University of Sydney
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Simulation of Networks

Simulation Task Set 1 – Theoretical Validation of Results

Task 1

Theoretical Validation: M/G/1 queue

Poisson Arrivals, rate λ packets per second
Packet lengths independent identically distributed, mean $\bar{h}$.
Occupancy $\rho = \lambda \bar{h}$

Mean queue length (including packet in service):

Negative-exponential packet length $\bar{Q} = \dfrac{\rho}{1-\rho}$

Constant packet length $\bar{Q} = \dfrac{\rho\left(1-\rho/2\right)}{1-\rho}$

Task 2.1

Theoretical Validation: M/M/N/N queue

Poisson Arrivals, rate λ packets per second
Call lengths independent identically distributed, negative exponential distribution, mean $\bar{h}$.
Offered traffic $A = \lambda \bar{h}$

Probability that a call is lost = $E_N(A) = \dfrac{\dfrac{A^N}{N!}}{\sum_{j=0}^{N} \dfrac{A^j}{j!}}$

Calculate recursively: $\dfrac{1}{E_N(A)} = 1 + \dfrac{N}{A}\dfrac{1}{E_{N-1}(A)}$ *with* $E_0(A) = 1$

Task 2.2

Theoretical Validation: M/G/N/N queue

Result is the same as that of task 2.1, irrespective of the distribution of call holding times.

Task 3

Theoretical Validation: M/G/1 queue

Assumptions as for task 1.

Negative-exponential packet length $\overline{T} = \frac{1}{1-\rho}\overline{h}$

Constant packet length $\overline{T} = \frac{\left(1-\rho/2\right)}{1-\rho}\overline{h}$

In general, for an M/G/1 queue with arbitrary distribution of packet lengths, with second moment $\overline{h^2}$

$$\overline{T} = \frac{\lambda\overline{h^2}}{2(1-\rho)} + \overline{h}$$

$$\overline{Q} = \frac{\lambda^2\overline{h^2}}{2(1-\rho)} + \rho$$

Task 4

Theoretical Validation: Polling Model

Assumptions:

N queues

All queues have mean message time $\overline{h}$ and second moment $\overline{h^2}$.

Queue i has Poisson arrival rate λ_i , and occupancy $\rho_i = \lambda_i \overline{h}$.

Total arrival rate to system is $\lambda = \sum_{i=1}^{N} \lambda_i$

Total system occupancy is $\rho = \sum_{i=1}^{N} \rho_i$

Polling delay is $\overline{s}$ $/N$ per station, and is constant (non-random).

Buffer lengths are effectively infinite.

Derived result: Mean cycle time $\overline{c} = \frac{\overline{s}}{1-\rho}$

mean number served per cycle at the i^{th} queue is $g_i = \lambda_i \overline{c}$

The following assumes that only 1 packet (maximum) can be served per poll at each queue.

Stability criteria: $g_i = \lambda_i \overline{c} < 1 \ \forall i$

(Note: make sure that your simulation chooses input values which satisfy these criteria)

If the system is symmetric, i.e. arrival rates are the same to each of the N queues:

Mean time until packet begins service = $\overline{W} = \frac{1}{1-g_i}\left[\frac{\lambda\overline{h^2}}{2(1-\rho)} + \frac{\overline{s}}{2}\frac{(1+\rho/N)}{(1-\rho)}\right]$

Mean time until packet completes service = $\overline{T} = \overline{W} + \overline{h}$

Mean queue length (including packet in service) at node i is $\overline{Q_i} = \lambda_i \overline{T}$

(note that as the polling delay $\overline{s} \to 0$, these results approach the M/G/1 results of tasks 1 and 3)

Task 5

Theoretical validation: Easy with no mobility! Hard with mobility!