Adaptive Local Ratio


$\phi - 1$

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Local Ratio

- ☆ Technique for algorithm design
- ☆ Related to Primal-Dual scheme
- ☆ Powerful, yet simple

Local Ratio

i) Decompose $w = w_1 + w_2 + \cdots + w_n$

ii) Construct a solution S

iii) Argue $w_k(S) \leq \rho\, w_k(A) \quad \forall A$

$\Rightarrow S$ is ρ-approximate

Local Ratio

- Large number of papers
- And many more PD papers!
- Weight decomp. using "simple" models

Optimizing the LR can pay off!

Talk Outline

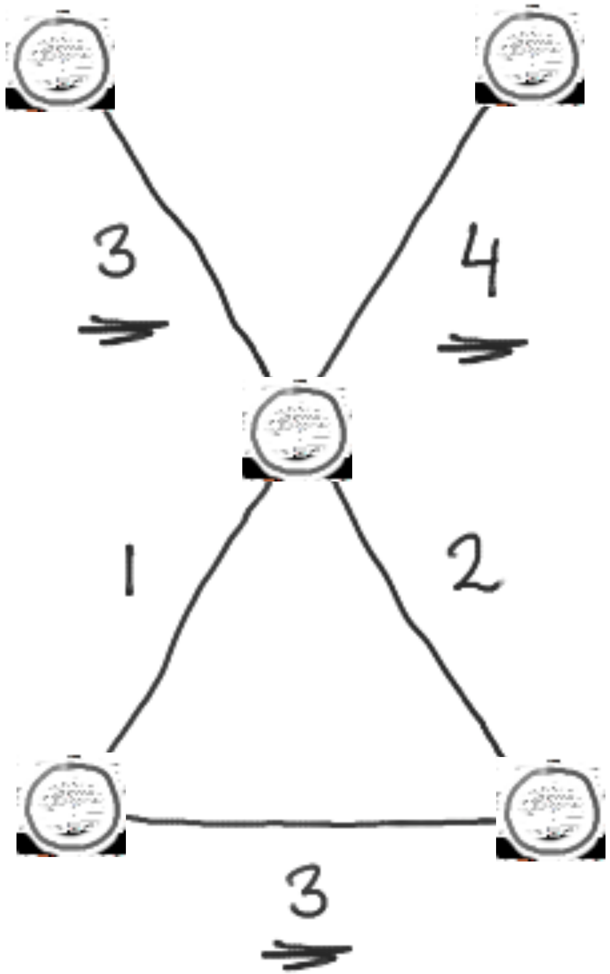
- ✓ Background
- ☆ Data Migration Problem
- ☆ Adaptive Local Ratio

Data Migration

Transfer graph $G=(V,E)$ $\begin{cases} V : \text{disks} \\ E : \text{transfers} \end{cases}$

Want to schedule the transfers $\equiv$ Partition E into matchings $M_1, \ldots M_k$

minimize $\sum_v \omega_v C_v$



$$\sum C_v = 2 \times 4 + 3 \times 3 = 17$$

Previous Results

- NP-hard
- 3-approx (LP rounding) [K03]
- shown to be tight [GHKS]
- 3-approx (primal-dual) [GM]

Talk Outline

- ✓ Background
- ✓ Data Migration Problem
- ☆ Adaptive Local Ratio

Local Ratio

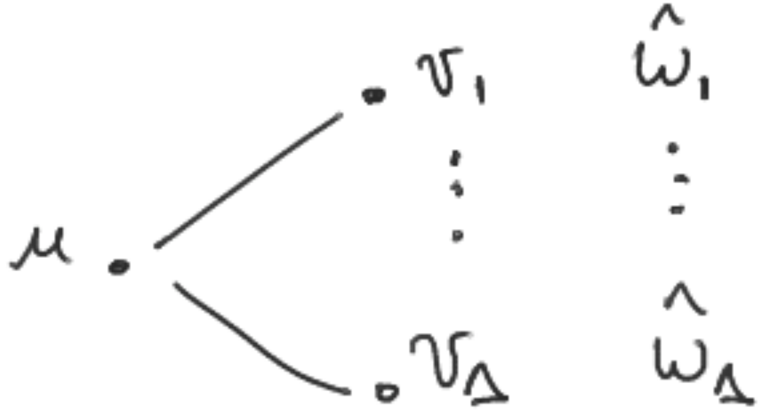
i) Decompose $\omega = \omega_1 + \omega_2 + \cdots + \omega_n$

ii) Construct a solution S

iii) Argue $\omega_k(S) \leq \rho\, \omega_k(A) \ \forall A$

$\Rightarrow$ S is ρ-approximate

Algorithmic Framework



$$w = w - \alpha \hat{w}$$

Lemma: $C_i \le \Delta + d_i - 1$

Find $\hat{w}$ so that $\hat{w}(S) \le \rho\, \hat{w}(A) \quad \forall A$

Bounding the Local Ratio

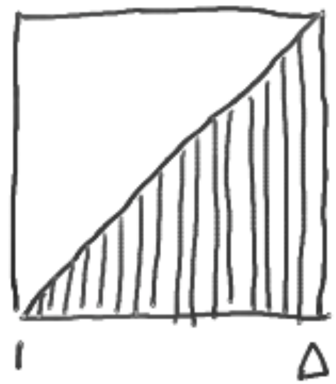
Lemma: $c_i \leq \Delta + d_i - 1$

$$\hat{w}(S) \leq \Delta^2 + \sum d_i$$

$\hat{w}_i = 1$

$$\hat{w}(A) \geqslant \sum d_i$$

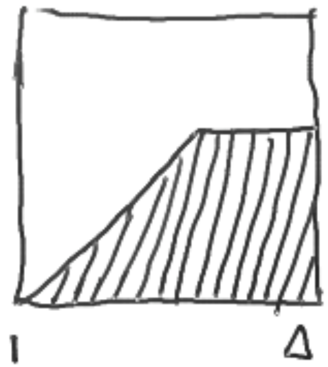
$$\hat{w}(A) \geqslant \frac{1}{2}\Delta(\Delta+1)$$



Using two models

$\hat{w}_i = 1$ or

$\hat{w}_i = \mathbb{I}\{d_i \geq d_j \ \forall j\}$



Lemma: $\hat{w}(S) \leq 2.82\, \hat{w}(A) \quad \forall A$

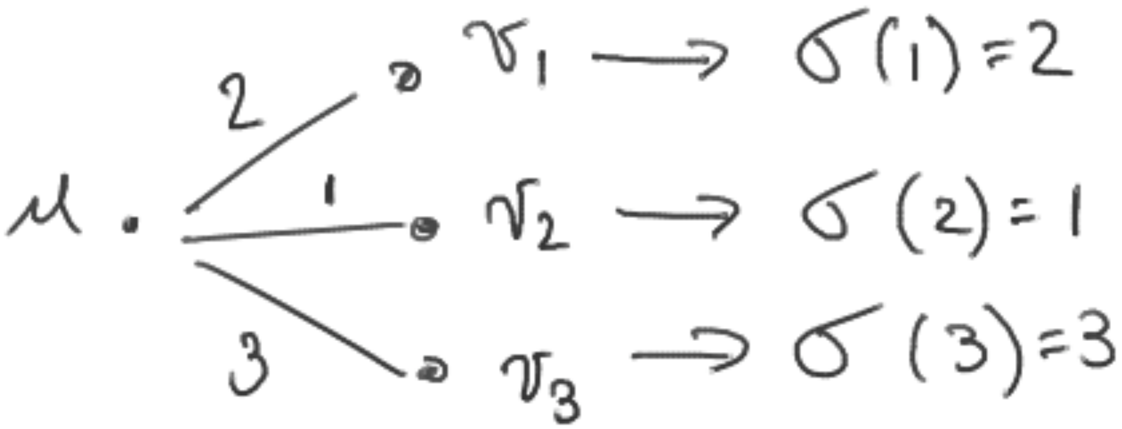
Using the best model

Given $d = (d_1, d_2, \ldots, d_\Delta)$

find $\hat{\omega} = (\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_\Delta)$

minimizing $\rho(d) = \dfrac{UB(d, \hat{\omega})}{LB(d, \hat{\omega})}$

$$UB(\hat{\omega}) = \sum_{i} \hat{\omega}_i \left(d_i + \Delta - 1 \right)$$



$$LB(\hat{\omega}) = \min_{\sigma \in P_\Delta} \sum_{i} \hat{\omega}_i \max(d_i, \sigma(i))$$

Towards an LP formulation

$$\rho(d) = \min_{\text{s.t. } \hat{\omega}_i \geq 0} \frac{UB(\hat{\omega})}{LB(\hat{\omega})} = \min \; UB(\hat{\omega})$$

$$\text{s.t.} \quad LB(\hat{\omega}) \geq 1, \quad \hat{\omega}_i \geq 0$$

$$UB(\hat{\omega}) = \sum_i \hat{\omega}_i (d_i + \Delta - 1)$$

$$LB(\hat{\omega}) = \min_{\sigma \in P_\Delta} \sum_i \hat{\omega}_i \max(d_i, \sigma(i))$$

Towards an LP formulation

$$\rho(d) = \min \sum \hat{\omega}_i (d_i + \Delta - 1)$$

$$\sum_i \hat{\omega}_i \max(d_i, \sigma(i)) \geqslant 1 \quad \forall \sigma \in P_\Delta$$

$$\hat{\omega}_i \geqslant 0$$

ELIPSOID METHOD IS NOT PRACTICAL

LP formulation

$$
\begin{aligned}
\rho(d) = \quad & \min \sum_i \hat{\omega}_i (d_i + \Delta - 1) \\
\text{s.t.} \quad & \sum (y_i - z_i) \geq 1 \\
& y_i - z_j \leq \hat{\omega}_i \max(d_i, j) \\
& \hat{\omega}_i, y_i, z_i \geq 0
\end{aligned}
$$

Bounding the Local Ratio

Def: $\rho = \sup_{d} \rho(d)$

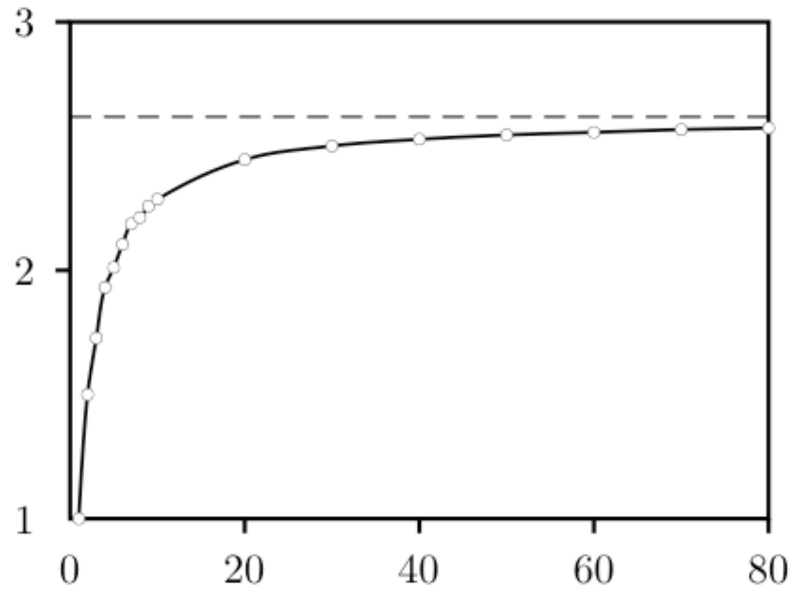
Thm: S is ρ-approximate and this is tight

Now we only need to bound ρ

Experimental Evaluation

Exhaustive search for small values of Δ

$$\rho_\Delta = \max_{d : |d| = \Delta} \rho(d)$$



$$\rho = \sup_d \rho(d)$$

$$\rho_\Delta = \max \alpha$$

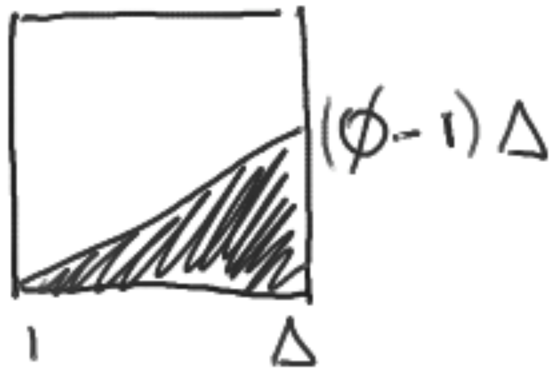
$$\sum_i x_{ij} \geq \alpha \qquad \forall j$$

$$\sum_j x_{ij} \leq \alpha \qquad \forall i$$

$$\sum_j x_{ij} \max(d_i, j) \leq d_i + \Delta - 1 \qquad \forall i$$

$$d_i, x_{ij}, \alpha \geq 0$$

Thm: $\rho = 1 + \phi \approx 2.61$



Intuition:

- weights exploit irregularities of d
- if d is "flat", only two models are useful

Talk Outline

- ✓ Background
- ✓ Summary of Results
- ✓ Adaptive Local Ratio

Concluding Remarks

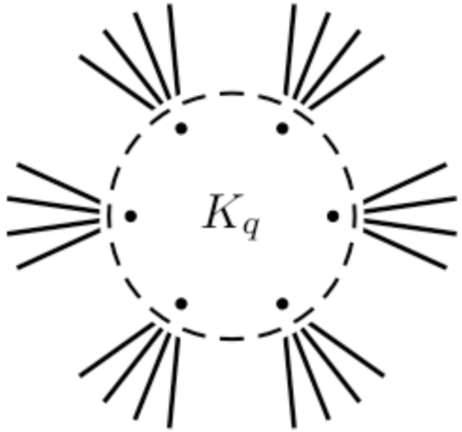
- Generalizations
- First LP-guided local-ratio Alg
- Optimizing the local ratio
  can make a difference

Thanks for

your attention!

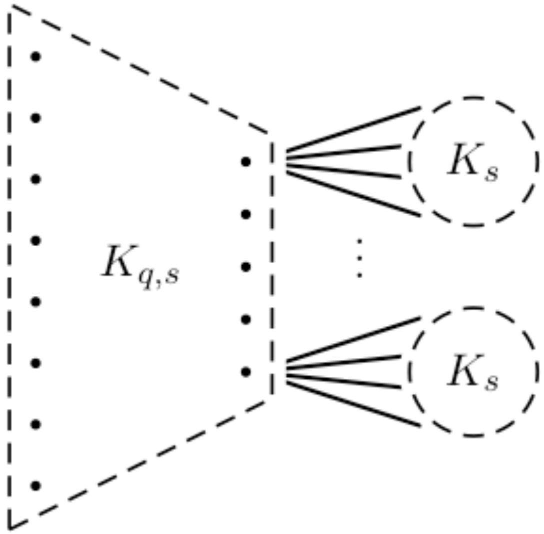
Do you really need $\ell(\cdot)$?

Forget about $\ell(\cdot)$ and just go with greedy



Sort $(u,v) \in E$ by

$$\begin{cases} \min(d_u, d_v) \\ \max(d_u, d_v) \end{cases}$$



Thanks for
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