**INTRODUCTION AND MOTIVATION**

**Bin Packing**

Space is a valuable commodity in many situations, from storing belongings in boxes to allocating jobs to processors, or advertising during television breaks.

In all of these situations, it is critical to optimize the use of space. The basic one-dimensional Bin Packing problem provides a framework for considering this type of problem. Given a list \( L \) of items, 

\[
L = \{a_1, a_2, \ldots, a_n\},
\]

each with a size in the range \((0,1]\), the problem seeks the minimum number of unit capacity bins the items can packed into, while obeying the capacity constraints.

**The Subset Sum Algorithm**

There are many heuristics used to solve this problem. One approach is to take one bin, pack it as full as possible, and then close it. This process then continues until no items are left to pack. This approach is termed the Subset Sum algorithm.

This heuristic however, does not perform well in the worst case. Consider the input shown in Figure 1 to the right, representative of the worst case. The Subset Sum algorithm uses 10 bins to pack all items (Figure 2), compared to 7 bins in the optimal packing (Figure 3).

**Variants of the Subset Sum Algorithm**

Two variants of the Subset Sum Algorithm for Bin Packing were proposed by Caprara and Pferschy [3]:

- The first, the Largest Items First Subset Sum (LSS), first opens bins for all large items (size greater than one half). The remaining space in these bins is packed using the usual Subset Sum algorithm. The items remaining after all of these bins are used are packed as usual.

- The second, Largest Remaining Item First Subset Sum (LRSS), packs the largest item remaining in the unpacked items as each bin is opened, with the remaining space being packed with Subset Sum.

These approaches perform better than the Subset Sum algorithm, as shown in Figure 4.

**Asymptotic Worst Case Ratio**

The asymptotic worst case ratio of an algorithm is a measurement of how badly an algorithm may perform in the worst case. It describes the maximum ratio of the number of bins used by the algorithm to that used in the optimal solution, as the number of bins used in the optimal solution approaches infinity.

The asymptotic worst case ratio for LSS is known to be between 1.37 and 1.44 [3]. For LRSS, the bounds are 1.30 and 1.44 [3].

**Project Aim**

The aim is to determine tight bounds on the asymptotic worst case ratio on the two variants of the Subset Sum algorithm for Bin Packing.

**LARGEST ITEMS FIRST**

**Weighting Function**

The different weighting cases are as follows:
- Large item: if no other items are packed in the bin, the size of the bin otherwise
- Small items when bin includes a large item: a proportion of the space remaining
- No large items in the bin: a proportion of the size of the bin if the bin is full enough; the size of the item otherwise
- If the bin is less than two thirds full: a proportion of two thirds based on the size of the item

**Result**

The worst case input that includes a large item is the same as for the Subset Sum. However, as large items must be packed optimally, the ratio is 1.202.

The worst case input that does not include a large item is known [4], and has a ratio of 1.376. In this packing, the first item is just over one third, and then each further item uses just over half of the space remaining in the bin.

**LARGEST REMAINING ITEM FIRST**

**Weighting Function**

The weighting function is similar to that for Largest Items First. However, the large item check is replaced for a largest item check, and thus there are no cases for when no large items are included.

**Result**

The worst case input is the same as for LSS. However, the asymptotic worst case ratio is 1.303. This improvement compared to LSS is due to the largest items being packed first into bins, and thus being packed optimally.

**CONCLUSION**

Our work has determined tight bounds on two variants of the Subset Sum algorithm for Bin Packing. We found that these were the previously known lower bounds.

We found that the worst case for Largest Items First includes no large items. Instead, the global worst case is equal to the previously known worst case in the absence of large items, with an asymptotic worst case ratio of 1.377.

We found the same worst case input for Largest Remaining Item First. However, it has a better asymptotic worst case ratio of 1.303.