Consider the following RE over the alphabet \{a,b,c\}

\[ \text{abb* | a(bc)*} \]

1) Devise an NFA accepting the same language. Use the construction methods we saw in lecture. The result should be non-deterministic with at least one epsilon-transition.
2) Transform the NFA into a DFA
3) Minimise the DFA, explaining all your steps
4) Implement a program that simulates the computation of your NFA. Your program should ask the user for a string of characters and return whether the string is accepted by the NFA and show the TRACE of the execution state by state. Explain how your program deals with non determinism. You must also provide testing runs with well chosen strings.
**QUESTION 1:**

Enter the alphabet as a string of characters:
\[ a, b, c \]

Enter the set of states \( Q \) as a comma separated list:
\[ \{0, 1, 2, 3, 4, 5, 6, 7\} \]

Enter the set of accepting states \( F \) as a comma separated list:
\[ \{6, 7\} \]

Enter the start state:
(0)

The NFA:

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->(0) \to (1) \to (2) \to (3) \to (4) \to (5) \to (6) \to (7)
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**QUESTION 2:**

**********NFA to DFA*********

STEP 1: We start with the epsilon closure of the start state.
We see where each set of states takes us via each transition in the alphabet. A set of states is taken via \( t \) to the union of all the states that are reached by any member of the set via \( t \).

- \( a \) takes \( \{0\}, \{2\}, \{4\} \) to \( \{2\}, \{5\} \)
- \( b \) takes \( \{0\}, \{2\}, \{4\} \) to \( \{1\} \)
- so we make \( \{0\}, \{2\}, \{4\} \) go to \{error\} via \( b \)
- \( c \) takes \( \{0\}, \{2\}, \{4\} \) to \( \{1\} \)
- so we make \( \{0\}, \{2\}, \{4\} \) go to \{error\} via \( c \)
- \( a \) takes \( \{2\}, \{5\} \) to \( \{1\} \)
- so we make \( \{2\}, \{5\} \) go to \{error\} via \( a \)
- \( b \) takes \( \{2\}, \{5\} \) to \( \{3\}, \{6\} \)
- \( c \) takes \( \{2\}, \{5\} \) to \( \{1\} \)
- so we make \( \{2\}, \{5\} \) go to \{error\} via \( c \)
- \( a \) takes \( \{3\}, \{5\} \) to \( \{1\} \)
- so we make \( \{3\}, \{5\} \) go to \{error\} via \( a \)
- \( b \) takes \( \{3\}, \{5\} \) to \( \{3\} \)
- \( c \) takes \( \{3\}, \{5\} \) to \( \{1\} \)
- so we make \( \{3\}, \{5\} \) go to \{error\} via \( c \)
- \( a \) takes \( \{5\}, \{7\} \) to \( \{1\} \)
- so we make \( \{5\}, \{7\} \) go to \{error\} via \( a \)
- \( b \) takes \( \{5\}, \{7\} \) to \( \{6\} \)
- \( c \) takes \( \{5\}, \{7\} \) to \( \{1\} \)
- so we make \( \{5\}, \{7\} \) go to \{error\} via \( c \)
a takes (6) to 11
so we make (6) go to [error] via a
b takes (6) to 11
so we make (6) go to [error] via b
c takes (6) to [5], [7]

STEP 2: So the states in equivalent DFA are:
[0], (1), (2), (3), (4), (5), (6), (7), [error]

STEP 3: Accept states of DFA are the above states
that include accept states of the NFA
the resulting accept states are:
(2, 5), (3), (5, 6), (5, 7)

STEP 4: Start state of DFA is the closure of the start
state of the NFA. The resulting start is:
(0, 1, 4)

Result:

\[
\begin{array}{cccc}
& a & (3, 5) & c \\
\omega(2, 5) & \text{[error]} & (3, 5) & \text{[error]} \\
\omega(3) & \text{[error]} & (3) & \text{[error]} \\
\omega(3, 6) & \text{[error]} & (5) & \text{[error]} \\
\omega(3, 7) & \text{[error]} & (5, 7) & \text{[error]} \\
(\omega(3, 6)) & \text{[error]} & (5, 7) & \text{[error]} \\
(\omega(3, 7)) & \text{[error]} & (5, 7) & \text{[error]} \\
\end{array}
\]

QUESTIONS:

********Minimising********

STEP ONE: accept states i.e. non accept states.
therefore the following are not equivalent:

[0], (1), (2), (3), (5), (6), (7), [error], [error], (0, 1, 4),

STEP TWO: for all a in alphabet, any pairs of states i.e.
where d(f, a) = t and d(f', a) = t' are not equivalent if t ≠ t'.
therefore we have the following:

\[
\begin{array}{cccc}
(3) \rightarrow (5, 7) & (3) \rightarrow (5, 7) & (3) \rightarrow (3, 5) \\
(3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) \\
(3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) \\
(3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) \\
(3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) \\
(3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) \\
(3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) & (3, 5) \rightarrow (3, 5) \\
\end{array}
\]
\{(3)\}, \{(3,5)\} \rightarrow a \rightarrow \{\text{error}\}, \{\text{error}\}\}
\{(3)\}, \{(2,5)\} \rightarrow b \rightarrow \{\text{error}\}, \{\text{error}\}\}
\{(3)\}, \{(2,5)\} \rightarrow c \rightarrow \{\text{error}\}, \{\text{error}\}\}
\text{since } \{(3)\} \cup \{(2,5)\} = \{\text{error}\}, \{\text{error}\}\}
\{(5,7)\}, \{(2,5)\} \rightarrow a \rightarrow \{\text{error}\}, \{\text{error}\}\}
\{(5,7)\}, \{(2,5)\} \rightarrow b \rightarrow \{\text{error}\}, \{\text{error}\}\}
\{(5,7)\}, \{(2,5)\} \rightarrow c \rightarrow \{\text{error}\}, \{\text{error}\}\}
\text{since } \{(5,7)\} \cup \{(2,5)\} = \{\text{error}\}, \{\text{error}\}\}

\text{Now these pairs are not equivalent:
\{(3)\}, \{\text{error}\}\} \rightarrow \{(5,7)\}, \{(0,1,4)\}\}
\{(6)\}, \{\text{error}\}\} \rightarrow \{(5,7)\}, \{(0,1,4)\}\}
\text{and } \text{error} = \{0,1,4\}, \{2,5\}\}
\{(6)\}, \{\text{error}\}\} \rightarrow \{(2,5)\}, \{6\}\}
\{(6)\}, \{\text{error}\}\} \rightarrow \{(2,5)\}, \{6\}\}
\{(6)\}, \{\text{error}\}\} \rightarrow \{(2,5)\}, \{6\}\}
\text{so these pairs are equivalent:
\{(0,1,4)\}, \{(0,7)\}\}
\{(0,1,4)\}, \{\text{error}\}\}
\{(1)\}\}
\{(3)\}, \{\text{error}\}\}
\{(3,6)\}\}
\{\text{error}\}\}
\{(3)\}, \{\text{error}\}\}
\{(3,5)\}\}
\{(5,7)\}\}
\text{STEP 3: now using the transitive property, we collect all the equivalence classes. The equivalence classes are:
\{(0,1,4)\}, \{(0,7)\}\}
\{\text{error}\}\}
\{(3,6)\}\}
\{\text{error}\}\}
\{(3)\}, \{\text{error}\}\}
\{(3,5)\}\}
\{(5,7)\}\}
\{\text{error}\}\}
\{\text{error}\}\}
\{\text{error}\}\}
\text{So the final set of states is:
\{(0,1,4)\}, \{(0,7)\}\}
\{(0,1,4)\}, \{\text{error}\}\}
\{(3,6)\}\}
\{\text{error}\}\}
\{(3)\}, \{\text{error}\}\}
\{(3,5)\}\}
\{(5,7)\}\}
\text{STEP 4: now we add all the transitions. any transition from any equivalence class will always take us to another equivalence class. the resulting transition function is:
\{(0,1,4)\} \rightarrow a \rightarrow \{2,5\}\}
\{\text{error}\}\}
\{(0,1,4)\} \rightarrow b \rightarrow \{3,6\}\}
\{\text{error}\}\}
\{(0,1,4)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(2,5)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{(2,5)\} \rightarrow b \rightarrow \{3,6\}\}
\{\text{error}\}\}
\{(2,5)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(3)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{(3)\} \rightarrow b \rightarrow \{3\}\}
\{(3)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(3,5)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{(3,5)\} \rightarrow b \rightarrow \{3\}\}
\{(3,5)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(5,7)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{(5,7)\} \rightarrow b \rightarrow \{3,6\}\}
\{\text{error}\}\}
\{(5,7)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{\text{error}\}\}
\text{STEP 5: the start state of the minimised dfa is the equivalence class of the start state of the original dfa: the resulting start state is:
\{(0,1,4)\}\}
\text{STEP 6: the accept states of the minimised dfa are the equivalence classes of the accept states of the original dfa: the resulting accept states are:
\{(2,5)\}, \{(3)\}, \{(3,5)\}, \{(5,7)\}\}
\text{Result:
\{(3,6)\} \rightarrow a \rightarrow \{2,5\}\}
\{\text{error}\}\}
\{(3,6)\} \rightarrow b \rightarrow \{3\}\}
\{\text{error}\}\}
\{(3,6)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(2,5)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{\text{error}\}\}
\{(2,5)\} \rightarrow b \rightarrow \{3,6\}\}
\{\text{error}\}\}
\{(2,5)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(3)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{\text{error}\}\}
\{(3)\} \rightarrow b \rightarrow \{3\}\}
\{\text{error}\}\}
\{(3)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(3,5)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{\text{error}\}\}
\{(3,5)\} \rightarrow b \rightarrow \{3\}\}
\{\text{error}\}\}
\{(3,5)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(5,7)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{\text{error}\}\}
\{(5,7)\} \rightarrow b \rightarrow \{3,6\}\}
\{\text{error}\}\}
\{(5,7)\} \rightarrow c \rightarrow \{\text{error}\}\}
\{(5,7)\} \rightarrow a \rightarrow \{\text{error}\}\}
\{\text{error}\}\}
\{(5,7)\} \rightarrow b \rightarrow \{3,6\}\}
\{\text{error}\}\}
\{(5,7)\} \rightarrow c \rightarrow \{\text{error}\}\}