Optimization Summer 2009	Homework 10	Khaled Elbassioni Julián Mestre

This assignment is **due on July 23** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else's solutions or any other source like the web.

Problem 1: Consider the partition LP formulation for the MST problem:

$$\begin{array}{lll} \text{minimize} & \sum_{e \in E} w(e)\, x_e & \\ \text{subject to} & \sum_{e \in E} x_e = n-1 & \\ & \sum_{e : |e \cap S_i| \le 1} x_e \ge k-1 & \forall \text{ partitions } S_1, \ldots, S_k \text{ of } V \\ & x_e \ge 0 & \end{array}$$

and the subtour LP formulation,

$$\begin{array}{lll} \text{minimize} & \sum_{e \in E} w(e)\, x_e & \\ \text{subject to} & \sum_{e \in E} x_e = n-1 & \\ & \sum_{e : |e \cap S| = 2} x_e \le |S| - 1 & \forall\, \emptyset \subset S \subset V \\ & x_e \ge 0 & \end{array}$$

Prove that they are equivalent—show that any vector x is feasible for first LP if and only if it is feasible for the second LP.

Problem 2: Consider the cut formulation for the MST problem strengthened with the additional constraint that $n-1$ edges must be chosen.

$$\begin{array}{lll} \text{minimize} & \sum_{e \in E} w(e)\, x_e & \\ \text{subject to} & \sum_{e \in E} x_e = n-1 & \\ & \sum_{e : |e \cap S| = 1} x_e \ge 1 & \forall\, \emptyset \subset S \subset V \\ & x_e \ge 0 & \end{array}$$

Prove that the integrality gap of this formulation is at least $2(1-3/n)$. *Hint: extend the example in Section 10.3 of the book.*