Tree-Constrained Matching

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Joint work with:

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K. Elbassioni (MPI-INF)
G. Klau (CWI)

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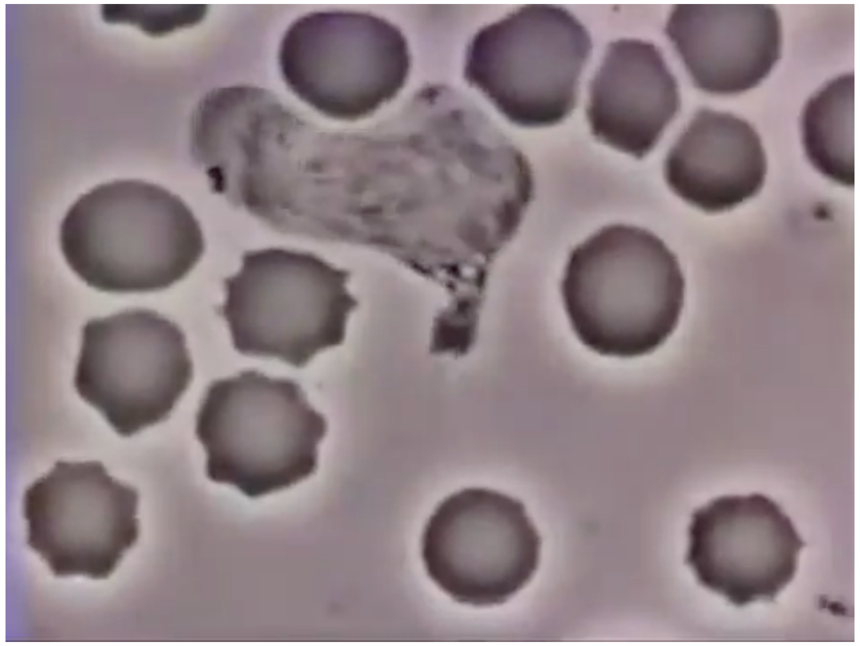

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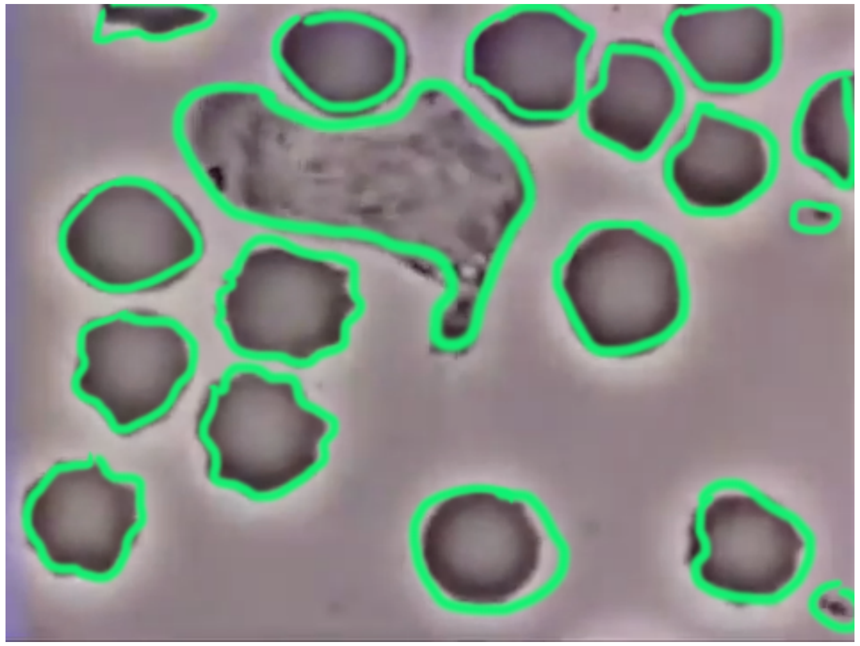

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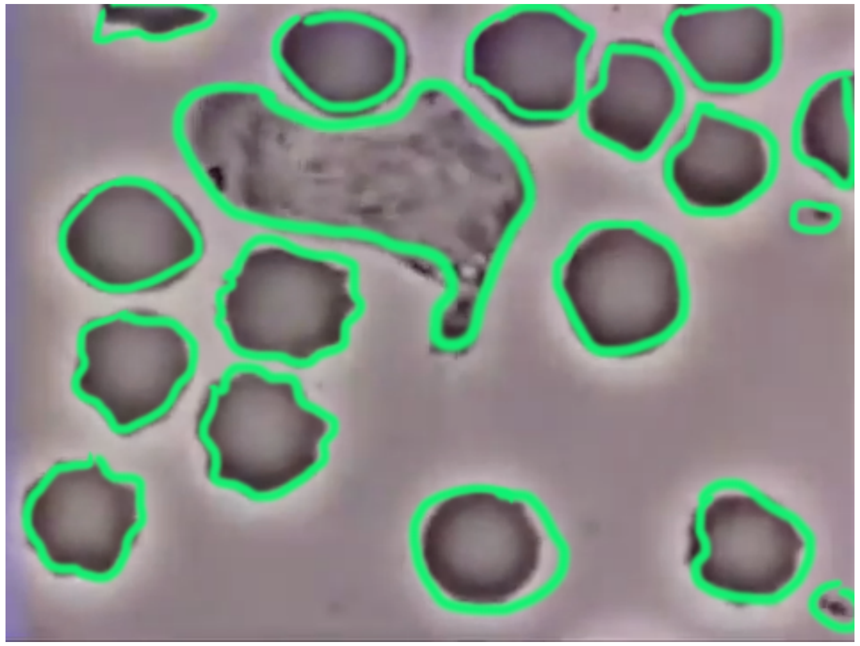

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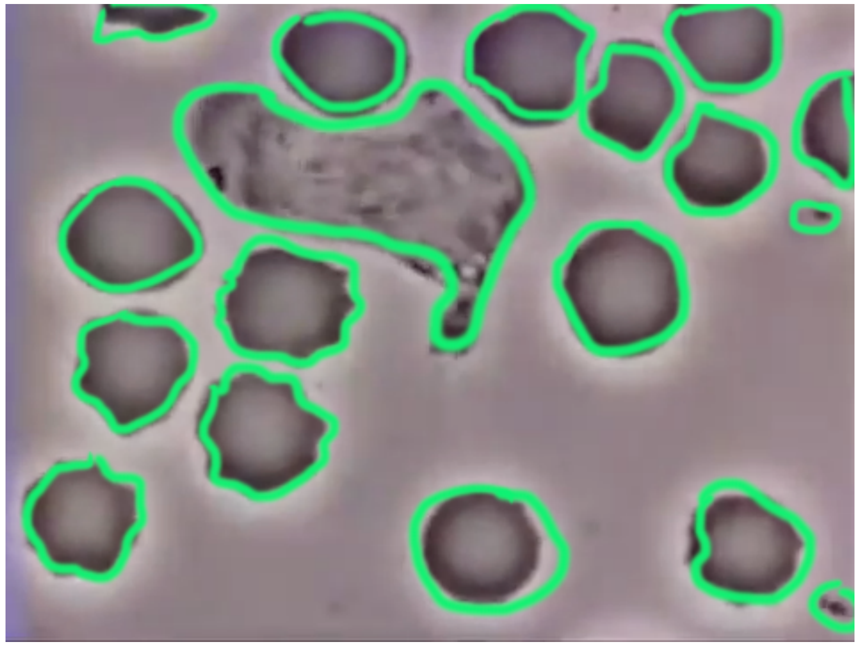

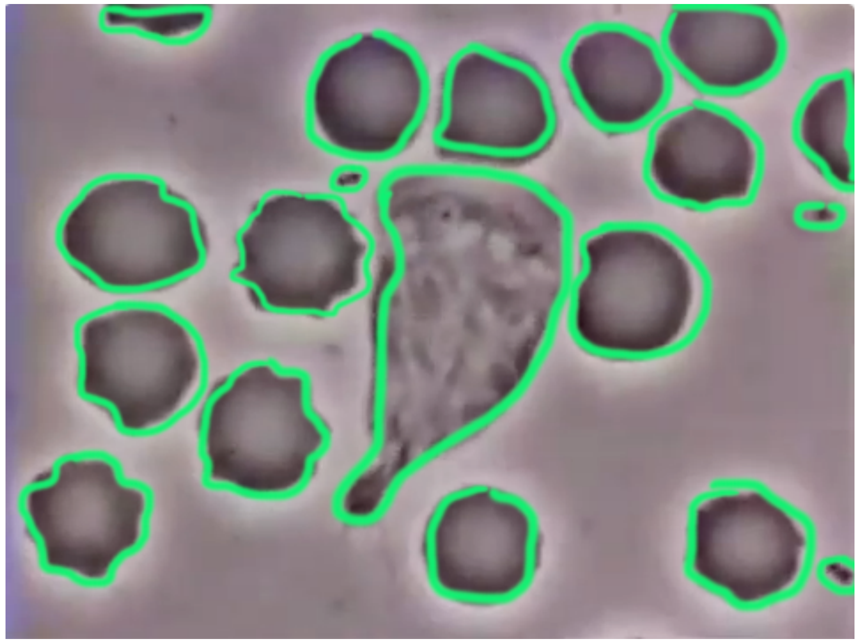

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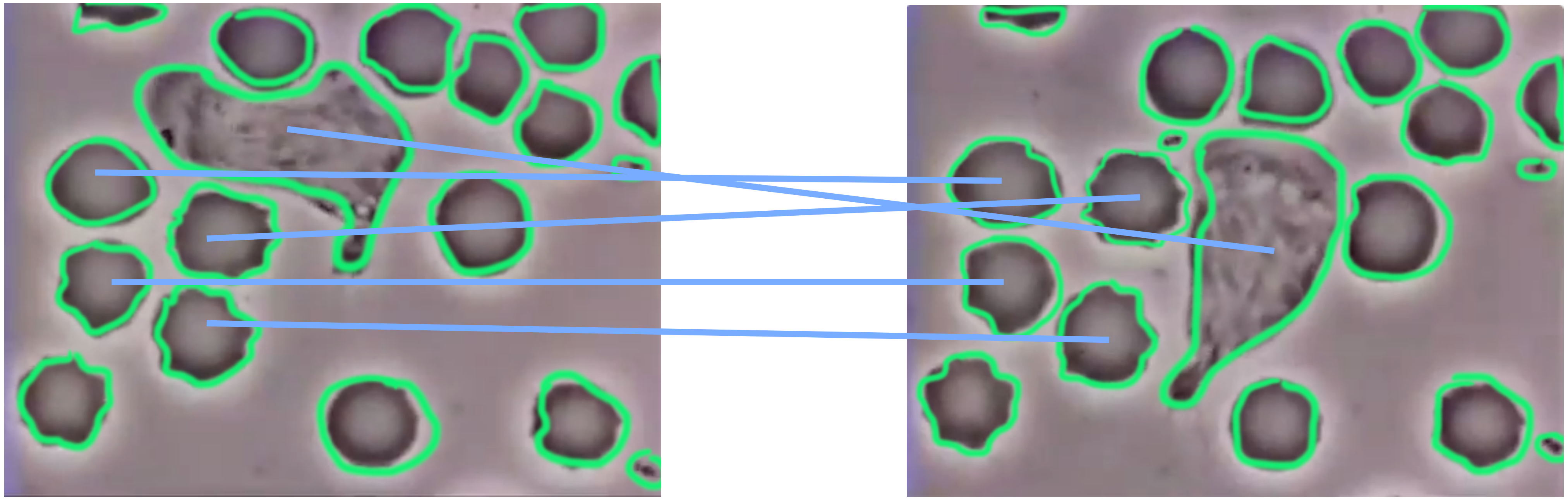

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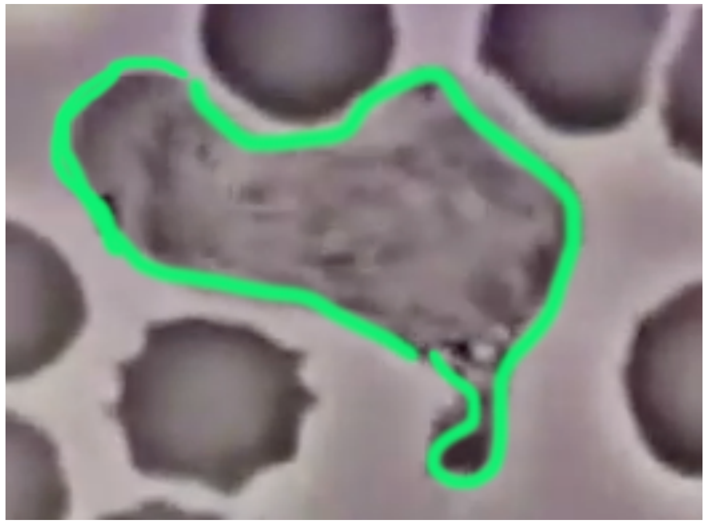

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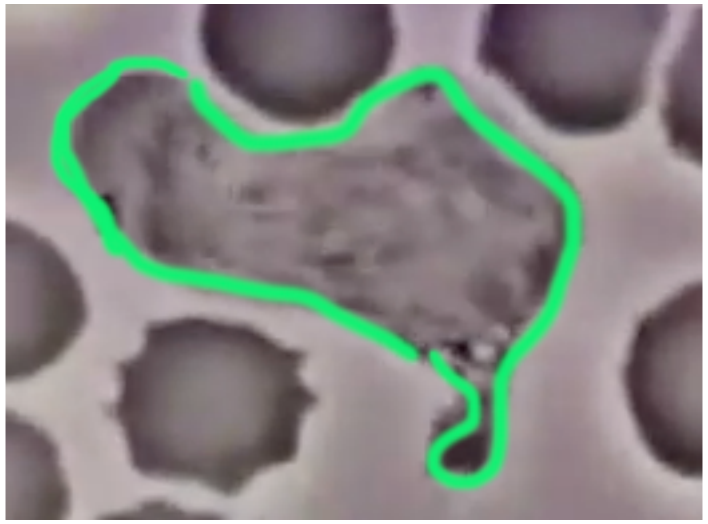

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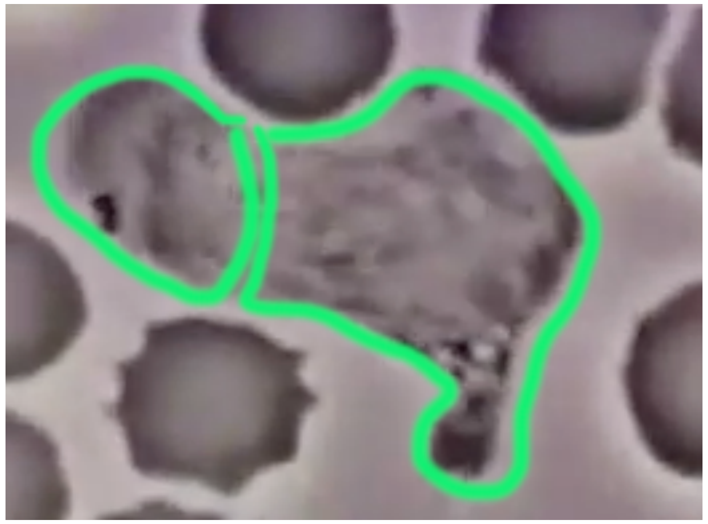
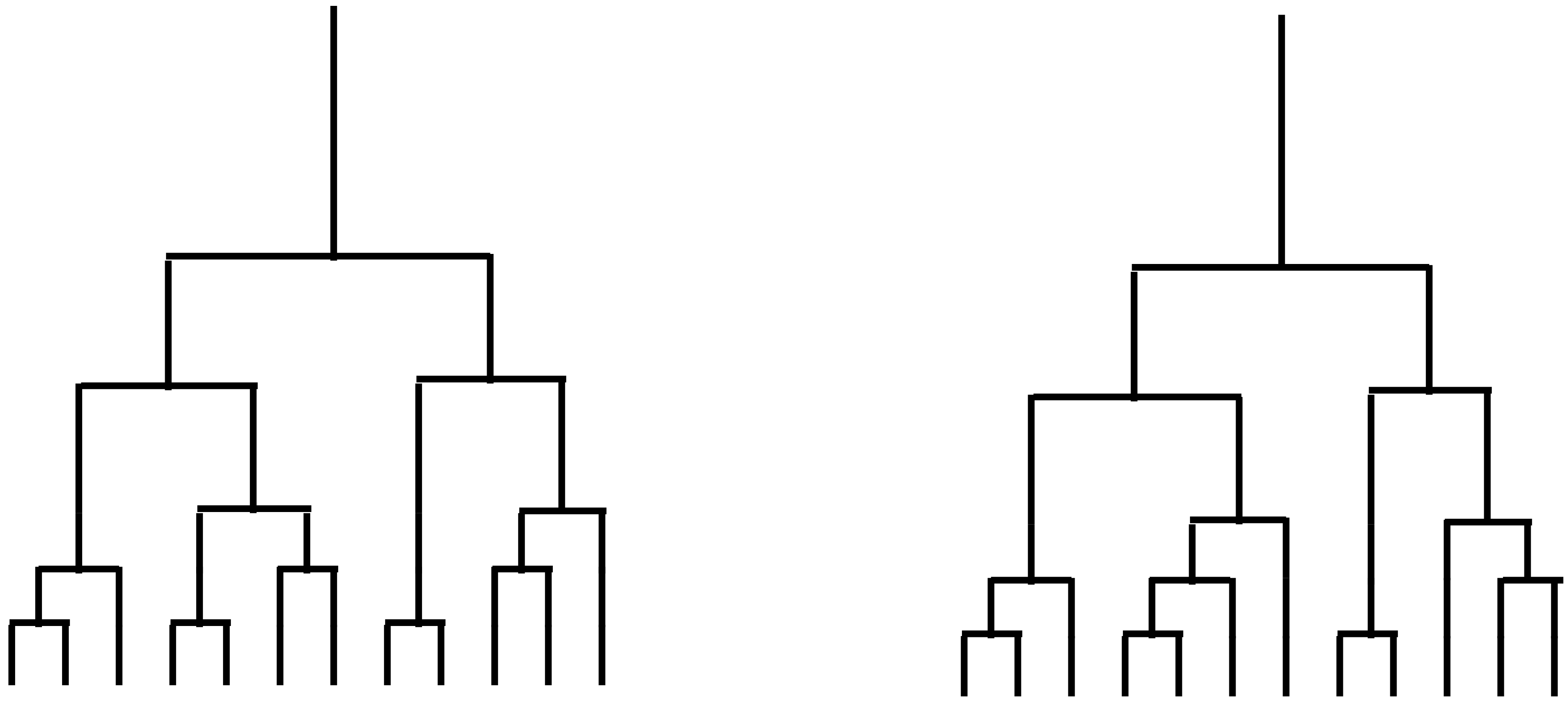


Hierarchical Clustering



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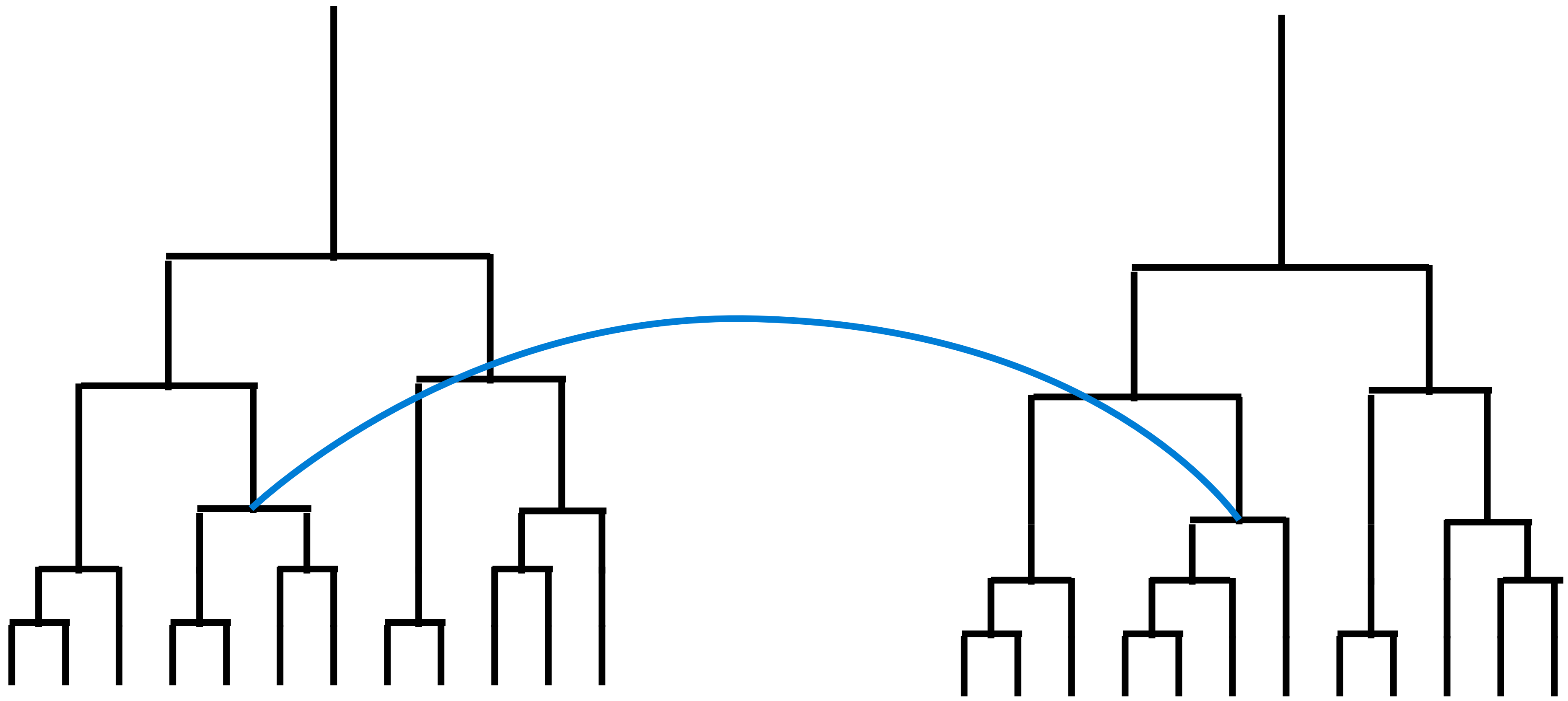

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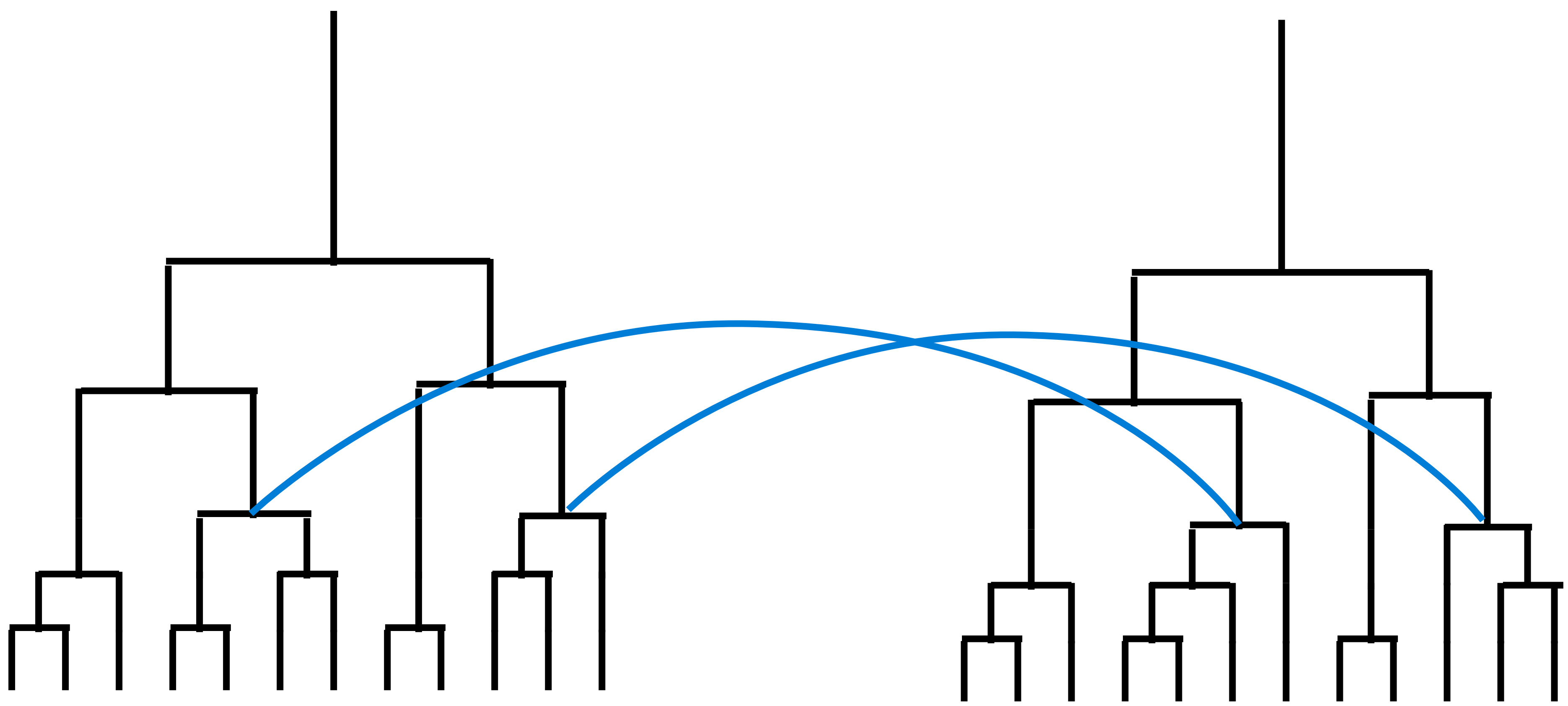

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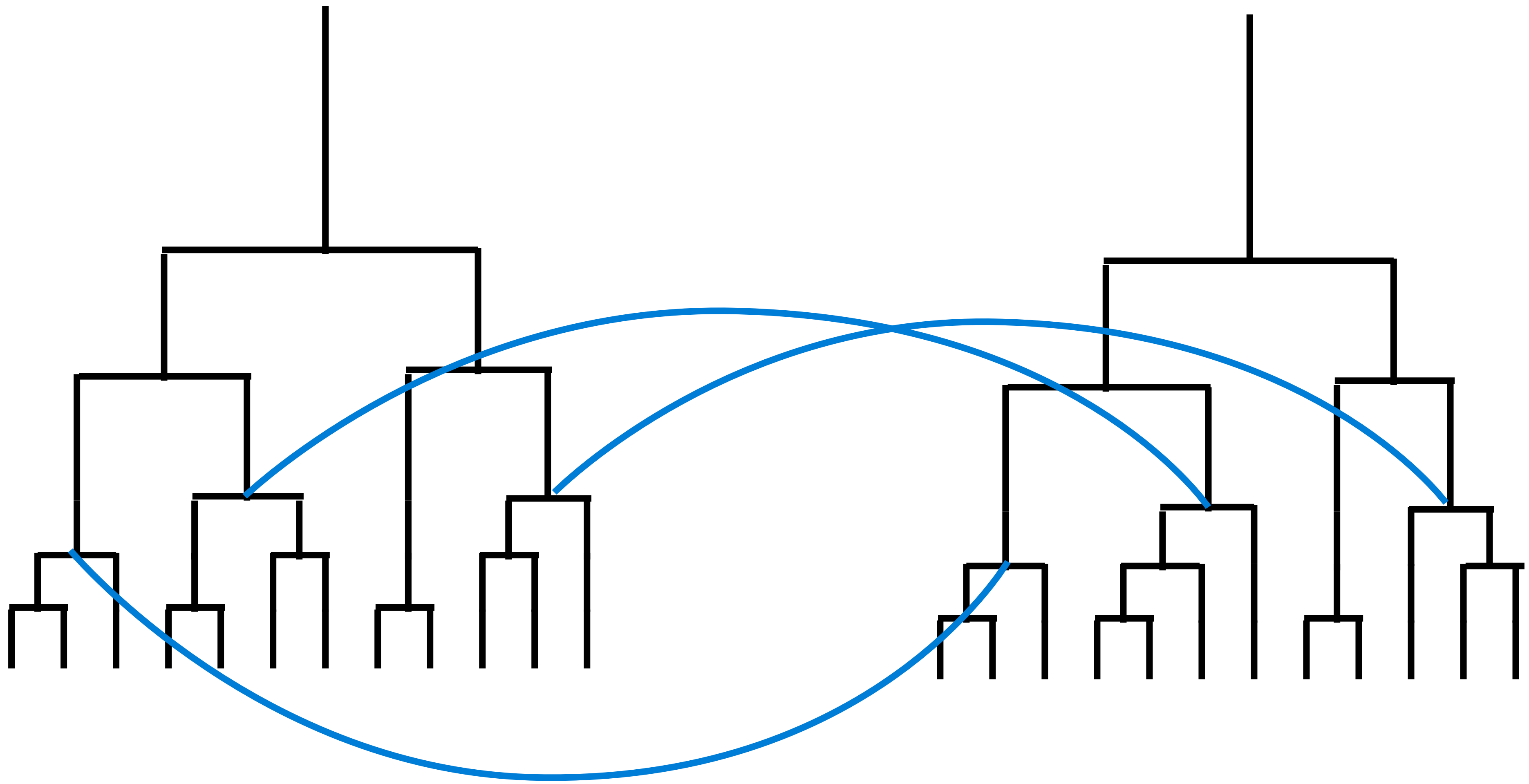

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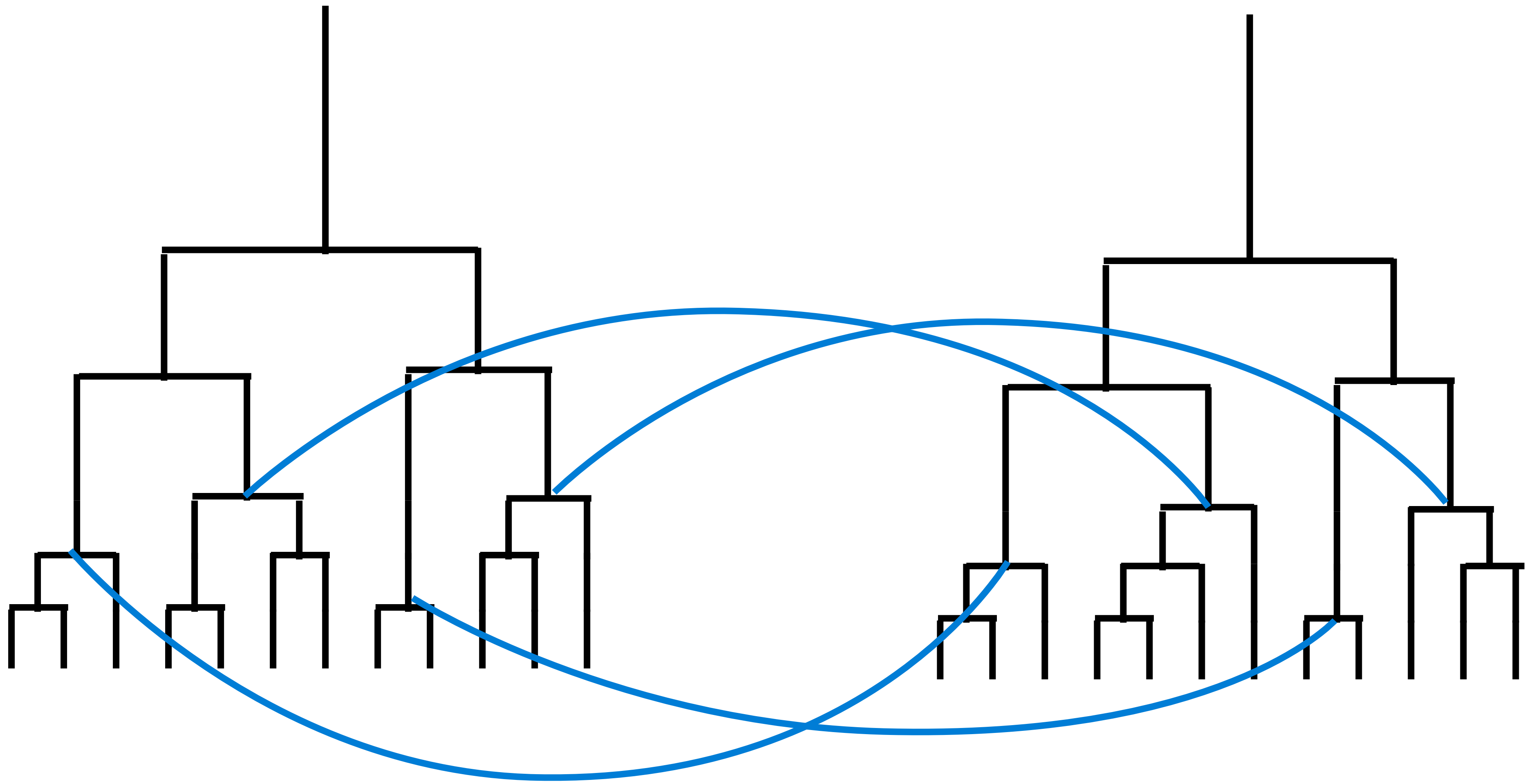
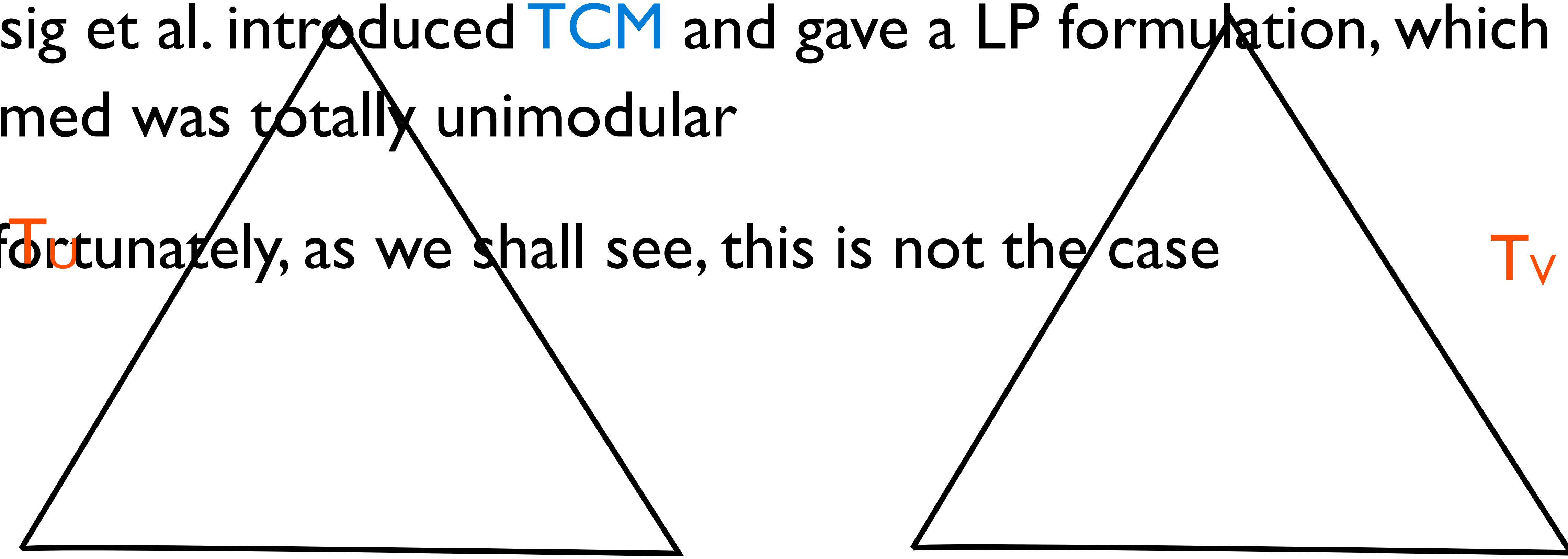


Tree-Constrained Matching

Given a weighted bipartite graph (U,V,E) and trees T_U and T_V over U and V, we want maximum weight matching M such that matched vertices in T_U and T_V are not comparable

Mosig et al. introduced TCM and gave a LP formulation, which they claimed was totally unimodular

Unfortunately, as we shall see, this is not the case



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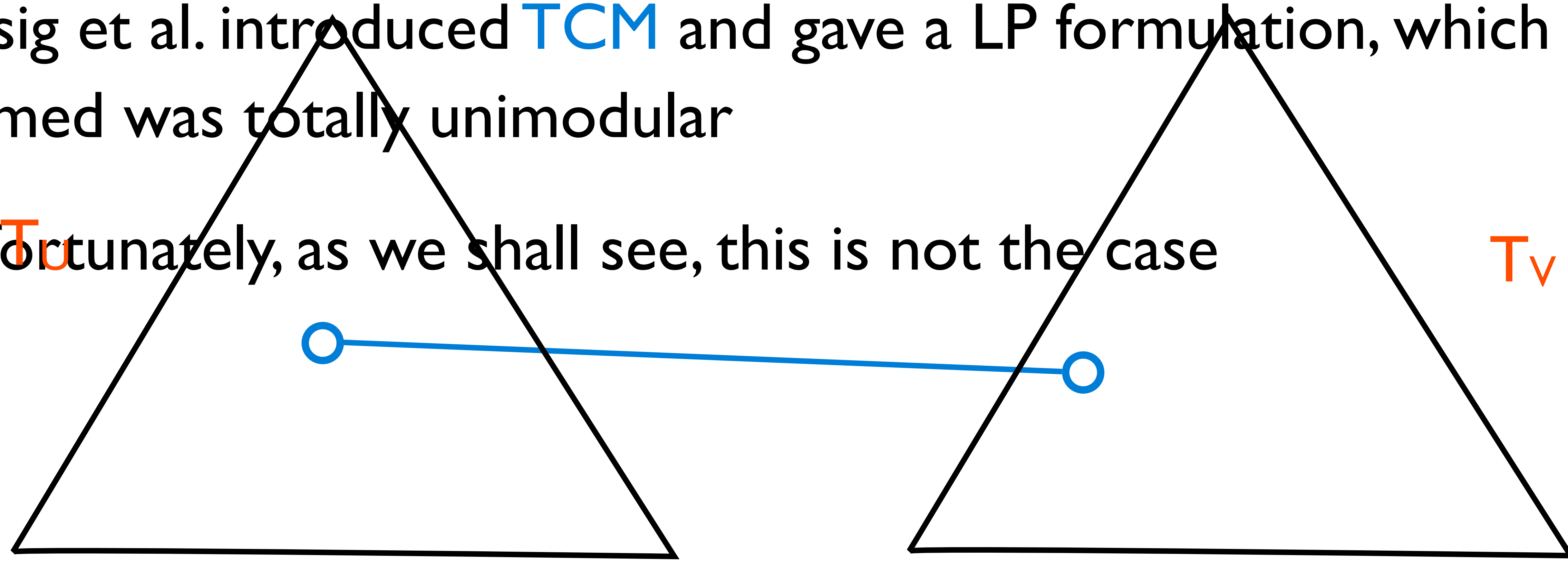

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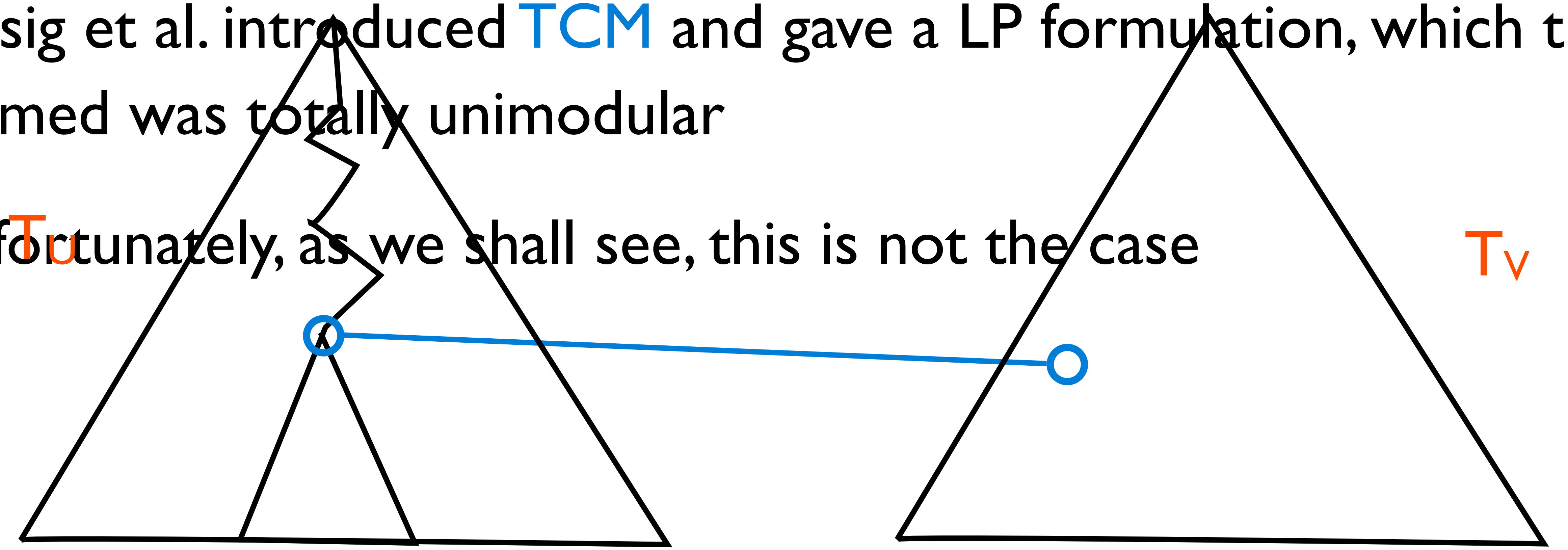

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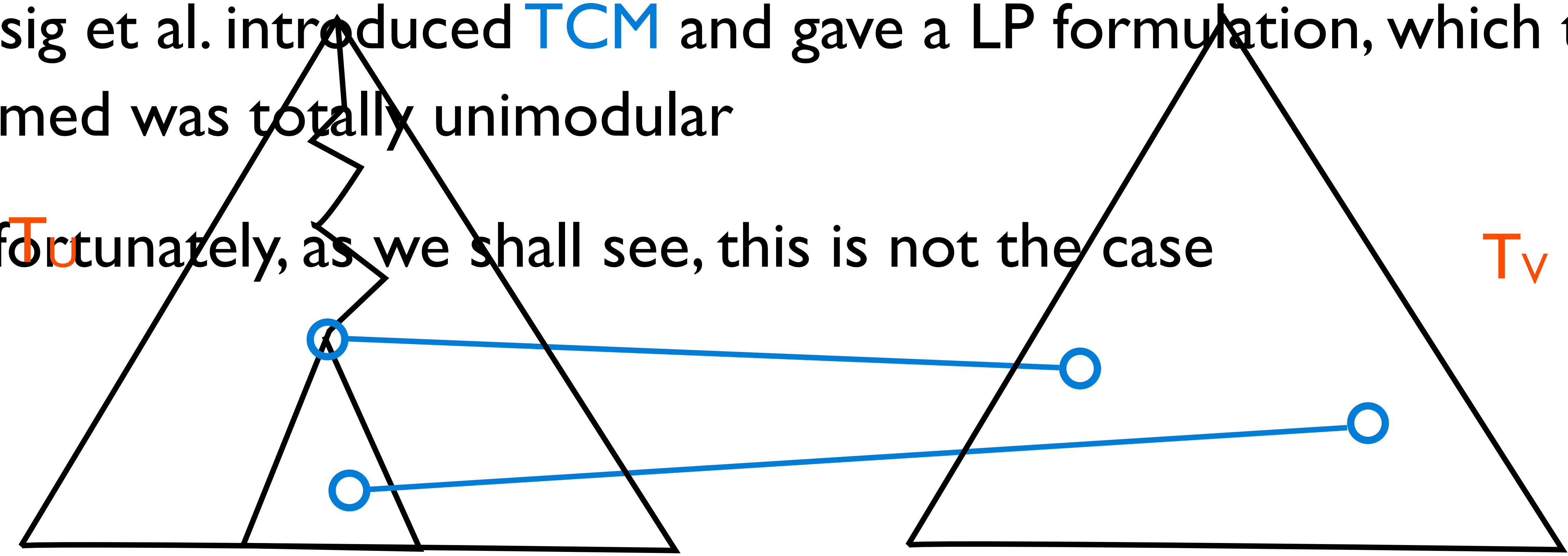

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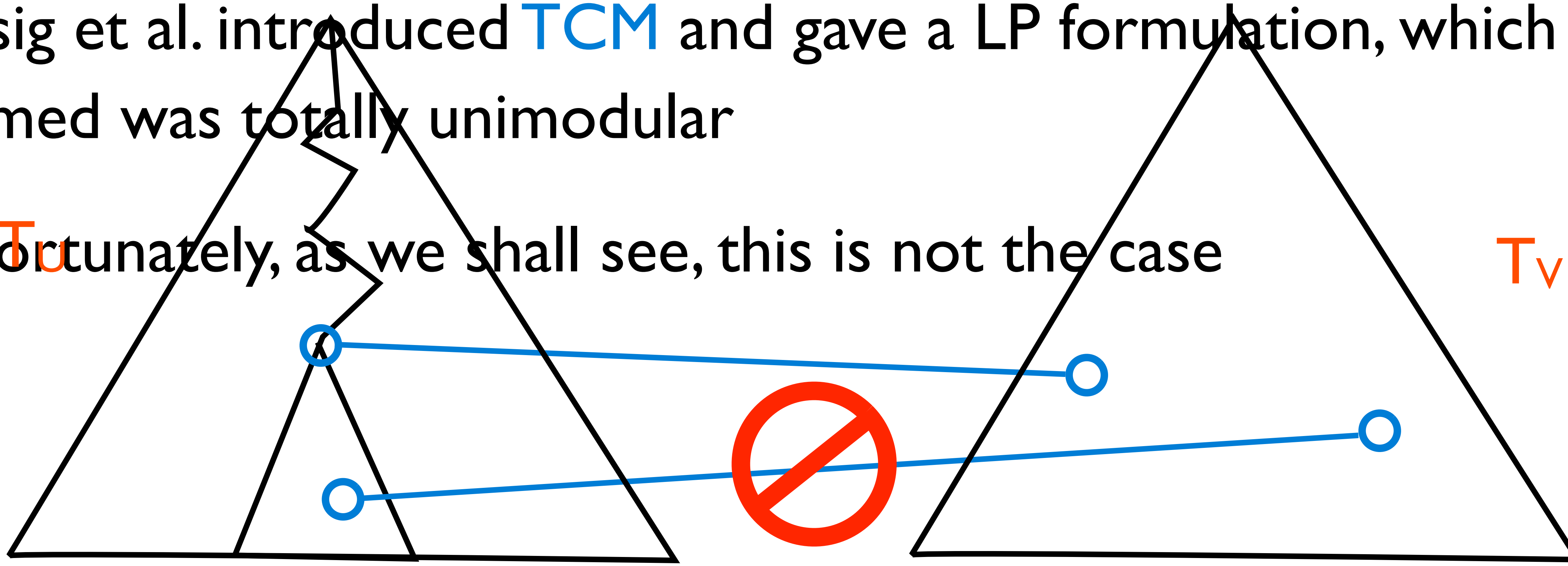

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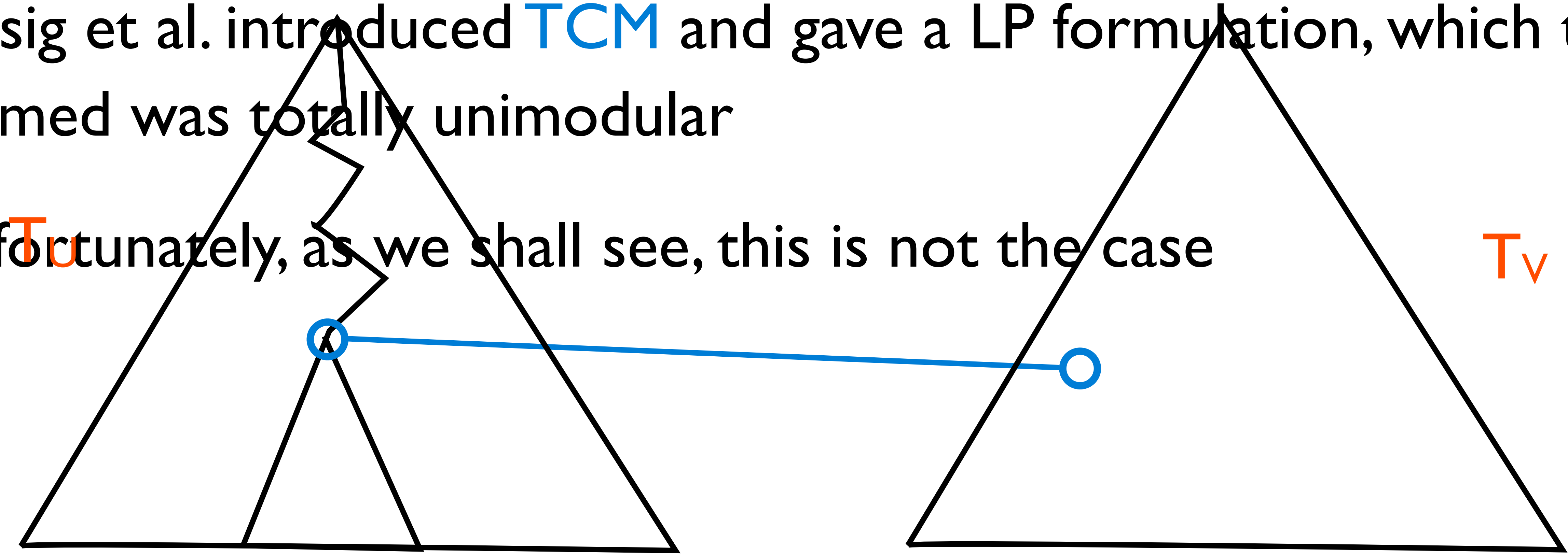

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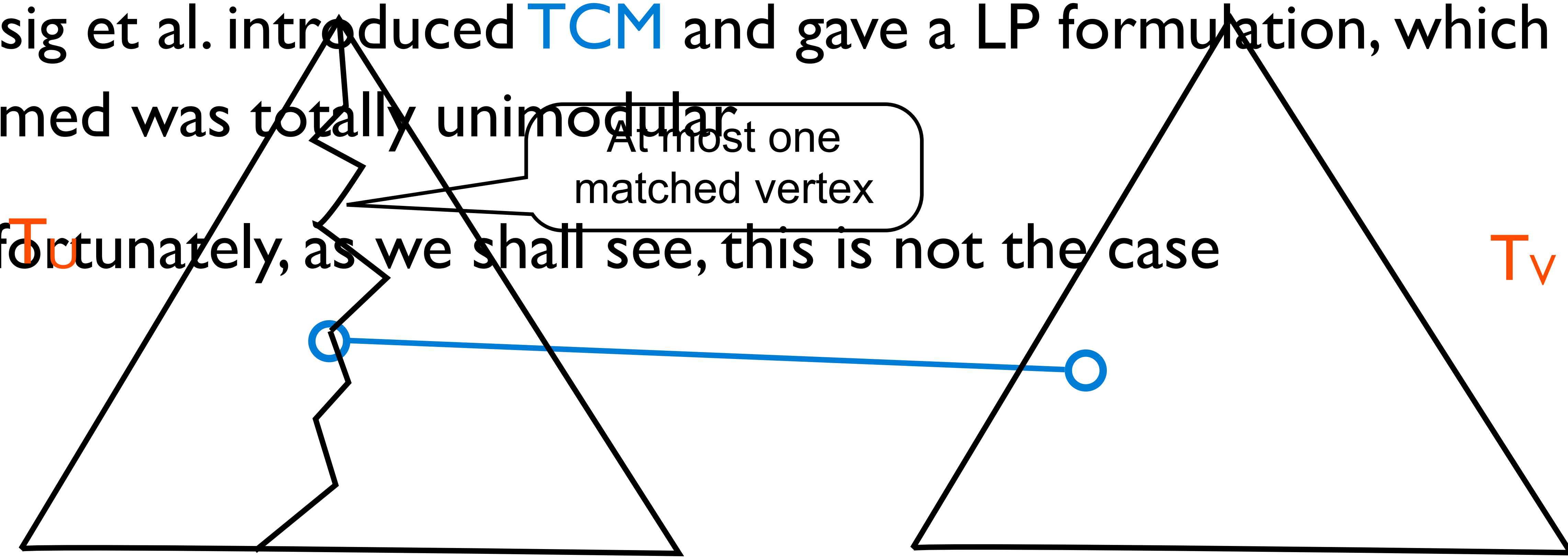

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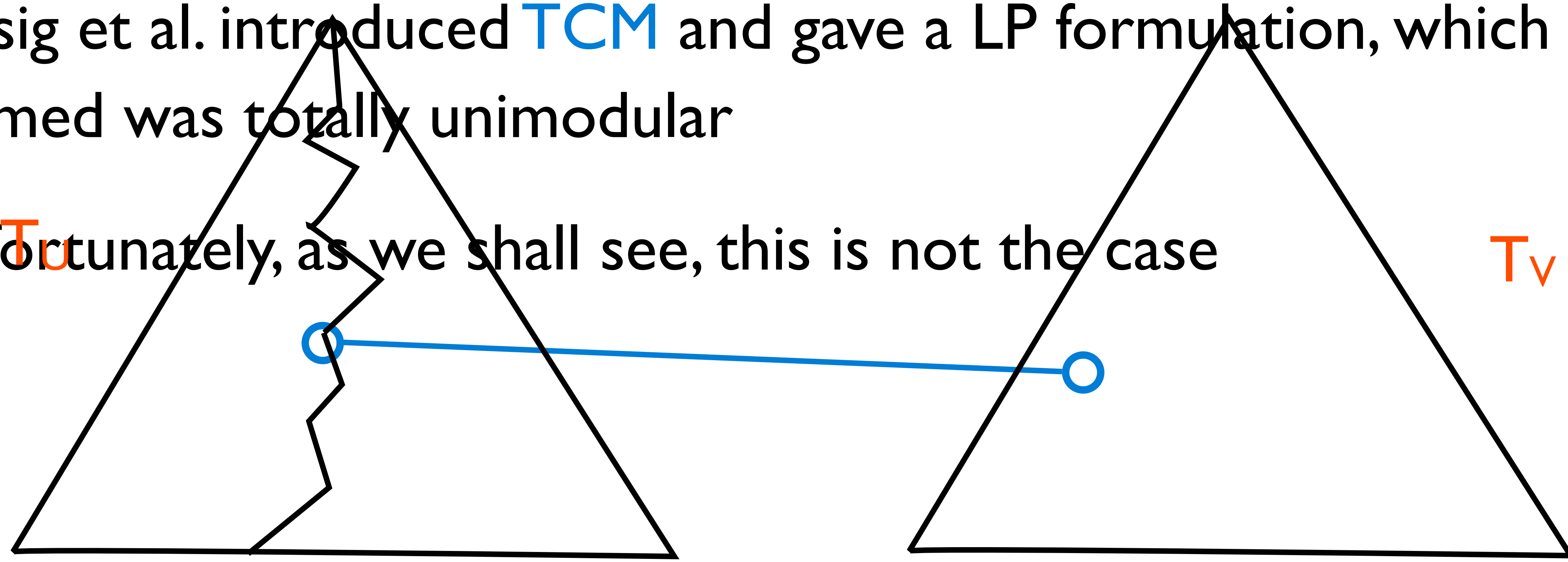

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Thus, there is a 4-approximation for TCM

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Generalization to posets

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Every edge (u,v) is assigned one interval in TU and one in TV

Matching is feasible ⇔ corresp. set of 2-intervals is independent

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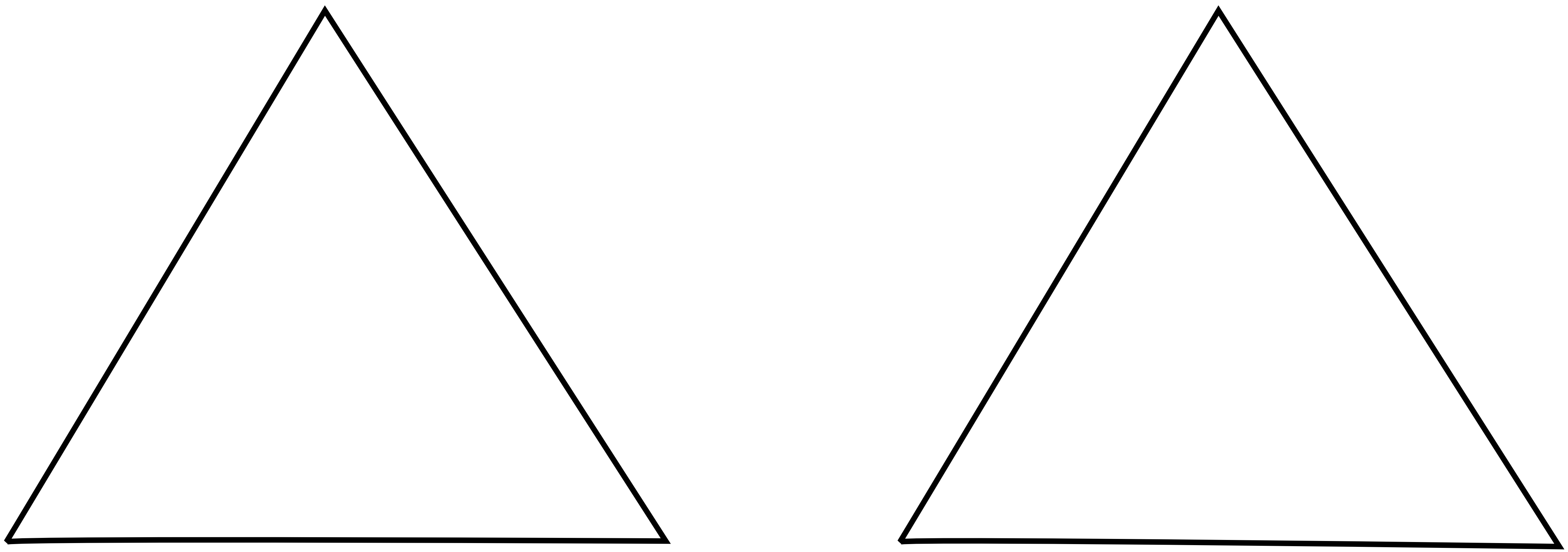

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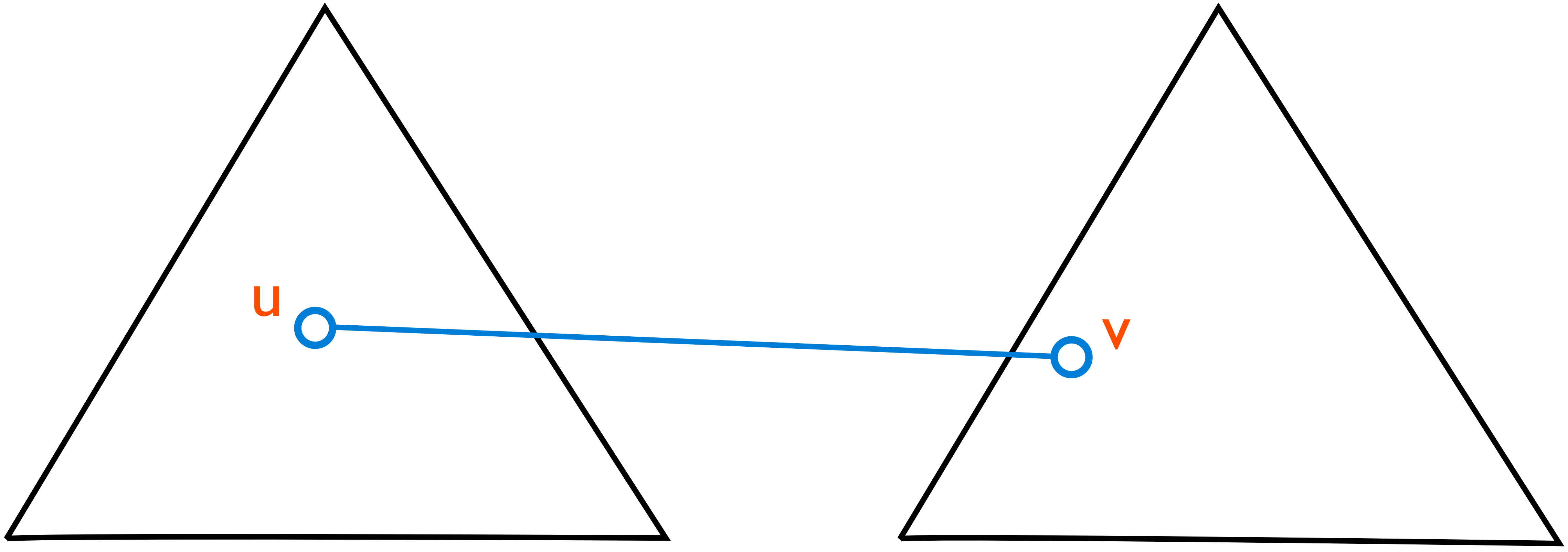

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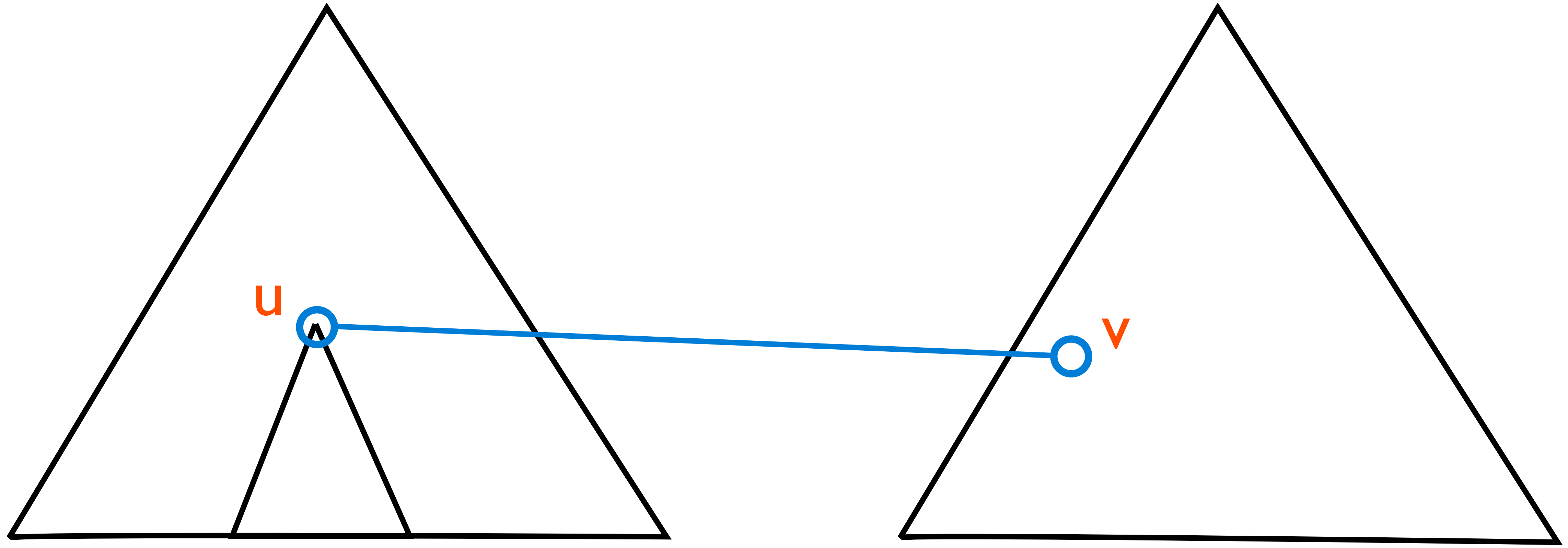

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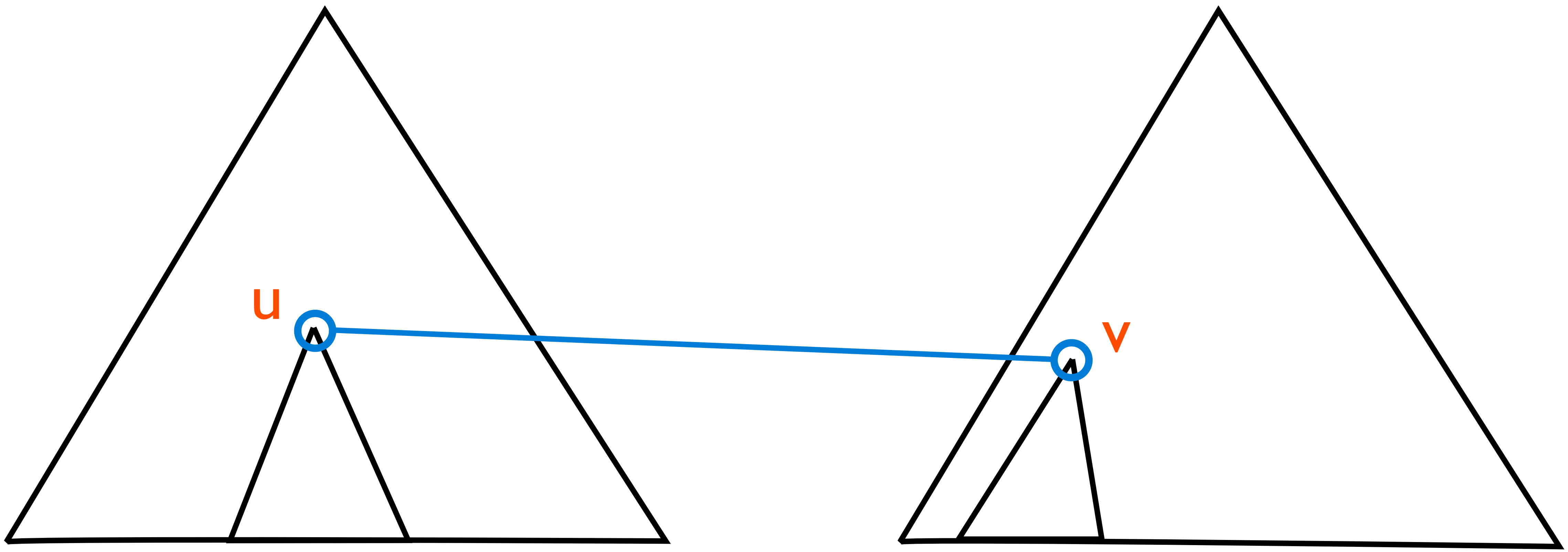

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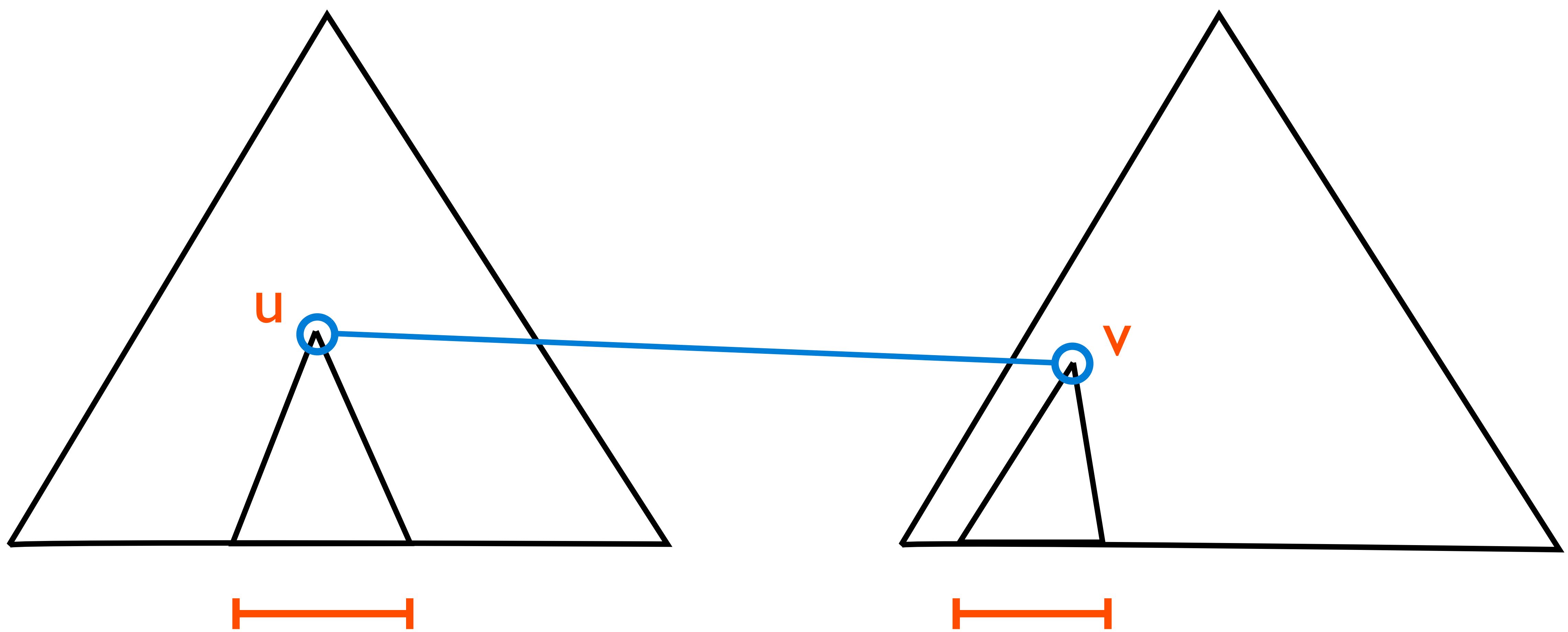

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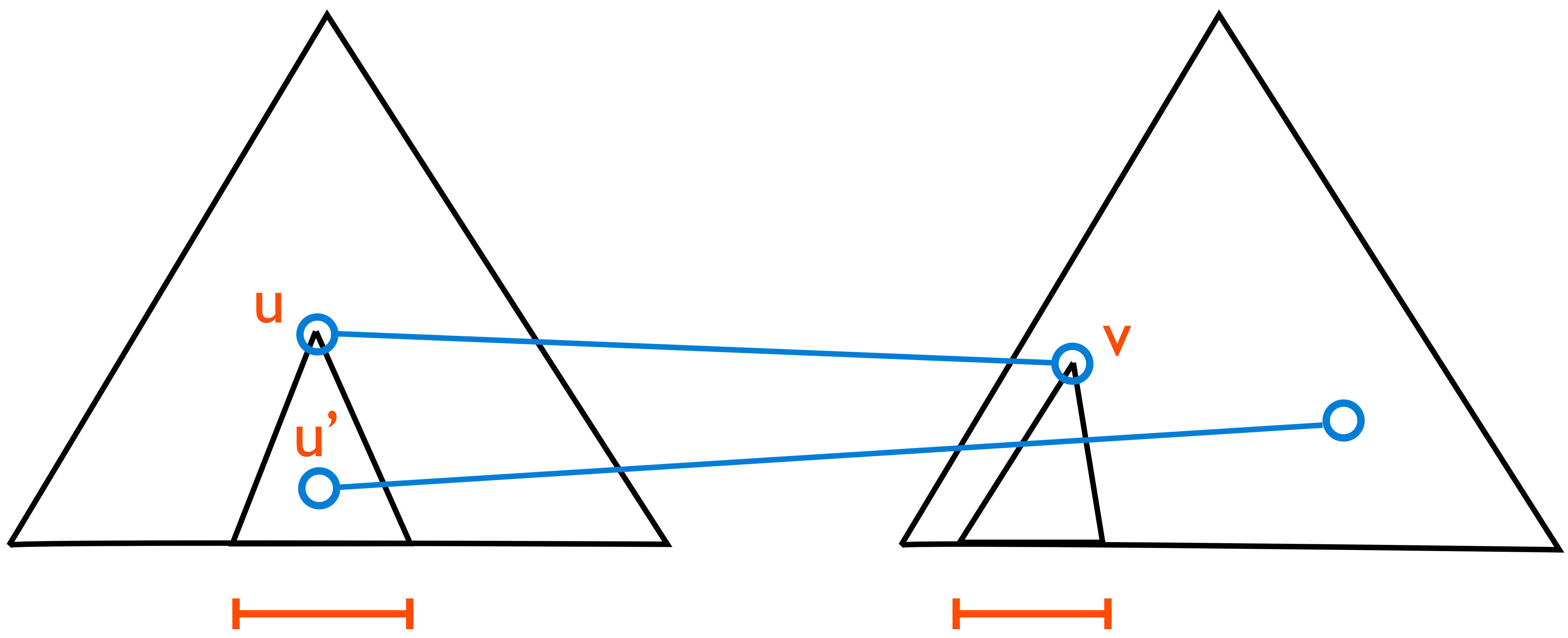

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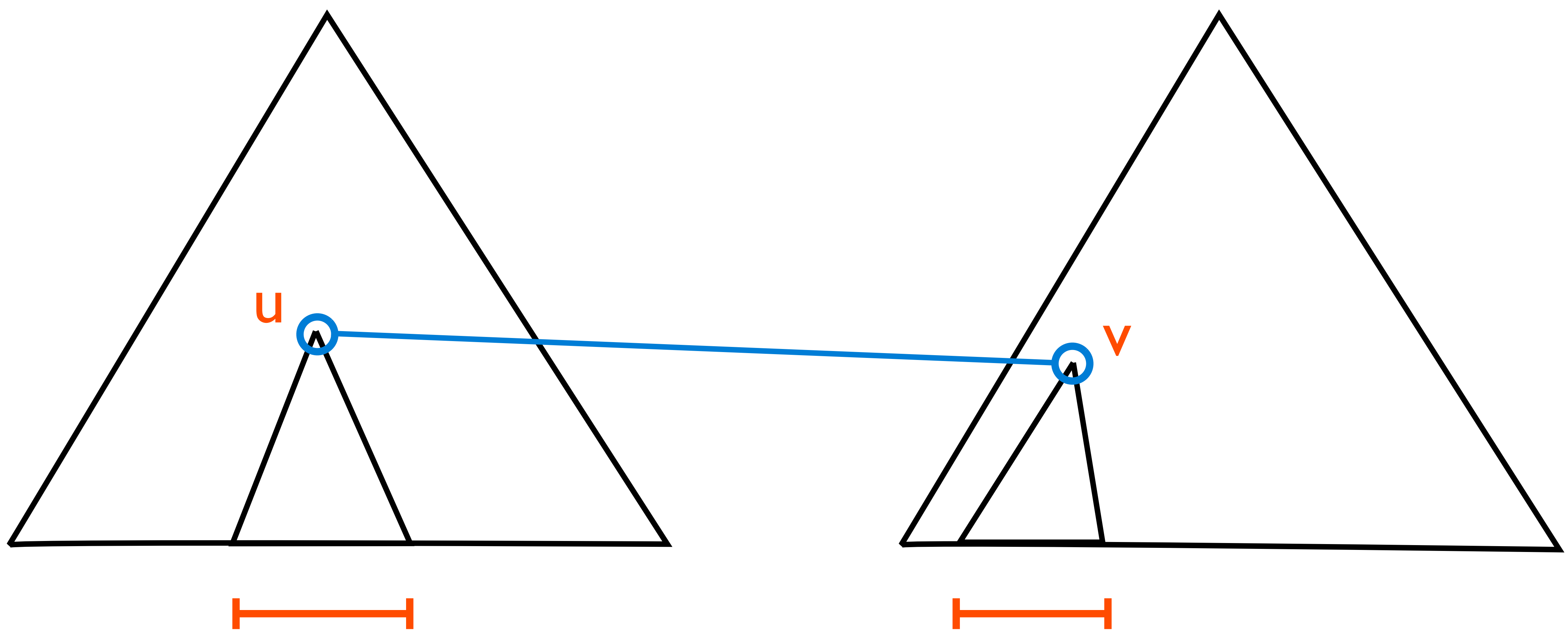

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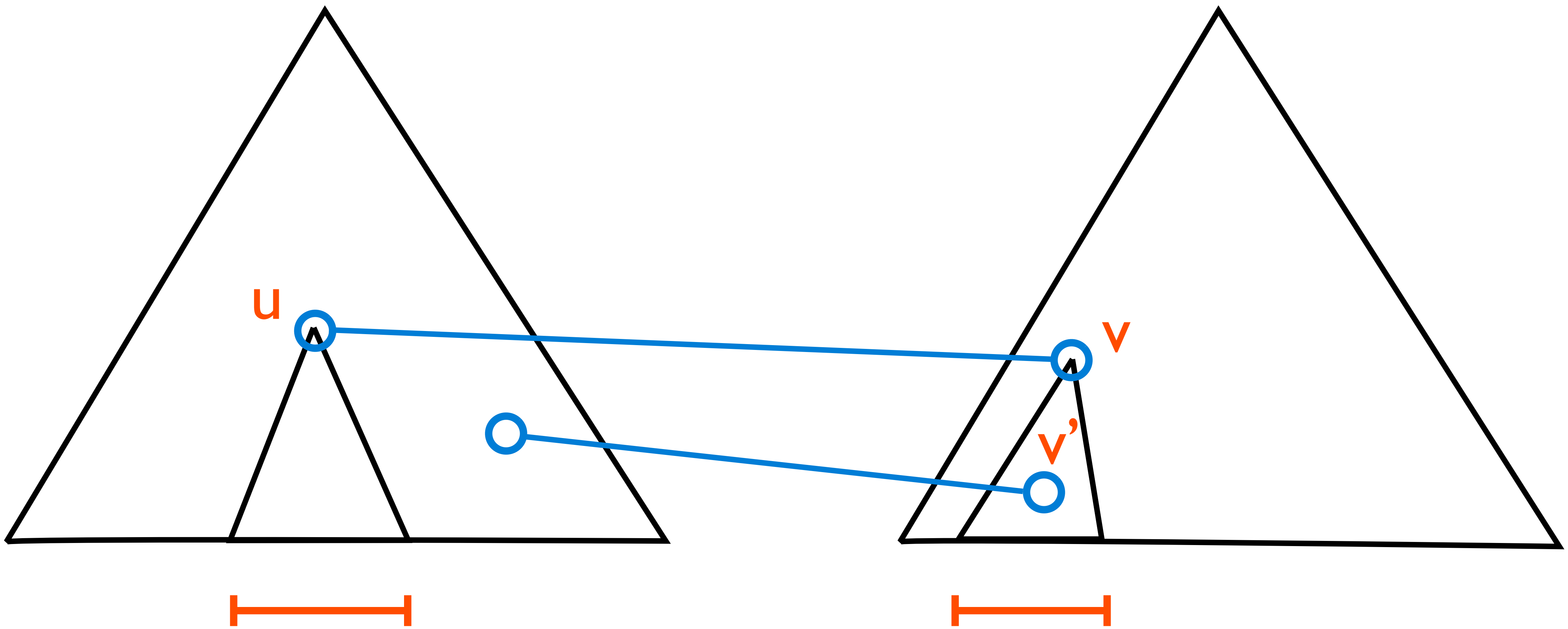

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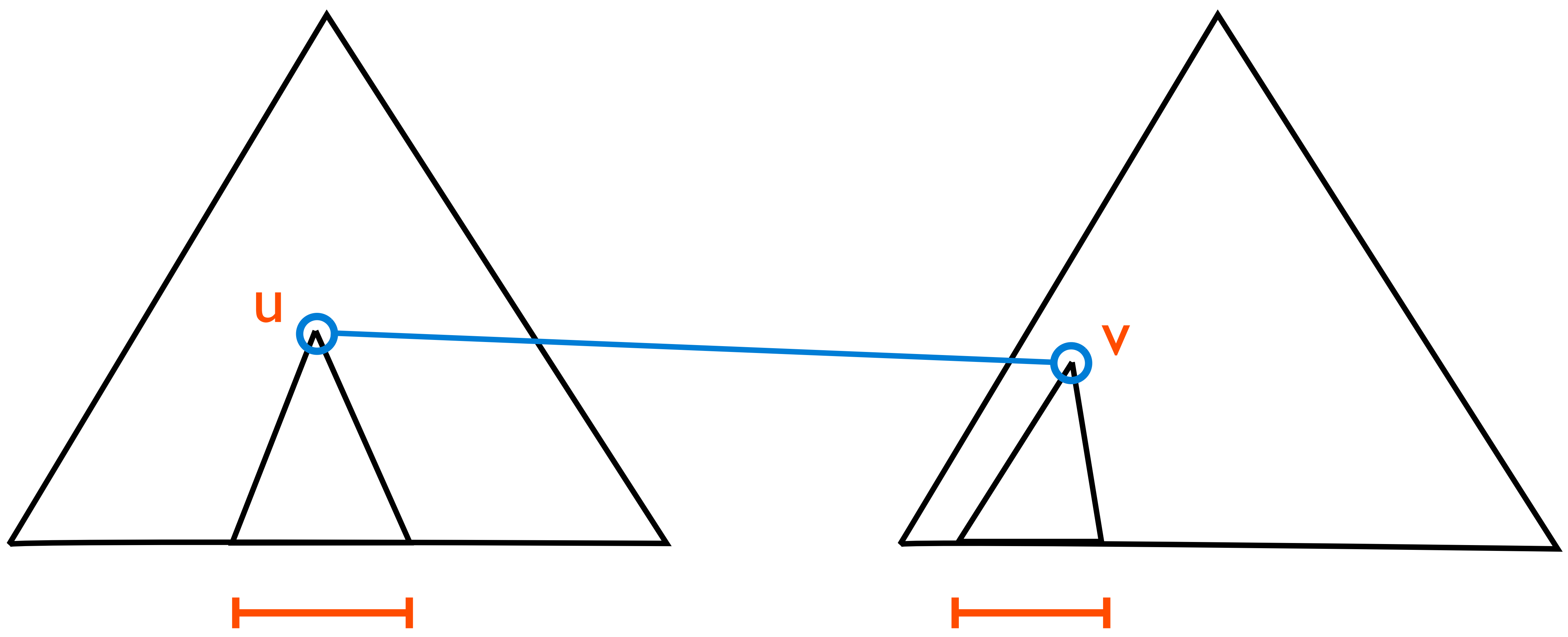

Bar-Yehuda et al. algorithm

minimize $\sum_u x_u$

subject to

$$\sum_{u\,:\,p \text{ in } u} x_u \leq 1 \quad \forall \text{ point } p$$

$$x_u \geq 0 \quad \forall \text{ d-interval } u$$

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MIS-d-interval(G)

1. let x be optimal LP solution
2. let S be the empty set
3. while G is non empty
4. let u minimize x(N(u))
5. add u to S
6. remove N(u) + u from G
7. return S

$\forall$ feasible $x : \exists\, u : x(N(u)) \leq 2d$

$\sum_u x_u \sum_{v \text{ in } N(u)} x_v$

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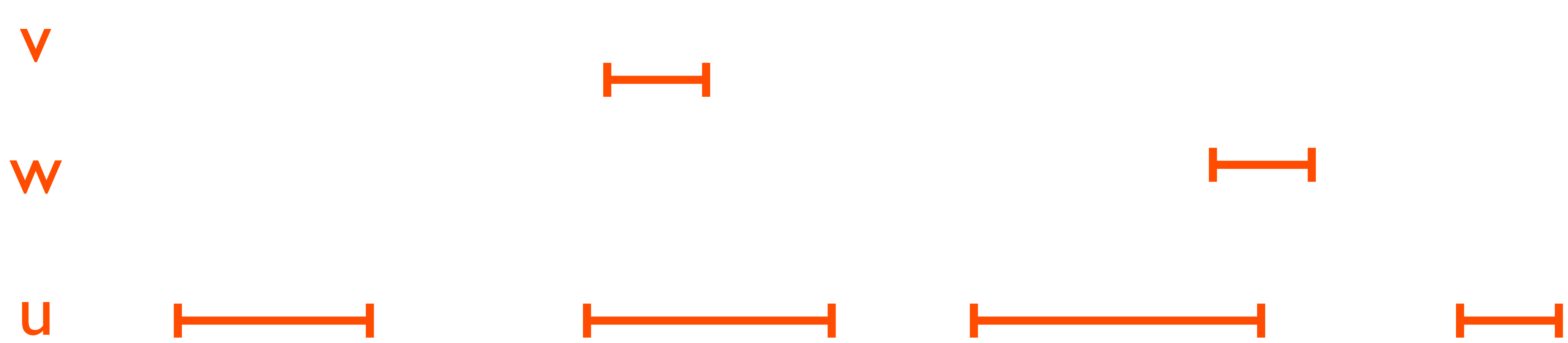

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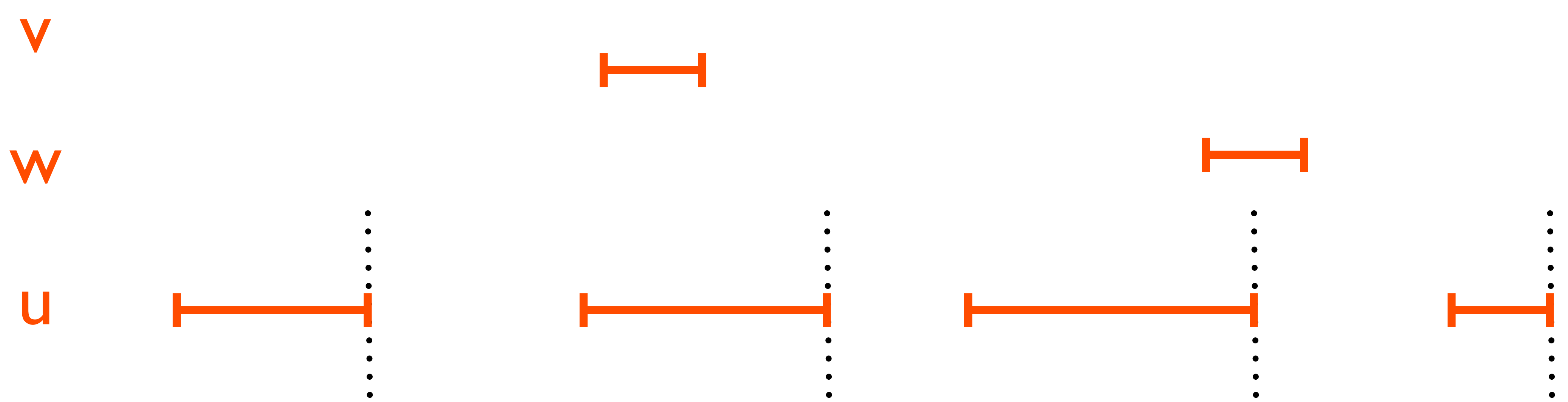

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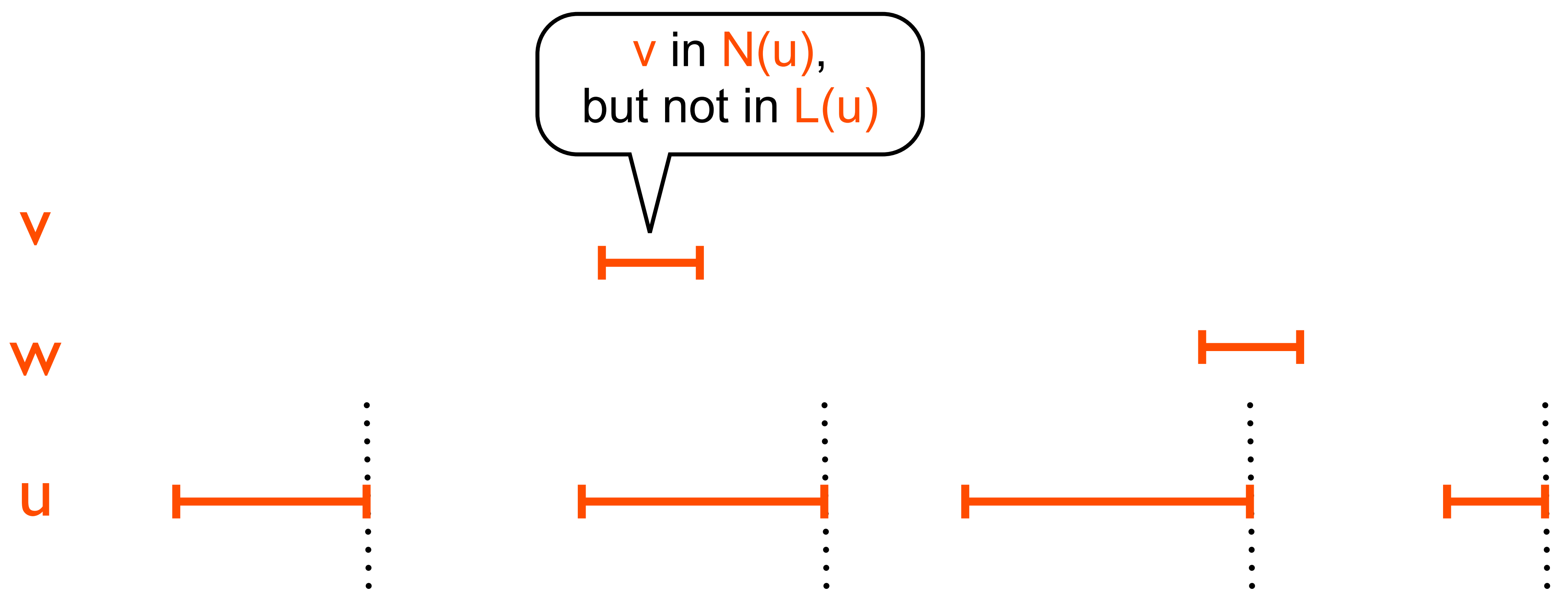

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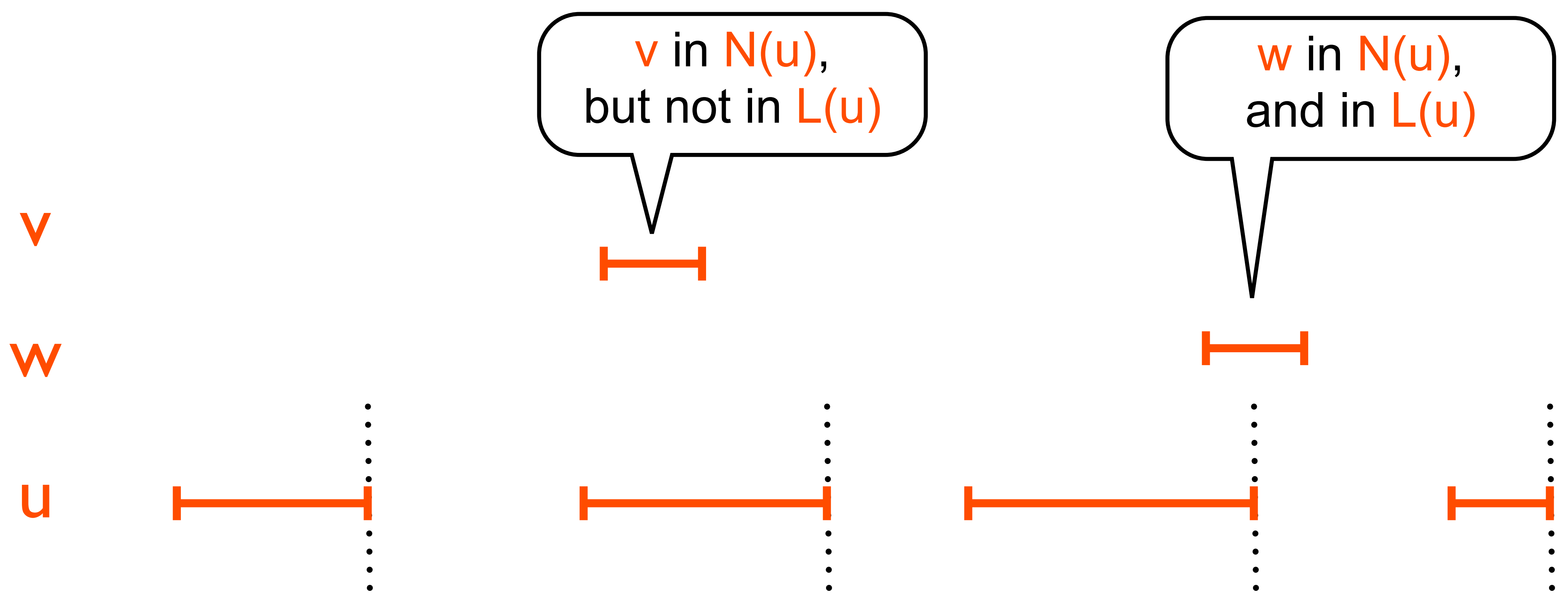
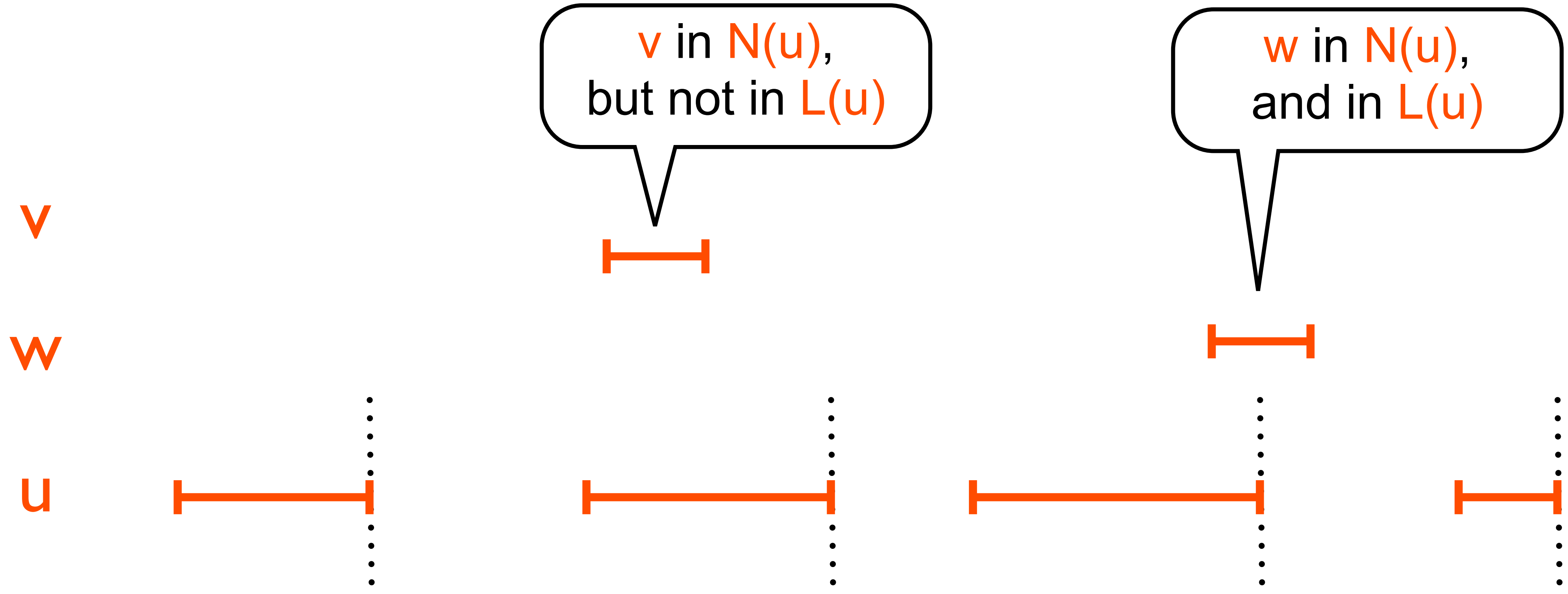


$\forall$ feasible $x : \exists\, u : x(N(u)) \leq 2d$

$$\sum_u x_u \sum_{v \text{ in } N(u)} x_v \leq 2 \sum_u x_u \sum_{v \text{ in } L(u)} x_v$$



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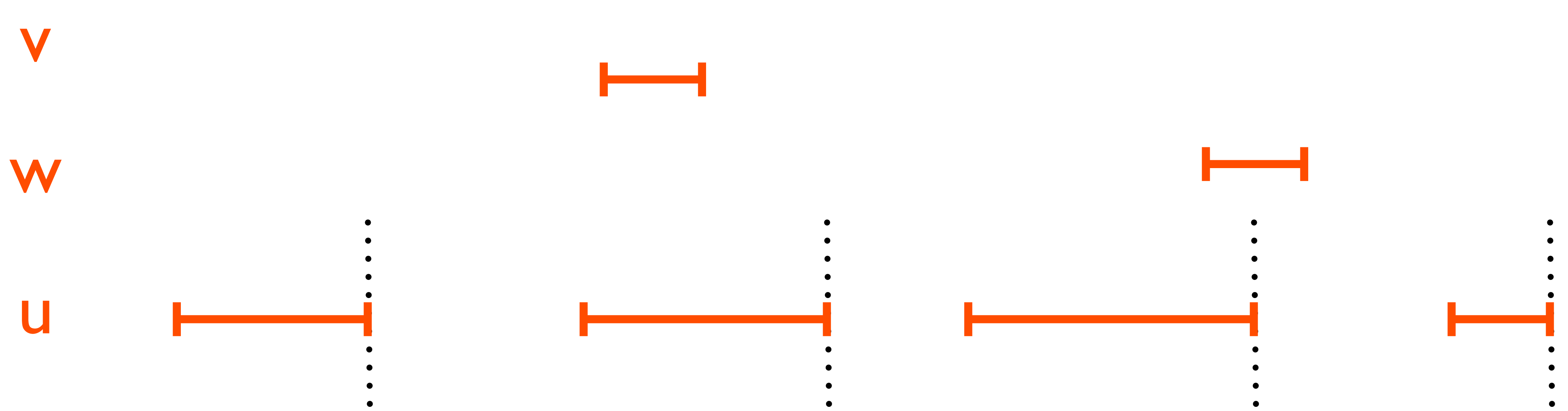

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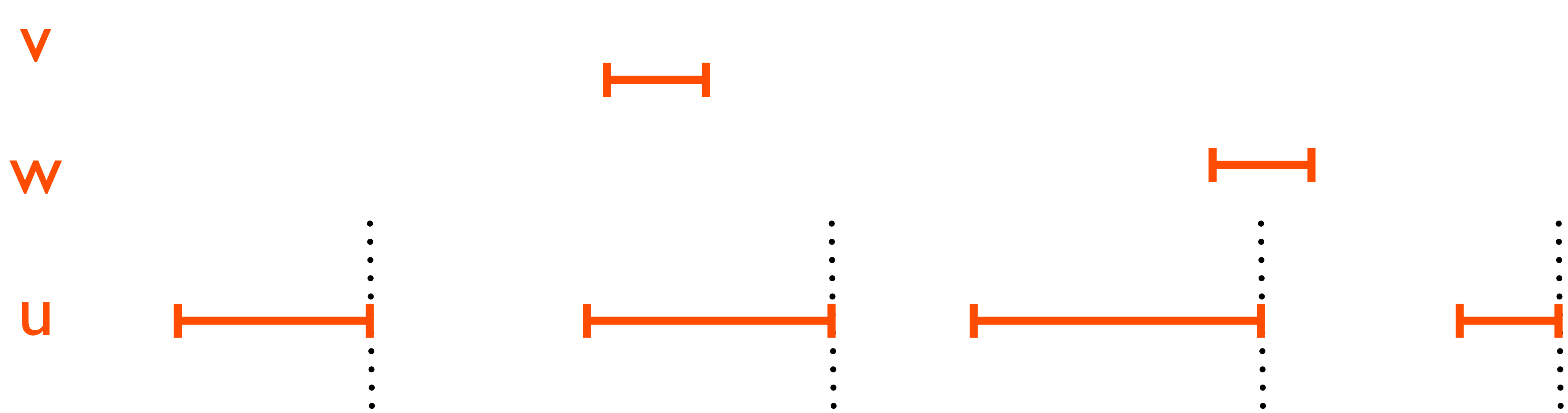
$\forall$ feasible x : $\exists$ u : x(N(u)) $\leq$ 2d

$$\begin{aligned}\textstyle\sum_u x_u \sum_{v \text{ in } N(u)} x_v &\leq 2 \textstyle\sum_u x_u \sum_{v \text{ in } L(u)} x_v \\ &\leq 2 \textstyle\sum_u x_u\, d \\ &\leq 2d \textstyle\sum_u x_u\end{aligned}$$



4-approximation for TCM

minimize $\sum_e x_e$

subject to

$\sum_{e \text{ on } P} x_e \leq 1 \quad \forall$ desc. path P

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N(e) is the set of edges in conflict with e

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For ease of analysis, assume that

- All edges $x_e > 0$
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$\forall$ basic feasible x : $\exists$ e : x(N(e) < 3

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- All edges $x_e > 0$
- No leaf is unmatched

If we have a leaf-to-leaf edge, we are done

$\forall$ basic feasible x : $\exists$ e : $x(N(e)) < 3$

For ease of analysis, assume that

- All edges $x_e > 0$
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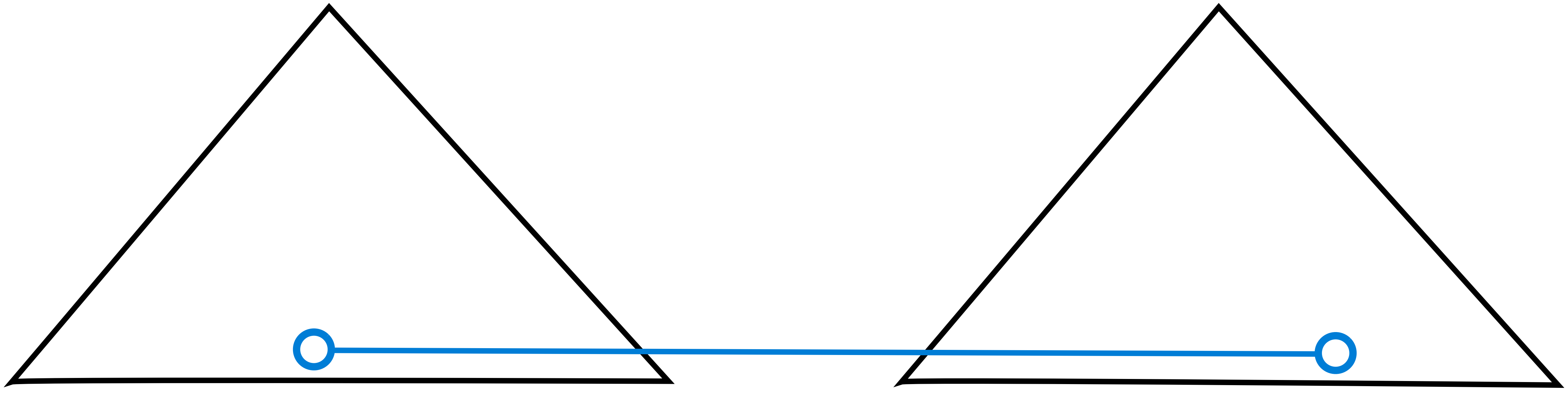

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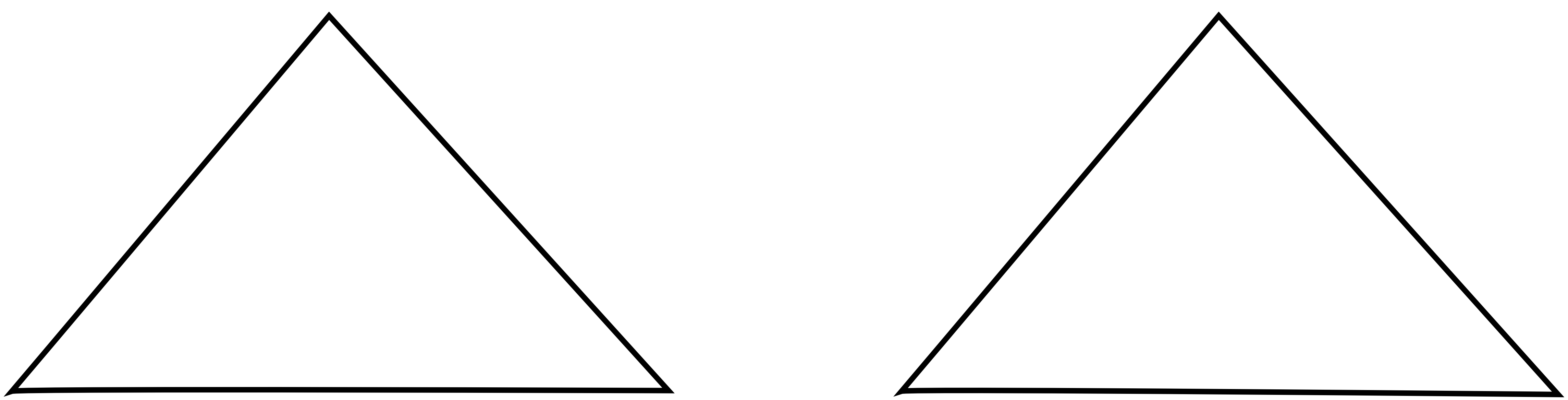

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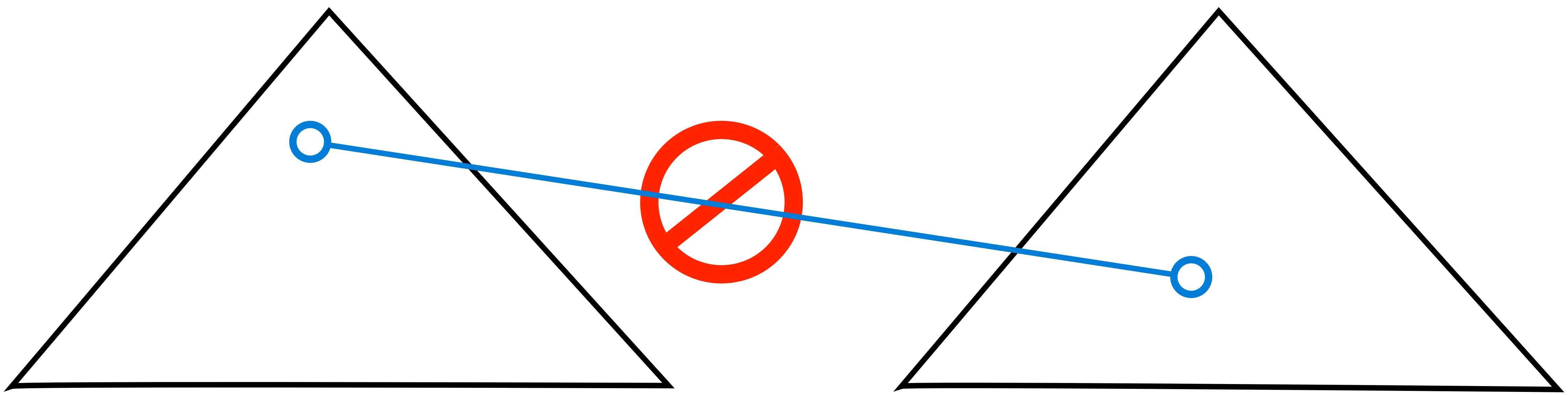

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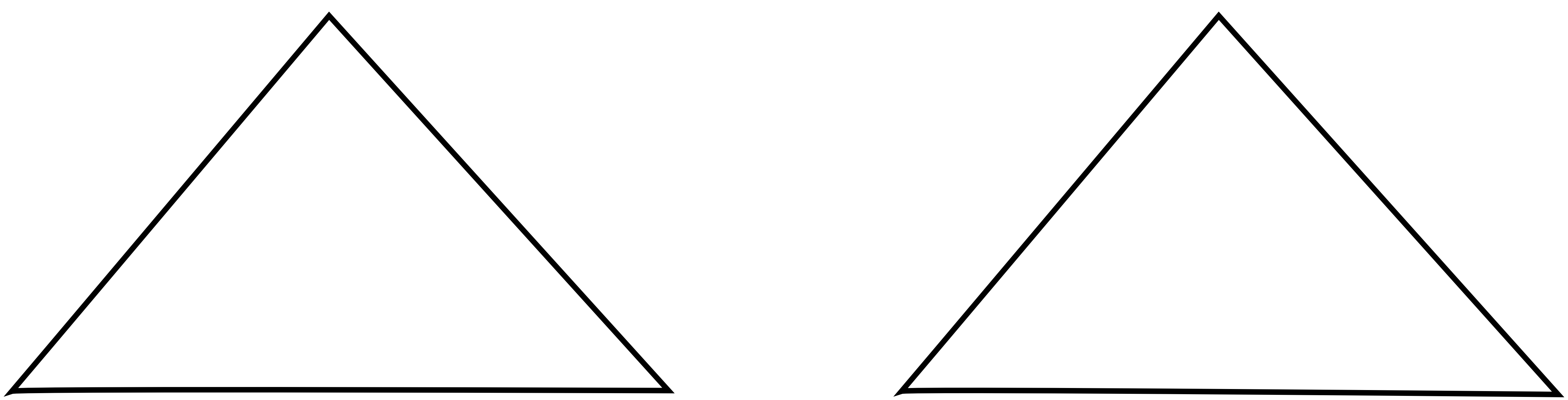

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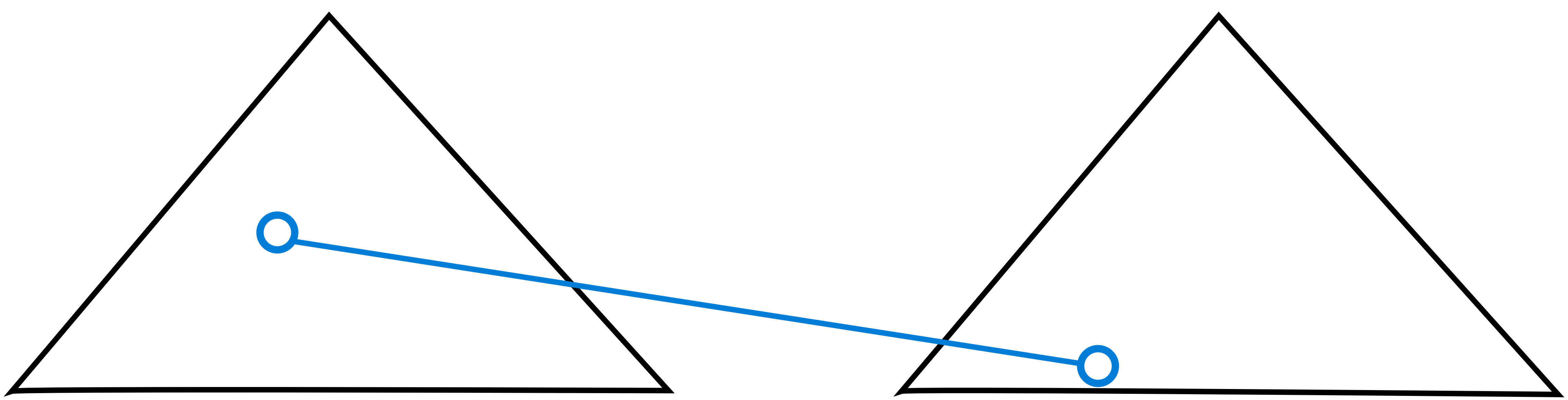

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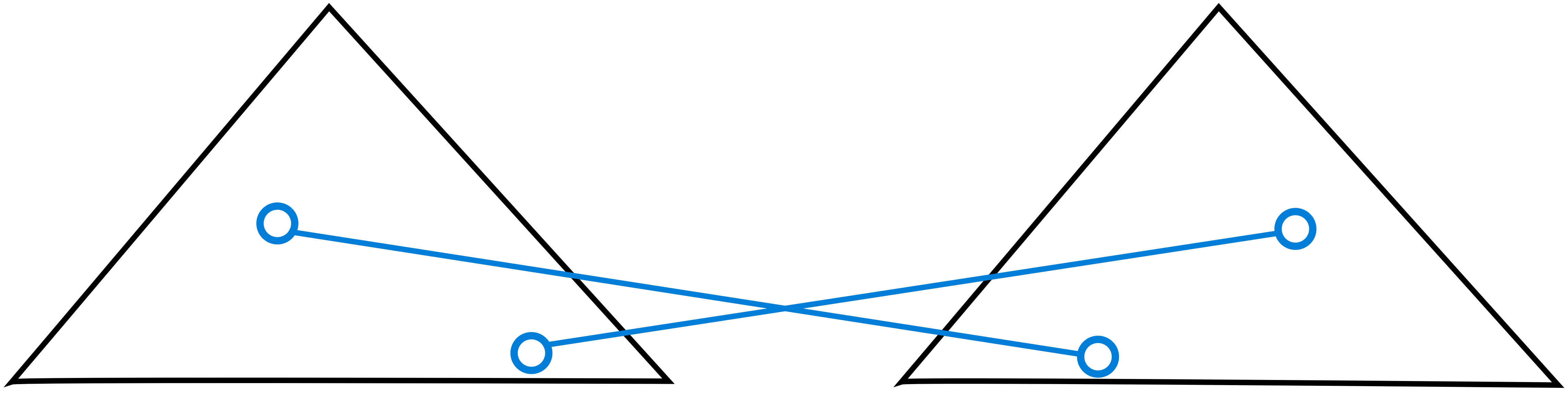

Generalization to posets

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It cannot be approximated to exp ($\log^{1-\varepsilon}$ rho), for any fixed $\varepsilon>0$, unless NP belongs to DTIME($n^{\text{polylog } n}$)

Thank you for your attention!