INTRODUCTION
- Floating point arithmetic susceptible to rounding errors - loss of accuracy.
- Dynamic error analysis an effective tool for measuring sensitivity.
- Implementation difficult - requires significant modifications to existing source code.

AIMS
- Automate the quantitative analysis of floating point rounding errors.
- Improved methods for the analysis of results.
- Demonstrate applications.

BACKGROUND
Floating Point Arithmetic:
- Binary IEEE754 Floating Point - subset of real numbers:
  \[ x = (-1)^a m \beta^e \]
- Implemented as finite precision rounded arithmetic system. Exact values are rounded approximations of inexact values:
  \[ \hat{x} = \mathbb{F}(x) = x(1 + \delta) \]
- Normalization stage can lead to cancellation

Monte Carlo Arithmetic:
- Track information lost - model inexactness using random perturbations:
  \[ \text{inexact}(x, t, \xi) = x + 2^{e_x} (-1)^{t} \xi \]
- Statistics on rounding error obtained through repeated computations
- Uniformly distributed random variable used for \( \xi \)
- Individual operators performed in terms of the inexact function:
  \[ x \circ y = \text{round}(\text{inexact}(\text{inexact}(x) \circ \text{inexact}(y))) \]

LIBRARY IMPLEMENTATION
- Operations converted with source to source compiler CILLY
- Generated file compiled with MCA library to produce executable:

METHODS FOR ANALYSIS OF RESULTS
- Results analysis in previous publications limited
- Approach may be more formally defined - provide better interpretation of MCA results
- Approach defines sensitivity to rounding error with two measurements:
  - The number of base-2 digits lost to rounding error, \( K \)
  - The minimum precision required to avoid a complete loss of significance, \( t_{\text{min}} \)
- Measurements are found by performing linear regression using relative standard deviation, \( \Theta \), as the dependant variable and, \( t \), as the exploratory variable:
  \[
  \log_{10}(\Theta) = \log_{10}(2^{K-t}) = -\log_{10}(2) t + \log_{10}(2^K)
  
  = mt + c
  \]
  - Robust regression methods are used to detect outlying data points
  - Slope is a known variable - reduces problem to a 1D optimization

TESTING & RESULTS
- Statistical results and new analysis techniques allow for the comparison of algorithms and detection of catastrophic cancellation:
- Analysis of results allows for optimization of algorithms:

CONCLUSIONS
- No automated methods for measuring sensitivity to rounding error available
- MCALIB – determine if single or double precision floating point arithmetic is required, compare implementations or optimize software for precision.
- First implementation of it’s type to perform these functions using automated dynamic analysis methods.
- Represents a revolution in the field of error analysis.