A Look at the Optimization Landscape of MIS

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Local Search

- Start from some feasible solution
- Improve it using incremental steps
- Until we reach a Local Optimum

Hope: local optima are close to global optimum

Physics analogy: Optimization Landscape

Local Search

- Where to start?
- How to move?
- When to stop?

Are these important?

Practitioner : Of course!

Worst case : Usually not

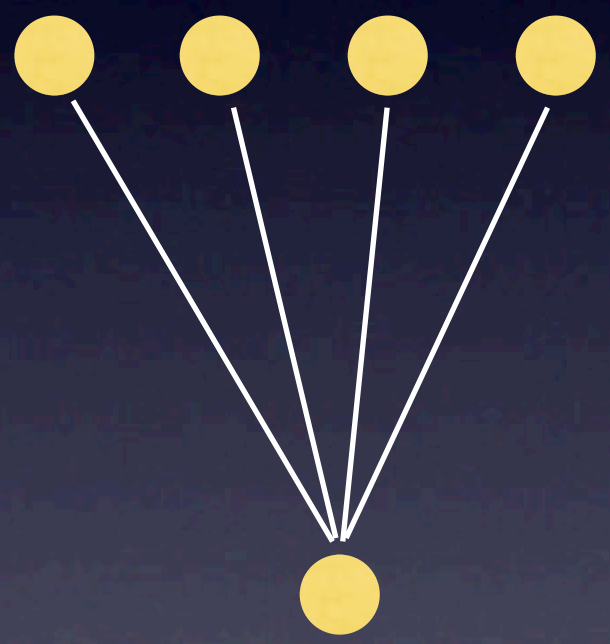
For maximum weight independent set
it matters... even in theory!

Maximum Independent Set

- Independent set: A subset of vertices such that no two are connected by an edge.
- Many applications: Edges denote conflicts.
- Notoriously hard: Unless NP $\subseteq$ ZPP, there is no $O(\Delta^{1-\varepsilon})$-approximation.

Special cases

- Δ-degree graphs
- K-set packing
- (D+1)-claw free graphs



Maximum size

- Greedy : D approximation [Folk]
- LS with two vertices : D/2 [BNR]
- LS with a few vertices : (D-1)/2 + ε [HS]

Maximum weight

- Greedy : D-approximation [Folk]
- LS with many neighbors : D-1 + ε [HA]
- Greedy + LS : 2 (D+1) /3 [HC]
- LS on squares : (D+1) /2 [B]

Avoid making small improvements "forever"

Round w(u) to w'(u), a multiple of M such that:

- Ensure progress at each step (big M)
- w(OPT)-w'(OPT) shouldn't change much (small M)

D-1+ε [HA]

- Start : Any set
- Move : Add a few vertices with positive gain
- While : The weight can be improved

2 (D+1) / 3 [HC]

- Start : Greedy choice
- Move : Use a claw of maximum gain
- While : The weight can be improved

(D+1)/2 [B]

- Start : Anywhere
- Move : Use a claw with positive gain on w^2
- While : w^2 can be improved

Analysis of LS^2

- Tight Example
- Nice claws
- If there is no nice claw then (D+1)-approx
- A nice claw always improves w^2