width increases from 5mm to 10mm. However, when the stiffener width increases further, the rate of improvement is much slower. The maximum ultimate shear strength seems to converge at the value of about 71kN irrespective of the stiffener dimensions. As can be seen in Fig. 21, the same phenomenon occurs for the members with the thickness of 1.5mm where the shear strength can only develop to a value of about 83kN which is equal to 90% of $V_y$.

As can be seen in Fig. 20, for $t=1.2\text{mm}$ and the stiffener depth ($b_{s1}$) of 5mm, post-buckling strength is observed when the stiffener width is very small ($b_{s2}=5\text{mm}$) or very large ($b_{s2}=30\text{mm}$). However, as shown in Fig. 21, for $t=1.5\text{mm}$, post-buckling strength does not exist for those stiffener dimensions. When the stiffener is very deep ($b_{s1}=180\text{mm}$), the increase of the stiffener width ($b_{s2}$) unexpectedly reduces the shear strength. Considering a 1.2mm thick member with the stiffener depth of $b_{s1}=180\text{mm}$, for $b_{s2}=5\text{mm}$, $b_{s2}=30\text{mm}$, the ultimate applied shear loads are $V_u=40.15\text{kN}$ and $V_u=37.13\text{kN}$ respectively.

![Figure 21: Summary of Shear Buckling Loads and Ultimate Shear Strengths for Sections with One Rectangular Stiffener (with $t=1.5\text{mm}$)](image)

**Shear Strength of Channel Section with Triangular Web Stiffeners**

Table 9 and Table 10 summarise the shear strengths, buckling loads and related data for sections with one and two triangular stiffeners respectively.

The designation ST20012-1TR5-V, for instance, is defined as follows:
- ‘ST200’ denotes the stiffened channel member with the overall section depth 200mm.
- ‘12’ is the thickness times 10 in mm.
- ‘1TR5’ indicates one (alternatively two) triangular stiffener(s) with its overall depth of 5mm.
- ‘V’ represents the pure shear load case.
### Table 9: Shear Strengths for Members with One Triangular Stiffeners (t=1.2mm)

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>Designation</th>
<th>Shear Load (V_{abq}, kN)</th>
<th>f_y (MPa)</th>
<th>V_{cr} (kN)</th>
<th>V_y (kN)</th>
<th>λ_v</th>
<th>V_{abq}/V_y</th>
<th>V_{abq}/V_{cr}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>C20012-V</td>
<td>37.92</td>
<td>593.30</td>
<td>15.96</td>
<td>82.22</td>
<td>2.27</td>
<td>0.46</td>
<td>2.38</td>
</tr>
<tr>
<td>5</td>
<td>ST20012-1TR5-V</td>
<td>38.38</td>
<td>593.30</td>
<td>17.83</td>
<td>82.22</td>
<td>2.15</td>
<td>0.47</td>
<td>2.15</td>
</tr>
<tr>
<td>10</td>
<td>ST20012-1TR10-V</td>
<td>38.36</td>
<td>593.30</td>
<td>25.94</td>
<td>82.22</td>
<td>1.78</td>
<td>0.47</td>
<td>1.48</td>
</tr>
<tr>
<td>20</td>
<td>ST20012-1TR20-V</td>
<td>43.95</td>
<td>593.30</td>
<td>50.88</td>
<td>82.22</td>
<td>1.27</td>
<td>0.53</td>
<td>0.86</td>
</tr>
<tr>
<td>30</td>
<td>ST20012-1TR30-V</td>
<td>55.45</td>
<td>593.30</td>
<td>65.94</td>
<td>82.22</td>
<td>1.12</td>
<td>0.67</td>
<td>0.84</td>
</tr>
<tr>
<td>40</td>
<td>ST20012-1TR40-V</td>
<td>62.88</td>
<td>593.30</td>
<td>79.28</td>
<td>82.22</td>
<td>1.02</td>
<td>0.76</td>
<td>0.79</td>
</tr>
<tr>
<td>50</td>
<td>ST20012-1TR50-V</td>
<td>67.50</td>
<td>593.30</td>
<td>91.08</td>
<td>82.22</td>
<td>0.95</td>
<td>0.82</td>
<td>0.74</td>
</tr>
<tr>
<td>60</td>
<td>ST20012-1TR60-V</td>
<td>69.98</td>
<td>593.30</td>
<td>107.09</td>
<td>82.22</td>
<td>0.88</td>
<td>0.85</td>
<td>0.65</td>
</tr>
<tr>
<td>70</td>
<td>ST20012-1TR70-V</td>
<td>68.76</td>
<td>593.30</td>
<td>119.60</td>
<td>82.22</td>
<td>0.83</td>
<td>0.84</td>
<td>0.57</td>
</tr>
<tr>
<td>80</td>
<td>ST20012-1TR80-V</td>
<td>64.75</td>
<td>593.30</td>
<td>121.17</td>
<td>82.22</td>
<td>0.82</td>
<td>0.79</td>
<td>0.53</td>
</tr>
<tr>
<td>90</td>
<td>ST20012-1TR90-V</td>
<td>64.27</td>
<td>593.30</td>
<td>99.38</td>
<td>82.22</td>
<td>0.91</td>
<td>0.78</td>
<td>0.65</td>
</tr>
<tr>
<td>100</td>
<td>ST20012-1TR100-V</td>
<td>61.37</td>
<td>593.30</td>
<td>83.38</td>
<td>82.22</td>
<td>0.99</td>
<td>0.75</td>
<td>0.74</td>
</tr>
</tbody>
</table>

### Table 10: Shear Strengths for Members with Two Triangular Stiffeners (t=1.2mm)

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>Designation</th>
<th>Shear Load (V_{abq}, kN)</th>
<th>f_y (MPa)</th>
<th>V_{cr} (kN)</th>
<th>V_y (kN)</th>
<th>λ_v</th>
<th>V_{abq}/V_y</th>
<th>V_{abq}/V_{cr}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>C20012-V</td>
<td>37.92</td>
<td>593.30</td>
<td>15.96</td>
<td>82.22</td>
<td>2.27</td>
<td>0.46</td>
<td>2.38</td>
</tr>
<tr>
<td>5</td>
<td>ST20012-2TR5-V</td>
<td>38.83</td>
<td>593.30</td>
<td>19.12</td>
<td>82.22</td>
<td>2.07</td>
<td>0.47</td>
<td>2.03</td>
</tr>
<tr>
<td>10</td>
<td>ST20012-2TR10-V</td>
<td>40.47</td>
<td>593.30</td>
<td>31.60</td>
<td>82.22</td>
<td>1.61</td>
<td>0.49</td>
<td>1.28</td>
</tr>
<tr>
<td>20</td>
<td>ST20012-2TR20-V</td>
<td>56.01</td>
<td>593.30</td>
<td>57.93</td>
<td>82.22</td>
<td>1.19</td>
<td>0.68</td>
<td>0.97</td>
</tr>
<tr>
<td>30</td>
<td>ST20012-2TR30-V</td>
<td>60.05</td>
<td>593.30</td>
<td>81.52</td>
<td>82.22</td>
<td>1.00</td>
<td>0.73</td>
<td>0.74</td>
</tr>
<tr>
<td>40</td>
<td>ST20012-2TR40-V</td>
<td>67.80</td>
<td>593.30</td>
<td>116.99</td>
<td>82.22</td>
<td>0.84</td>
<td>0.82</td>
<td>0.58</td>
</tr>
<tr>
<td>50</td>
<td>ST20012-2TR50-V</td>
<td>68.57</td>
<td>593.30</td>
<td>153.30</td>
<td>82.22</td>
<td>0.73</td>
<td>0.83</td>
<td>0.45</td>
</tr>
<tr>
<td>60</td>
<td>ST20012-2TR60-V</td>
<td>70.58</td>
<td>593.30</td>
<td>174.78</td>
<td>82.22</td>
<td>0.69</td>
<td>0.86</td>
<td>0.40</td>
</tr>
<tr>
<td>70</td>
<td>ST20012-2TR70-V</td>
<td>68.26</td>
<td>593.30</td>
<td>166.05</td>
<td>82.22</td>
<td>0.70</td>
<td>0.83</td>
<td>0.41</td>
</tr>
<tr>
<td>80</td>
<td>ST20012-2TR80-V</td>
<td>63.79</td>
<td>593.30</td>
<td>127.71</td>
<td>82.22</td>
<td>0.80</td>
<td>0.78</td>
<td>0.50</td>
</tr>
<tr>
<td>90</td>
<td>ST20012-2TR90-V</td>
<td>60.31</td>
<td>593.30</td>
<td>89.99</td>
<td>82.22</td>
<td>0.96</td>
<td>0.73</td>
<td>0.67</td>
</tr>
<tr>
<td>100</td>
<td>ST20012-2TR100-V</td>
<td>60.08</td>
<td>593.30</td>
<td>80.79</td>
<td>82.22</td>
<td>1.01</td>
<td>0.73</td>
<td>0.74</td>
</tr>
</tbody>
</table>

### Figure 22: Summary of Shear Buckling Loads and Ultimate Shear Strengths for Sections with One and Two Triangular Stiffeners (with t=1.2mm)
Fig. 22 shows the shear strengths and buckling capacities of a plain channel section and channel sections with one and two triangular stiffeners. The overall depth of a stiffener \(d\) varies from 5mm to 100mm. For a member with one triangular stiffener, the buckling capacity keeps increasing to the maximum value of about 120kN at \(d=80\)mm and then starts dropping. The shear strength gradually rises to the largest value of approximately 70kN at \(d=60\)mm and only slightly reduces after that. Except for the cases of relatively small stiffeners (\(d=5\) and 10mm), the shear strength is always less than the buckling capacity implying that no post-buckling strength is achieved.

When two triangular stiffeners are employed, the buckling capacity is generally higher than the capacity of a member with one triangular stiffener except at \(d \geq 90\)mm. The difference varies from a small value when \(d=5\)mm to a value up to 40% at \(d=60\)mm. However, despite this very high variation of the buckling capacity, the shear strength is not significantly different. It is noted that the ultimate shear strength for either one or two triangular stiffeners irrespective of the stiffener dimensions is 70.5kN which is equal to the ultimate value observed for a 1.2mm thick member with one rectangular stiffener.

Fig. 23 shows the buckling capacity and shear strength curves for 1.5mm thick channel members with one and two triangular stiffeners. The capacities for a plain channel member are also plotted for comparison. Generally, the change of both strength and buckling load for members with a thickness of 1.5mm is very similar to the variation occurring in Fig. 22 for 1.2mm thick cases. At \(d=60\)mm, the use of two triangular stiffeners increases the buckling load to a very high value of about 319kN, approximately three times higher than that of a plain channel member but the ultimate shear strength is only improved by 37%. It is also evident that the shear strength curves approach more closely to the line of shear yielding \(V_y\). The upper limiting value of shear strength is about 81.5kN.

For both 1.2mm and 1.5mm thick members with one and two triangular stiffeners, post-buckling strength only exists when the stiffeners are very small (\(d=5\)mm). This is different from the rectangular stiffener cases where post-buckling strength may not occur in the members with tiny stiffeners.

The strengths of members with one and two triangular stiffeners are plotted in comparison with the DSM shear strength curves as shown in Fig. 24 and Fig. 25 respectively. The shear strength points for \(d=5\)mm and 10mm lie above the DSM shear curve without TFA meaning that TFA occurs and contributes to the shear load carrying capacity of sections. For larger values of \(d\), due to the higher values of \(V_c\), the data shift significantly to the left hand side. Further, as the difference in shear strength for members with one and two triangular stiffeners is not significant, the difference in their locations is mainly governed by the slenderness, i.e. the elastic shear buckling capacity. Nevertheless, the DSM curve seems to be able to follow the variation of these points.
Figure 24: DSM Shear Curves and Ultimate Shear Strengths for Sections with One Triangular Stiffener (with t=1.2mm)

Figure 25: DSM Shear Curves and Ultimate Shear Strengths for Sections with Two Triangular Stiffeners (with t=1.2mm)
DISCUSSION

The Influence of Initial Geometric Imperfection on Shear Strength of Plain Channel Sections

The initial geometric imperfection is required for the nonlinear analysis since the out-of-plane deformations cannot occur in a perfectly flat plate under in-plane loading conditions. However, large initial imperfections may result in high values of out-of-plane bending stresses which reduce the strength of structures.

Fig. 26 shows the shear strength curves for a plain channel member in which the thickness is varied. Two values of initial imperfection are employed, namely 0.0015\(t\) and 0.15\(t\). It can be seen that for thin channel sections \((t \leq 1.9 \text{mm})\), the change of the imperfection values does not significantly affect the ultimate shear strength. For a thicker section, the reduction of the ultimate applied shear load due to the larger imperfection is about 5%.

Understanding Tension Field Action

The Tension Field Action (TFA) has been understood as the post-buckling strength, the capacity which a cold-formed member can develop after buckling. Accordingly, the TFA does not exist if the ultimate shear strength is less than the buckling load which commonly occurs for a stiffened member. The post-buckling strengths of the 1.2mm and 1.5mm thick members with rectangular stiffeners are summarized in Table 11. The variables are the stiffener dimensions.

It is interesting to observe in Table 11 that for the same thickness, the post-buckling strengths are similar for different values of stiffener width \((b_{s2})\) when the stiffener depth is large and \(b_{s2}\) is greater than 5mm. This fact proves that the development of the TFA in the vertical plate of the stiffener does not significantly rely on the restraints at the two edges, i.e. the stiffness of the stiffener width plates. This may be because the stress states of the elements in the shear field at ultimate are self-equilibrated as pointed out by Lee and Yoo (1998). Therefore, it is not unusual that the horizontal plates of the stiffener do not play an important role in the post-buckling shear strength of cold-formed members where the stiffener depth is relatively large.
The Effect of Stiffener Geometry

Table 12: Shear Buckling Loads and Shear Strengths for Members Stiffened by Small Stiffeners

<table>
<thead>
<tr>
<th>Designation</th>
<th>t = 1.2mm</th>
<th>t = 1.5mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b2, 5mm</td>
<td>b2, 10mm</td>
</tr>
<tr>
<td>ST200-RE-b1=5mm</td>
<td>6.093</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b1=10mm</td>
<td>1.079</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b1=20mm</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b1=30mm</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b1=40mm</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b1=50mm</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b1=60mm</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b2=80mm</td>
<td>5.470</td>
<td>0.00</td>
</tr>
<tr>
<td>ST200-RE-b2=100mm</td>
<td>9.988</td>
<td>4.313</td>
</tr>
<tr>
<td>ST200-RE-b2=120mm</td>
<td>12.75</td>
<td>9.384</td>
</tr>
<tr>
<td>ST200-RE-b2=180mm</td>
<td>21.17</td>
<td>18.48</td>
</tr>
</tbody>
</table>

Table 12 summarizes the buckling capacities ($V_c$) and ultimate shear strengths ($V_u$) of 1.2mm and 1.5mm thick members stiffened by small rectangular or triangular stiffeners. As demonstrated by Pham SH, Pham CH and Hancock (2012a,b), a lipped channel section with a rectangular stiffener is more effective in increasing the buckling load ($V_c$) than a section with a triangular stiffener with approximately the same amount of steel. This point is clearly proven by the values shown in Table 12. However, it is of interest to observe that the ultimate shear strength ($V_u$) is almost the same for both rectangular and triangular stiffeners. Consequently, the channel member with small triangular stiffeners has more post-buckling strength ($V_{pb}$) than the member with a small rectangular stiffener with the same overall stiffener depth.

Discussion on FEM Results

All the numerical modelling results for the FEM analyses are plotted in Fig. 27. The predominantly shear test series by Pham and Hancock (2009a,b) are also included. The dot-dashed curve is the DSM shear curve with TFA where a scale factor of 0.9 is applied. It is evident that the data follows the shear strength curve except for some points marked as A1 and A2. The curve with a reduction factor seems to better fit the data rather than the unscaled curve. The reason for the reduction factor is the original DSM shear strength curve is based on the tests where the bolting at the end sections may provide a higher degree of restraint compared with the idealized simply supported boundary conditions used in the FEM.
The data in the A2 region belongs to the cases where the stiffener width is very small ($b_{s2}=5\text{mm}$ and $b_{s2}=10\text{mm}$), the stiffener depth is less than 60mm and the thickness is 1.2mm. These points seem not to follow the DSM curve with TFA and therefore need to be re-calibrated. Region A1 includes considerable data and reflects a trend which the current DSM curve has not completely covered. The points which trend upwards in the A1 region may be due to inelastic reserve capacity in shear.

The Relations Between the $V_{cr}$ Determined by the SAFSM and the FEM

As discussed previously, the buckling load ($V_{cr}$) is easier to determine by means of the SAFSM. However, it is necessary to calibrate the buckling load obtained by the SAFSM to utilize in the DSM design formulas for shear when a channel member is stiffened.
Fig. 28 compares the buckling loads determined by means of the SAFSM (bfinst7) and the FEM (ABAQUS) for 1.2mm thick members where the stiffener depth varies from 5mm to 180mm. The stiffener widths are 5mm, 15mm and 30mm for Fig. 28a, Fig. 28b and Fig. 28c respectively. The green curves depict the buckling capacities determined by the bfinst7.cpp program at the minima in the shear signature curves if they occur. The blue curves represent the buckling loads at the half-wave length (HWL) of 200mm whereas the red curves show the results of buckling analyses conducted by ABAQUS for 200mm length members.

It can be seen that the $V_{cr}$ at the 200mm HWL determined by SAFSM is not necessarily always less than the value obtained by the FEM despite the stiffer boundary condition assumption for the latter. For stiffened members, the shear signature curves determined by the SAFSM tend to minimize at the HWL which may be much smaller or greater than the HWL of 200mm, therefore, the buckling loads achieved at the 200mm HWL may vary significantly. It is of interest to observe that when $b_{s2}=15mm$ and $30mm$ where the minima always occur, the buckling capacities determined by the SAFSM at minimum points and by the FEM tend to be in the same ratio.
Fig. 29 shows the positions of all available data in comparison with the DSM shear strength curve with TFA. The data includes all the members shown in Fig. 27 and 2mm thick stiffened members with the stiffener width of 30mm. The buckling load incorporated in the section slenderness ($\lambda_v$) determination is obtained at the minimum point (marked as ◆) whenever it occurs. Otherwise, the value at the 200mm HWL (marked as ◆) is chosen instead. The DSM shear strength curves with and without a scale factor of 0.9 are included.

As can be seen, for the sections for which the buckling loads are able to be determined at the minima, the data simply shift to the right due to the increase of the slenderness. For the sections for which the minimum point cannot be identified in the shear signature curve, the data vary arbitrarily.

CONCLUSIONS

General

The proposals for the Direct Strength Method (DSM) for shear with TFA have been recently approved by the AISI Specification Committee for including in the NAS S100-2012. These proposals were based on the series of the predominantly shear tests conducted at the University of Sydney for plain channel sections and Supacee® sections with small intermediate stiffeners. Their application for sections with substantial longitudinal stiffeners needs to be verified.

This report successfully sets up a finite element model of a cold-formed simply supported beam under pure shear. More than 200 FE models were analysed by means of the ABAQUS program for shear strength determination of cold-formed channel members with complex web stiffeners. The results were systematically summarized and discussed. Further, some possible issues were pointed out for future research.

Finite Element Analysis

The numerical investigations of the shear strength were utilized using the ABAQUS program. The report outlines the conceptual ideas and the processes to build a finite element model of a short cold-formed steel beam which is simply supported and subjected to pure shear loads. The validations prove that the FE model produces accurate although slightly conservative results compared with tests.

Shear strength analyses were performed for channel members with three types of web stiffeners. The results including the buckling loads and the shear strengths were compared with those for a plain channel member and also plotted against the DSM shear strength curves. Generally, the occurrence of either the rectangular or triangular stiffeners increases the buckling load and ultimate shear strength for a stiffened channel member, especially for a relatively thin member. However, the role of the stiffeners in improving the buckling
load is significantly more than that for the shear strength. For a relatively thick structure, the stiffener does not bring significant benefit in terms of shear strength.

For stiffened members undergoing pure shear, the classical formula to determine the shear strength at yielding, \( V_y = 0.6 f_y b_w t_w \) where \( b_w \) is the projected depth of the stiffened web may need to be improved, especially for the cases of substantially stiffened webs.

Generally, the formulae for DSM design for shear are capable of covering the sections with complex web stiffeners. However, there are still some specific cases where the DSM shear strength curve may not be applicable, especially for sections with small stiffeners.

**ACKNOWLEDGEMENT**

Funding provided by the Australian Research Council Discovery Project Grant DP110103948 has been used to perform this project. The first author was supported by an Australian Government AusAid Scholarship.
REFERENCES


Hancock, G. J. and Pham, C. H. (2012). “Direct Strength Method of Design for Shear of Cold-formed Channels based on a Shear Signature Curve”, Proceedings, 21st International Specialty Conference on Cold-Formed Steel Structures, St Louis, Missouri, USA, October.


Pham, C. H. and Hancock, G. J. (2009b). “Experimental Investigation of High Strength Cold-Formed SupaCee® Sections in Combined Bending and Shear”, Research Report R907, The University of Sydney, December.


Pham S. H., Pham C. H., Hancock G. J. (2012a). “Shear Buckling of Channel Sections with Complex Web Stiffeners”, Research Report, School of Civil Engineering, University of Sydney, R924, January.

Pham S. H., Pham C. H., Hancock G. J. (2012b). “Shear Buckling of Thin-Walled Channel Sections with Complex Stiffened Webs”, Proceedings, 21st International Specialty Conference on Cold-Formed Steel Structures, St Louis, Missouri, USA, October.


Yu, C. and Schafer, B. W. (2006). “Finite Element Modeling of Cold-Formed Steel Beams: Validation and Application”, Proceedings, Eighteen International Specialty Conference on Cold-Formed Steel Structures, University of Missouri-Rolla, Orlando, Florida, USA, pp. 89-103.

APPENDIX : SHEAR STRENGTH ANALYSIS RESULTS FOR STIFFENED MEMBERS

Shear Strength Data for 1.2mm Thick Members

Figure 2-1: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=5mm, t=1.2mm).

Figure 2-2: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=10mm, t=1.2mm).
Figure 2-3: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=15mm, t=1.2mm).

Figure 2-4: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=30mm, t=1.2mm).
Figure 2-5: DSM Shear Curves and Ultimate Shear Strengths for Members with One Triangular Stiffener (t=1.2mm).

Figure 2-6: DSM Shear Curves and Ultimate Shear Strengths for Members with Two Triangular Stiffeners (t=1.2mm).
Shear Strength Data for 1.5mm Thick Members

Figure 2-7: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=5mm, t=1.5mm).

Figure 2-8: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=10mm, t=1.5mm).
Figure 2-9: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (b_{wz}=15mm, t=1.5mm).

Figure 2-10: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (b_{wz}=30mm, t=1.5mm).
Figure 2-11: DSM Shear Curves and Ultimate Shear Strengths for Members with One Triangular Stiffener (t=1.5mm).

Figure 2-12: DSM Shear Curves and Ultimate Shear Strengths for Members with Two Triangular Stiffeners (t=1.5mm).
Shear Strength Data for 2.4mm Thick Members

Figure 2-13: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=5mm, t=2.4mm).

Figure 2-14: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=10mm, t=2.4mm).
Figure 2-15: DSM Shear Curves and Ultimate Shear Strengths for Members with Rectangular Stiffener (bs2=30mm, t=2.4mm).