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Abstract

Katsuno and Mendelzon suggested eight postulates for characterizing the semantics of update where the principle of minimal change is embedded [Katsuno and Mendelzon, 1991a]. In this paper, we propose a new method for knowledge base update based on persistence. We examine the relations between the two different update semantics by investigating the satisfiability of Katsuno and Mendelzon’s update postulates under our persistent update semantics. We show that the persistence-based update operator satisfies most of Katsuno and Mendelzon’s update postulates and sometimes satisfies all of them, from which we can see the essential distinction between the persistent update and the minimality-based update.

Key words: knowledge representation, persistence, update

1 Introduction

Katsuno and Mendelzon suggested eight postulates for characterizing the semantics of update where the principle of minimal change is embedded [Katsuno and Mendelzon, 1991a]. In this paper, we propose a new method for knowledge base update based on persistence by applying the Persistence Set Approach (PSA) presented in [Zhang and Foo, 1993]. Although the basic motivations between the persistence principle and minimal change for representing update are quite different, we examine the relations between them by investigating the satisfiability of Katsuno and Mendelzon’s update postulates under our persistent update semantics. We show that the persistence-based update operator satisfies their postulates (U1) – (U4), (U6) and (U8), and satisfies (U5) and (U7) sometimes, from which we can see the essential distinction between the persistent update and the minimality-based update.

The paper is organized as follows. We first review Katsuno-Mendelzon’s update postulates in section 2. In section 3 we propose a persistent semantics for propositional knowledge base update. In section 4, we then examine the satisfiability of Katsuno-Mendelzon’s update postulates under our persistent update semantics in detail, and present the main properties of the persistent update. Finally, we describe
the conclusion in section 5.

2 Propositional Knowledge Base Update: Review

Let $\mathcal{L}$ be a finitary propositional language. We represent a knowledge base by a propositional formula $\psi$. A propositional formula $\phi$ is complete if $\phi$ is consistent and for any propositional formula $\mu$, $\phi \models \mu$ or $\phi \models \neg \mu$. $\text{Models}(\psi)$ denotes the set of all models of $\psi$, i.e., all interpretations of $\mathcal{L}$ in which $\psi$ is true.

The motivation of Katsuno and Mendelzon’s proposal for update postulates is an observation on the difference between revision and update. Gardenfors et al. proposed a theory for belief revision, i.e., the AGM logic [Gardenfors, 1988], in which there are some postulates that should be satisfied by any reasonable revision operator. Formally, given a knowledge base $\psi$ and a formula $\mu$, $\psi \circ \mu$ denotes the revision of $\psi$ by $\mu$, that is, the new knowledge base obtained by adding new knowledge $\mu$ to the old knowledge base $\psi$. Katsuno and Mendelzon proved that the AGM revision postulates can be equivalently presented as follows [Katsuno and Mendelzon, 1991b]:

(R1) $\psi \circ \mu$ implies $\mu$.
(R2) If $\psi \land \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \land \mu$.
(R3) If $\nu$ is satisfiable then $\psi \circ \mu$ is also satisfiable.
(R4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$.
(R5) $(\psi \circ \mu) \land \phi$ implies $\psi \circ (\mu \land \phi)$.
(R6) If $(\psi \circ \mu) \land \phi$ is satisfiable then $\psi \circ (\mu \land \phi)$ implies $(\psi \circ \mu) \land \phi$.

Katsuno and Mendelzon revealed that the above postulates for revision are inappropriate for update, and then proposed eight postulates for characterizing the semantics of update. Let $\psi \bowtie \mu$ denote the update of $\psi$ by $\mu$. Then Katsuno-Mendelzon’s update postulates can be presented as follows [Katsuno and Mendelzon, 1991a]:

(U1) $\psi \bowtie \mu$ implies $\mu$.
(U2) If $\psi$ implies $\mu$ then $\psi \bowtie \mu \equiv \psi$. 
(U3) If both \( \psi \) and \( \mu \) are satisfiable then \( \psi \circ \mu \) is also satisfiable.
(U4) If \( \psi_1 \equiv \psi_2 \) and \( \mu_1 \equiv \mu_2 \) then \( \psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2 \).
(U5) \( (\psi \circ \mu) \land \phi \) implies \( \psi \circ (\mu \land \phi) \).
(U6) If \( \psi \circ \mu_1 \) implies \( \mu_2 \) and \( \psi \circ \mu_2 \) implies \( \mu_1 \) then \( \psi \circ \mu_1 \equiv \psi \circ \mu_2 \).
(U7) If \( \psi \) is complete then \( (\psi \circ \mu_1) \land (\psi \circ \mu_2) \) implies \( \psi \circ (\mu_1 \lor \mu_2) \).
(U8) \( (\psi_1 \lor \psi_2) \circ \mu \equiv (\psi_1 \circ \mu) \lor (\psi_2 \circ \mu) \).

Update operators satisfying these postulates can be characterized in terms of the following representation theorem. An update assignment is a function which assigns to each interpretation \( I \) a preorder \( \leq_I \) over the set of interpretations of \( \mathcal{L} \). We say that this assignment is faithful iff for any interpretation \( J \), if \( I \neq J \) then \( I \leq_I J \) and \( J \not\leq_I I \).

**Theorem 1** [Katsuno and Mendelzon, 1991a] An update operator \( \circ \) satisfies \((U1)-(U8)\) iff there exists a faithful assignment that maps each interpretation to a partial preorder \( \leq_I \) such that:

\[
\text{Models}(\psi \circ \mu) = \bigcup_{I \in \text{Models}(\psi)} \text{Min}(\text{Models}(\mu), \leq_I),
\]

where \( \text{Min}(\text{Models}(\mu), \leq_I) \) means the set of elements of \( \text{Models}(\mu) \) that are minimal under preorder \( \leq_I \). \( \square \)

Katsuno-Mendelzon’s update postulates characterize the update semantics for a class of update operators that are based on the principle of minimal change in knowledge base update. One of these update operators satisfying \((U1)-(U8)\) is, for example, the Possible Models Approach (PMA) update operator [Winslett, 1988], which is defined as follows. Let \( I, J \) be two interpretations of \( \mathcal{L} \), and \( \text{Diff}(I, J) \) the set of propositional letters that have different truth values in \( I \) and \( J \) respectively. \( \psi \circ_{\text{pma}} \mu \) means updating knowledge base \( \psi \) with \( \mu \) under the PMA semantics. Then

\[
\text{Models}(\psi \circ_{\text{pma}} \mu) = \bigcup_{I \in \text{Models}(\psi)} \text{Min}(\text{Models}(\mu), \leq_{I, \text{pma}}),
\]
where $J_1 \leq_{I,pma} J_2$ iff $Diff(I, J_1) \subseteq Diff(I, J_2)$. In other words, $\text{Models}(\psi \circ_{pma} \mu)$ is the set of models of $\mu$ that are minimally different from $I$ (or say “closest” to $I$).

To see the difference between revision and update, let us consider an example that was first described by Katsuno and Mendelzon [Katsuno and Mendelzon, 1991a]. Suppose a propositional language $\mathcal{L}$ where there are only two propositional letters $\text{Ontable}(\text{Book})$ and $\text{Ontable}(\text{Cup})$. A knowledge base $\psi \equiv (\text{Ontable}(\text{Book}) \land \neg \text{Ontable}(\text{Cup})) \lor (\neg \text{Ontable}(\text{Book}) \land \text{Ontable}(\text{Cup}))$, which states that “the book or the cup is on the table, but not both”. Assume a robot receives an order to put the book on the table. According to the AGM revision postulates, we have $\psi \circ_{agm} \text{Ontable}(\text{Book}) \equiv \text{Ontable}(\text{Book}) \land \neg \text{Ontable}(\text{Cup})$, whereas the PMA update operation gives the result $\psi \circ_{pma} \text{Ontable}(\text{Book}) \equiv \text{Ontable}(\text{Book})$. From our intuition, it seems that the PMA represents a more reasonable result than the AGM revision does, since we do not know the definite position of the cup, why should we conclude that the cup is not on the table after putting the book on the table?

3 A Persistent Semantics for Update

In this section, we present a persistent semantics for propositional knowledge base update. We consider the occurrence of domain constraints in our problem domain. Let $C$ be a consistent set of propositional formulas that represents all domain constraints of the world. Thus, for any knowledge base $\psi$, we require $\psi \models C$. Let $I$ be an interpretation of $\mathcal{L}$. We say that $I$ is a state of the world if $I \models C$. A knowledge base $\psi$ can be treated as a description of the world, where $\text{Models}(\psi)$ is the set of possible states of the world with respect to $\psi$.

As it will be shown next, in our formalism, updating knowledge base $\psi$ with $\mu$ is defined by updating all possible states of the world with respect to $\psi$ with $\mu$, and this is achieved by the Persistence Set Approach (PSA) as described in [Zhang and Foo, 1991].

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1Here, we adopt Katsuno and Mendelzon’s definition to present the PMA where domain constraints are ignored. If there exists a set of propositional formulas, $C$, to represent the domain constraints, we should require that for any knowledge base $\psi$, $\psi \models C$, and for any update operator $\circ$, $S \in \text{Models}(\psi \circ \mu)$ implies $S \models C$. This will be presented in the following section.
1993]. We first give the definition of persistence set as follows.

**Definition 1** Let $I$ be a state of the world, i.e., $I \models C$, $\mu$ a propositional formula and consistent with $C$. We define the *persistence set* of $I$ with respect to $\mu$, denoted as $\Delta(\mu, I)$, as follows:

1. $\Delta_0 = I - \{f \mid \{\mu\} \cup C \models \neg f\}$,

2. $\Delta_1 = \Delta_0 - \{f \mid$ there exists propositional letters $f_1, \ldots, f_n$, where $f_i$ or $\neg f_i$ in $I$ for each $i \in N = \{1, \ldots, n\}$, such that $\{\mu\} \cup C \models [\neg]f \lor \forall_{i \in N} \neg f_i$ but $C \not\models [\neg]f \lor \forall_{i \in N} \neg f_i$ and $\{\mu\} \cup C \not\models \forall_{i \in N} \neg f_i$, and for any proper subset $M$ of $N$, $\{\mu\} \cup C \not\models [\neg]f \lor \forall_{j \in M} \neg f_j\}$,

3. $\Delta_i = \Delta_{i-1} - \{f_1, \ldots, f_k \mid$ there exists some subset $F \subseteq I - \Delta_{i-1}$ such that $(F \cup \Delta_{i-1} - \{f_1, \ldots, f_k\}) \cup C \models f_1 \land \cdots \land f_k$ but $(\Delta_{i-1} - \{f_1, \ldots, f_k\}) \cup C \not\models f_i$ where $1 \leq i \leq k$ \},

4. $\Delta(\mu, I) = \bigcap_{i=0}^{\infty} \Delta_i$.

The notation $[\neg]$ means that the negation sign $\neg$ may or may not appear. $\square$

We now explain this definition in detail. We propose a *persistence principle* for updating a state with a formula, which says that a fact persists if it is not logically relevant to those facts that must change or may be subject to change because of updating the state with a formula. We formalize this principle from a viewpoint of purely logical syntax and first order semantics.

Consider a state of the world $I$ and a formula $\mu$ where $\mu$ is consistent with $C$. Obviously, after updating $I$ with $\mu$, a fact $f$ in $I$ must change if $f$ is inconsistent with $\mu$ with respect to $C$, i.e., $\{\mu\} \cup C \models \neg f$. We call such a fact *non-persistent* with respect to $\mu$ (i.e., condition 1 in the above definition). On the other hand, suppose there exists some fact $f$ where $f$ or its negation $\neg f$ appears in a disjunction that is entailed by $\mu$ (or together with $C$)\(^2\), we say that $f$ is *indefinitely affected* by $\mu$.

\(^2\)Of course, the disjunction should be non-trivial in our sense, i.e., $A \models B \lor C$, but $A \not\models B$ and $A \not\models C$. 

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The intuitive meaning of an indefinite effect is that after updating \( I \) with \( \mu \), the satisfaction of \( \mu \) (together with \( C \)) in the resulting state may or may not cause a change of the truth value of \( f \), but we do not know which is the case (i.e., condition 2). Moreover, for a fact \( f \), if there exists a subset \( F \) of \( I \) in which some fact is non-persistent or indefinitely affected by \( \mu \), and \( f \) is entailed by \( F \) (or together with \( C \)), then \( f \) is *implicitly affected* by \( \mu \) if there is no other justification for \( f \), because after updating \( I \) with \( \mu \), \( f \) may lose its justification. In this case, it is incautious to assume that \( f \) persists (i.e., condition 3). We call those facts that are either implicitly or indefinitely affected by \( \mu \) *mutable*. Finally, for a fact \( f \), if for its every justification there exists some mutable fact in the justification, \( f \) is *mutable* (i.e., condition 3). Thus, a fact in \( I \) *must* persist if it is neither non-persistent nor mutable. We take all such facts to form the persistence set, as presented in the above definition (condition 4).

**Theorem 2** [Zhang and Foo, 1993] For any state of the world \( I \) and any propositional formula \( \mu \) that is consistent with \( C \), the persistence set \( \Delta(\mu, I) \) is unique.

**Proof.** For any \( i \) we prove the uniqueness of \( \Delta_i \) by induction on \( i \) whence \( \Delta(\mu, I) \) is unique. Obviously, \( \Delta_0 \) and \( \Delta_1 \) are unique. Now consider \( \Delta_i \). Suppose that there are two different \( \Delta_i \) and \( \Delta'_i \) such that there exists some \( f \not\in \Delta_i \) but \( f \in \Delta'_i \). By inductive hypothesis, however, we would have \( f \in \Delta_{i-1} \) and

\[
f \in \{ f_1, \cdots, f_k \} \text{ there exists some } F \subseteq I - \Delta_{i-1} \text{ such that }\\( F \cup \Delta_{i-1} - \{ f_1, \cdots, f_k \} \cup C \models f_1 \land \cdots \land f_k \text{ but }\\\Delta_{i-1} - \{ f_1, \cdots, f_k \} \cup C \not\models f_l \text{ where } 1 \leq l \leq k,\]
\]

and

\[
f \not\in \{ f_1, \cdots, f_k \} \text{ there exists some } F \subseteq I - \Delta_{i-1} \text{ such that }\\( F \cup \Delta_{i-1} - \{ f_1, \cdots, f_k \} \cup C \models f_1 \land \cdots \land f_k \text{ but }\\\Delta_{i-1} - \{ f_1, \cdots, f_k \} \cup C \not\models f_l \text{ where } 1 \leq l \leq k,\]
\]

this is contradictory. □

So far, the possible state resulting from updating \( I \) with \( \mu \) is defined as follows.
Definition 2 Let $I$ be a state of the world, i.e., $I \models C$, and $\mu$ a propositional formula that is consistent with $C$. An interpretation $I'$ of $\mathcal{L}$ is a possible state of the world resulting from updating $I$ with $\mu$, iff

1. $I' \models C$,
2. $I' \models \mu$, and
3. $\Delta(\mu, I) \subseteq I'$.

Denote the set of all possible states of the world resulting from updating $I$ with $\mu$ as $Update(\mu, I)$. □

Now we can define our persistence-based update operator for knowledge base update based on the PSA.

Definition 3 Let $\psi$ be a knowledge base, i.e., $\psi \models C$, $\mu$ a propositional formula. $\psi \circ_{psa} \mu$ denotes the persistent update of $\psi$ with $\mu^3$, where

1. If $\psi$ implies $\mu$ or $\psi$ is inconsistent then $\psi \circ_{psa} \mu \equiv \psi$, otherwise
2. $Models(\psi \circ_{psa} \mu) = \bigcup_{I \in Models(\psi)} Update(\mu, I)$. □

In the above definition, condition 1 says that if $\psi$ implies $\mu$, then nothing is changed since the knowledge $\mu$ has been represented by knowledge base $\psi$; or if $\psi$ is inconsistent, then any update can not change it into a consistent knowledge base. Condition 2 says that if $\psi$ is consistent and does not imply $\mu$, then $\psi$ should be changed, and this change follows the persistence principle as defined previously. Obviously, our persistent update semantics differs from those based on the principle of minimal change. In our definitions for update, there is no similar representation for minimal change. The following example shows the difference between our method and those minimality-based approaches, for example, the PMA.

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$^3$Here we only consider the well-defined update, that is, $\mu$ is consistent with $C$.  

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Example 1. Consider a propositional language \( \mathcal{L} \) in which there are only two propositional letters \( \text{Ontable}(\text{Book}) \) and \( \text{Ontable}(\text{Cup}) \). Suppose the current knowledge base \( \psi \) is

\[-\text{Ontable}(\text{Book}) \land -\text{Ontable}(\text{Cup})\]

that is, neither the book nor the cup is on the table at the moment. Assume the robot receives an instruction to put the book or cup on the table, i.e., updating \( \psi \) with \( \mu \), where \( \mu \equiv \text{Ontable}(\text{Book}) \lor \text{Ontable}(\text{Cup}) \). Let us see how our method works for this example. Since \( \psi \) is complete, the only state of the current world is

\[\{ -\text{Ontable}(\text{Book}), -\text{Ontable}(\text{Cup}) \} .\]

From the condition 2 of Definition 1, it is easy to see that both \( -\text{Ontable}(\text{Book}) \) and \( -\text{Ontable}(\text{Cup}) \) are mutable. Thus,

\[\psi \circ_{pma} \mu \equiv \text{Ontable}(\text{Book}) \lor \text{Ontable}(\text{Cup}) .\]

Applying the PMA, on the other hand, we have the following result:

\[\psi \circ_{pma} \mu \equiv (-\text{Ontable}(\text{Book}) \land \text{Ontable}(\text{Cup})) \lor (\text{Ontable}(\text{Book}) \land -\text{Ontable}(\text{Cup})) .\]

which says that after the update, either the book or cup is on the table, but not both. From our intuition, however, it seems that there is not enough logical reason to infer that only one of the book and the cup can be placed on the table after updating \( \psi \) with \( \text{Ontable}(\text{Book}) \lor \text{Ontable}(\text{Cup}) \). This example reveals that the persistence principle is more conservative than the minimal change generally. \( \square \)

4 The Persistence-based Update and Katsuno-Mendelzon’s Postulates

From the previous sections, we can see that the basic motivations of the persistence-based update and minimality-based update are quite different. However, as will be
seen in this section, we can examine the relations between them by investigating the satisfiability of Katsuno-Mendelzon’s update postulates under our persistent update semantics. We first give the main result of this section as follows.

**Theorem 3** The persistence-based update operator $\circ_{pua}$ satisfies Katsuno-Mendelzon’s postulates (U1) - (U4), (U6) and (U8).

**Proof.** (U1) - (U4) and (U8) are directly follow from the definitions of persistent update Definition 1, 2 and 3. Now we prove (U6). First we prove that if $\psi \circ_{pua} \mu_1$ implies $\mu_2$, then $\psi \models \mu_2$ or $\{\mu_1\} \cup C \models \mu_2$.

Ignoring the trivial case, we assume that $\psi$ is consistent. From Definition 2 and 3, we know that

$$Models(\psi \circ_{pua} \mu_1) = \bigcup_{\models S \in Models(\psi)} Update(\mu_1, S)$$

$$= \bigcup_{\models S \in Models(\psi)} Models(\{\mu_1\} \cup C \cup \Delta(\mu_1, S))^4.$$

Thus, if $\psi \circ_{pua} \mu_1$ implies $\mu_2$, we have $\{\mu_1\} \cup C \cup \Delta(\mu_1, S) \models \mu_2$ for any $S \in Models(\psi)$. There are then three cases: (1) $\{\mu_1\} \cup C \models \mu_2$; (2) $C \cup \Delta(\mu_1, S) \models \mu_2$; or (3) $\{\mu_1\} \cup C \cup \Delta(\mu_1, S) \models \mu_2$ where $\{\mu_1\} \cup C \not\models \mu_2$ and $C \cup \Delta(\mu_1, S) \not\models \mu_2$.

Case (1) is our result. Now we consider case (2). Since for any $S \in Models(\psi)$ $C \cup \Delta(\mu_1, S) \models \mu_2$, also we know that $S \models C$ and $S \models \Delta(\mu_1, S)^5$, it follows that $S \models \mu_2$ and hence $\psi \models \mu_2$. Now we show that case (3) will never occur. Suppose that case (3) holds, then we would have $\{\mu_1\} \cup C \models \mu_2 \lor \neg \Delta(\mu_1, S)$. Since $C \not\models \mu_2 \lor \neg \Delta(\mu_1, S)$, it follows that each $f \in \Delta(\mu_1, S)$ is indefinite, this is contradictions with the fact that for every $f \in \Delta(\mu_1, S)$ $f$ is persistent.

So far, we can prove (U6) by using the above result. From $\psi \circ_{pua} \mu_1 \models \mu_2$ and $\psi \circ_{pua} \mu_2 \models \mu_1$, there are four cases: $\{\mu_1\} \cup C \models \mu_2$ and $\{\mu_2\} \cup C \models \mu_1$; $\{\mu_1\} \cup C \models \mu_2$ and $\psi \models \mu_1$; $\{\mu_2\} \cup C \models \mu_1$ and $\psi \models \mu_2$; or $\psi \models \mu_2$ and $\psi \models \mu_1$. For the first case, we have $C \models \mu_1 \equiv \mu_2$, and then $\Delta(\mu_1, S) = \Delta(\mu_2, S)$ for each $S \in Models(\psi)$, so $\psi \circ_{pua} \mu_1 \equiv \psi \circ_{pua} \mu_2$ according to the definitions of persistence update. Consider

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$^4$For simplicity, here we treat $\{\mu_1\} \cup C \cup \Delta(\mu_1, S))$ to be the conjunction of all elements of $\{\mu_1\} \cup C \cup \Delta(\mu_1, S)$.

$^5$As before, here we take $\Delta(\mu_1, S)$ to be the conjunction of all elements of $\Delta(\mu_1, S)$. 
the second case, since $\psi \models \mu_1$, $\psi \circ_{psa} \mu_1 \equiv \psi$. From $\psi \models \mu_1$ and $\psi \models C$, it follows $\text{Models}(\psi) \subseteq \text{Models}(\{\mu_1\} \cup C)$, and hence $\psi \models \mu_2$ from $\{\mu_1\} \cup C \models \mu_2$. So $\psi \circ_{psa} \mu_1 \equiv \psi \circ_{psa} \mu_2 \equiv \psi$. The third case is similar to the second. Directly from the fourth case we have $\psi \circ_{psa} \mu_1 \equiv \psi \circ_{psa} \mu_2 \equiv \psi$. □

Generally, (U5) and (U7) are not satisfied under the persistent semantics because the intuitive meaning of (U5) and (U7) is to represent a minimal change for update while the persistent update does not follow this principle generally. This can be shown by the following example.

**Example 2.** We first show that (U5) is not satisfied under the persistent update. Consider a propositional language $\mathcal{L}$ where there are six propositional letters $a, b, c, d, e$ and $f$. Assume the set of domain constraints $C$ is presented as follows:

$$\{a \supset c, b \supset c \lor e, c \land d \supset f\}.$$  

Let $\psi$ be a knowledge base where $\psi \equiv a \land \neg b \land c \land d \land e \land f$, $\mu \equiv b$ and $\phi \equiv a$. Since $\psi$ is complete, there is only one model of $\psi$, say $I$,

$$I = \{a, \neg b, c, d, e, f\}.$$  

Now consider two updates of $\psi$ with $\mu$ and $\mu \land \phi$ respectively. In accordance with Definition 1, we have

$$\Delta(\mu, I) = \{a, d\}^6,$$

$$\Delta(\mu \land \phi, I) = \{a, c, e\}^7.$$  

\[^6\text{Obviously, } \neg b \text{ is non-persistent with respect to } \mu \equiv b. \text{ Since } \{b\} \cup C \models c \lor e, \text{ and } c \text{ and } e \text{ are indefinitely affected by } b. \text{ Again, since } \{c, d\} \cup C \models f \text{ and } c \text{ is indefinitely affected by } b, \text{ it follows that } f \text{ is implicitly affected by } b. \text{ Hence, only } a \text{ and } d \text{ are persistent.}\]

\[^7\text{Notice that } c \text{ and } e \text{ are persistent in this case. } \mu \land \phi \equiv a \land b. \text{ Although } \{a \land b\} \cup C \models c \lor e, \text{ and } \{a \land b\} \cup C \models c. \text{ So } c \lor e \text{ is a trivial disjunction with respect to } \{a \land b\} \cup C. \text{ In accordance with Definition 1, neither of } c \text{ and } e \text{ is indefinitely affected by } a \land b. \text{ On the other hand, since } \{a \land b\} \cup C \models c, \text{ and } c \supset \neg d \lor f, \text{ it follows that } \{a \land b\} \cup C \models \neg d \lor f. \text{ So both } d \text{ and } f \text{ are indefinitely affected by } a \land b.\]
Thus, from Definition 2, it follows that there exists a possible state $I'$ resulting from updating $\psi$ with $\mu$, where

$$I' = \{a, b, c, d, \neg e, f\},$$

such that $I' \in Update(\mu, I) \cap Models(\phi)$, but $I' \notin Update(\mu \land \phi, I)$, that is $(\psi \circ_{psa} \mu) \land \phi$ does not imply $\psi \circ_{psa} (\mu \land \phi)$.

Now let us consider (U7). In the same language $\mathcal{L}$, suppose the set of domain constraints $C$ is

$$\{a \supset \neg c, a \supset b \lor e, b \supset c \lor f, b \supset a \lor d\}.$$  

Let $\psi \equiv \neg a \land \neg b \land c \land d \land e \land f, \mu_1 \equiv a$ and $\mu_2 \equiv b$. Again, since $\psi$ is complete, there is only one model of $\psi, I$, where

$$I = \{\neg a, \neg b, c, d, e, f\}.$$  

Consider three updates of $\psi$ with $\mu_1$, $\mu_2$ and $\mu_1 \lor \mu_2$ respectively. From Definition 1, we have

$$\Delta(\mu_1, I) = \{d, f\},$$

$$\Delta(\mu_2, I) = \{e\}, \text{ and}$$

$$\Delta(\mu_1 \lor \mu_2, I) = \{c, f\}.$$  

Thus, from Definition 2, it follows that there exists a state $I'$, where

$$I' = \{a, b, \neg c, d, e, f\},$$

such that $I' \in Update(\mu_1, I) \cap Update(\mu_2, I)$, but $I' \notin Update(\mu_1 \lor \mu_2, I)$\footnote{\mu_1 \lor \mu_2 \equiv a \lor b. Notice that $\{a \lor b\} \cup C \models b \lor e$ and $\{a \lor b\} \cup C \models a \lor d$, and both $b \lor e$ and $a \lor d$ are non-trivial disjunctions with respect to $\{a \lor b\} \cup C$. Hence all of $a, b, d$ and $e$ are indefinitely affected by $a \lor b$.}. That is,

$$(\psi \circ_{psa} \mu_1) \land (\psi \circ_{psa} \mu_2) \text{ does not imply } \psi \circ_{psa} (\mu_1 \lor \mu_2). \quad \square$$

\footnote{In particular, for any $I'' \in Update(\mu_1 \lor \mu_2, I), I'' \models \neg a$ since $c \in \Delta(\mu_1 \lor \mu_2, I)$, and there is a constraint $a \supset \neg c$.}
Although the persistent update operator does not satisfy the postulates (U5) and (U7) in the general case, we have the following theorem, which says that under some conditions, (U5) and (U7) are still satisfied by the persistent update operator.

**Theorem 4** Let $C$ be the set of domain constraints. Then the following properties hold:

(U5’) If $\psi_{\text{psa}} \mu \models [\neg] \phi$ but $\psi \not\models \phi$, then $(\psi_{\text{psa}} \mu) \land \phi$ implies $\psi_{\text{psa}} (\mu \land \phi)$.

(U7’) If $\psi_{\text{psa}} \mu_1 \models [\neg] \mu_2$ (or $\psi_{\text{psa}} \mu_2 \models [\neg] \mu_1$), then $(\psi_{\text{psa}} \mu_1) \land (\psi_{\text{psa}} \mu_2)$ implies $\psi_{\text{psa}} (\mu_1 \lor \mu_2)$.

**Proof.** We first prove (U5’). Suppose $\psi$ is consistent. If $\psi_{\text{psa}} \mu \models \neg \phi$, then $\text{Models}((\psi_{\text{psa}} \mu) \land \phi) = \emptyset$, the result holds. Now suppose $\psi_{\text{psa}} \mu \models \phi$. From the proof of (U6), $\psi_{\text{psa}} \mu \models \phi$ implies $\models \phi$ or $\mu \cup C \models \phi$. Since $\psi \not\models \phi$, it must be the case that $\mu \cup C \models \phi$. That is $C \models (\mu \land \phi)$, it hence follows $C \models (\mu \land \phi) \equiv \mu$ and then $\psi_{\text{psa}} (\mu \land \phi) \equiv \psi_{\text{psa}} \mu$, which is implied by $(\psi_{\text{psa}} \mu) \land \psi$.

We now prove (U7’). We consider the non-trivial case that $\psi$ is consistent. If $\psi_{\text{psa}} \mu_1 \models \neg \mu_2$, then $\text{Models}((\psi_{\text{psa}} \mu_1) \land (\psi_{\text{psa}} \mu_2)) = \emptyset$, the result holds. Suppose $\psi_{\text{psa}} \mu_1 \models \mu_2$, from the proof of (U6), we know that $\psi \models \mu_2$ or $\mu_1 \cup C \models \mu_2$. From $\psi \models \mu_2$ and Definition 3, we have $\psi_{\text{psa}} \mu_2 \equiv \psi$, and also $\psi_{\text{psa}} (\mu_1 \lor \mu_2) \equiv \psi$. Hence $(\psi_{\text{psa}} \mu_1) \land (\psi \lor \mu_2) \equiv (\psi_{\text{psa}} \mu_1) \land (\psi_{\text{psa}} (\mu_1 \lor \mu_2))$, the result holds. Now suppose $\mu_1 \cup C \models \mu_2$. This follows that $C \models (\mu_1 \lor \mu_2)$, and also $C \models \mu_2 \equiv (\mu_1 \lor \mu_2)$. Hence $(\psi_{\text{psa}} \mu_1) \land (\psi_{\text{psa}} \mu_2) \equiv (\psi_{\text{psa}} \mu_1) \land (\psi_{\text{psa}} (\mu_1 \lor \mu_2))$, which implies $(\psi_{\text{psa}} (\mu_1 \lor \mu_2))$. □

Intuitively, the above theorem tells us that if there does not exist the indefinite and implicit effects of update, the persistent update operator satisfies (U5) and (U7), and then satisfy all Katsuno-Mendelson’s postulates. Furthermore, from Katsuno and Mendelson’s representation theorem (Theorem 1), it follows that the indefinite and implicit effects of persistent update is unexpressible by any model-based minimality principle, which is the essential distinction between the persistent update and the minimalism-based update.
The fact that the AGM revision postulates are inappropriate for minimality-based update [Katsuno and Mendelzon, 1991a] is paralleled by the following corollary which says that the AGM revision postulates are also inappropriate for the persistence-based update.

**Corollary 1** $\diamond_{psa}$ satisfies the AGM revision postulates (R1) and (R4), but violates (R2), (R3), (R5) and (R6).

**Proof.** The satisfaction of (R1) and (R4) by $\diamond_{psa}$ directly follows from the definitions of persistent update. It is easy to show that $\diamond_{psa}$ violates (R2), (R3) and (R5). $\diamond_{psa}$ does not satisfy (R2) obviously. For (R3), suppose $\psi$ is inconsistent, then $\psi \diamond_{psa} \mu \equiv \psi$, that is inconsistent. (R5) is the same as (U5), and we have shown that $\diamond_{psa}$ violates it.

Now we show that $\diamond_{psa}$ violates (R6) too. We only need give one example to show the case. Let us consider Example 2 presented previously once again. As described before, the persistence set with respect to $\mu$ and $\mu \land \phi$ are $\Delta(\mu, I) = \{a, d\}$ and $\Delta(\mu \land \phi, I) = \{a, c, e\}$ respectively. It is easy to see that $I' = \{a, b, c, \neg d, e, f\}$ is a possible state resulting from updating $\psi$ with $\mu \land \phi$ such that $I' \in Update(\mu \land \phi, I)$, but $I' \notin Update(\mu, I) \cap Models(\phi)$ since $d$ is persistent with respect to $\mu$ while it is indefinitely affected by $\mu \land \phi$. Hence $\psi \diamond_{psa} (\mu \land \phi)$ does not imply $(\psi \diamond_{psa} \mu) \land \phi$. □

5 Conclusion

In this paper, we proposed a persistent semantics for propositional knowledge base update. We compared our method with the minimality-based update approach by investigating the satisfiability of Katsuno-Mendelzon’s update postulates under our persistent update semantics, from which we can see the essential distinction between these two different approaches for knowledge base update.

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