Optimization Summer 2009	Homework 1	Khaled Elbassioni Julián Mestre

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This assignment is **due on April 30** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else's solutions or any other source like the web.

Problem 1: Find a quadratic function $f(x_1, x_2)$ such that the optimization problem

$$\begin{array}{lll} \text{maximize} & f(x_1, x_2) & \\ \text{subject to} & 0 \le x_i \le 1 & \text{for } i = 1, 2 \end{array}$$

has bounded objective value, but no basic solution is optimal.

Problem 2: Find a convex set $S \subseteq R^2_+$ and $\mathbf{c}$ such that the optimization problem

$$\begin{array}{ll} \text{minimize} & \mathbf{c}'\mathbf{x} \\ \text{subject to} & \mathbf{x} \in S \end{array}$$

has bounded objective value, but no optimal solution.

Problem 3: Consider the optimization problem

$$\begin{array}{lrl} \text{minimize} & x_1 + x_2 & \\ \text{subject to} & s\,x_1 + t\,x_2 \le 1 & \\ & x_i \ge 0 & \text{for } i = 1, 2 \end{array}$$

Find the range of s and t that make the program: infeasible, bounded feasible, and unbounded feasible.

Problem 4: Let $S = \{\mathbf{x} \in R^n \mid \mathbf{A}\mathbf{x} \ge \mathbf{b}\}$ be a polyhedron. Let $B(\mathbf{y}, r)$ be a ball of radius r around a point $\mathbf{y}$; that is, the set of points at distance at most r from y. Consider the optimization problem

$$\begin{array}{lr} \text{maximize} & r \\ \text{subject to} & B(\mathbf{y}, r) \subseteq S \\ & \mathbf{y} \in R^n \\ & r \ge 0 \end{array}$$

Give a linear programming formulation for this problem.