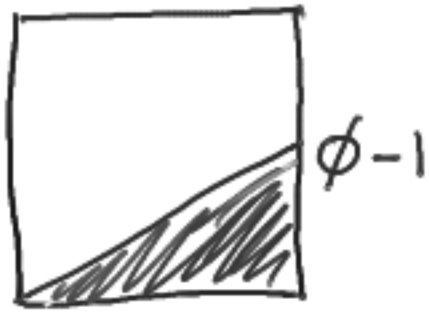
Adaptive Local Ratio



Julián Mestre

Talk Outline

- Background
- Data Migration
- Adaptive Local Ratio

Primal-dual & Local-Ratio

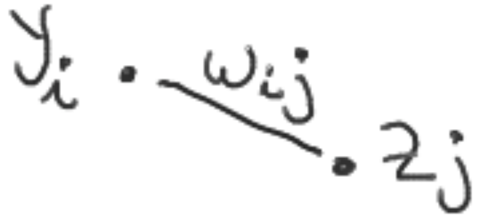
Won't assume any prev. Knowledge

Bad news: we'll use these a lot

Good news: easy to understand!

Minimum Perfect Matching

 $w_{ij} \in \mathbb{R}^+$



$$\min \sum_{(i,j)\in M} w_{ij}$$

s.t.

M is perfect

$$\max \sum_i (y_i - z_i)$$

s.t.

$$y_i - z_j \le w_{ij} \quad \forall i,j$$

$$\sum_{(i,j)\in M} w_{ij} \geq \sum_{(i,j)\in M} (y_i - z_j) = \sum_i (y_i - z_i)$$

Minimum Perfect Matching

$$\min \sum_{i,j} w_{ij} x_{ij}$$

s.t.

$$\sum_i x_{ij} \leq 1$$

$$\sum_j x_{ij} \geq 1$$

$$x_{ij} \in \{0,1\}$$

$$\Longrightarrow$$

$$\max \sum_i (y_i - z_i)$$

s.t.

$$y_i - z_j \leq w_{ij}$$

$$y_i, z_i \geq 0$$

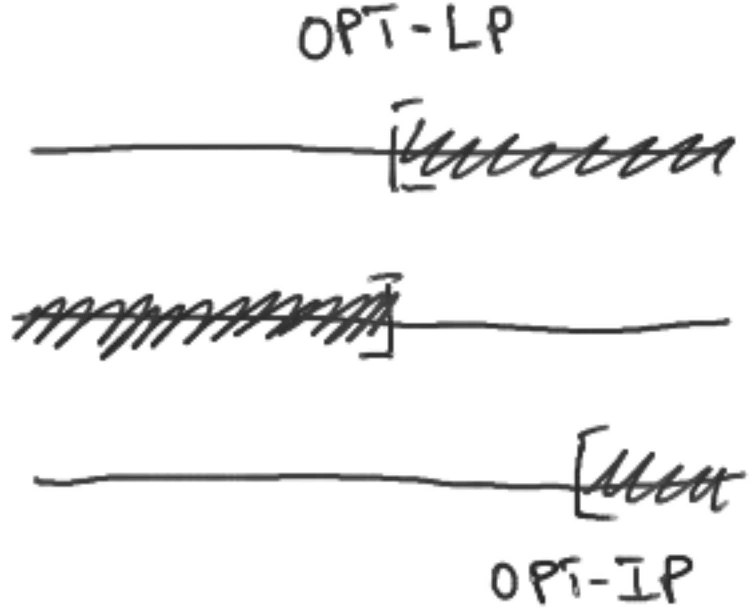
! LP duality !

LP Duality

Primal LP

Dual LP

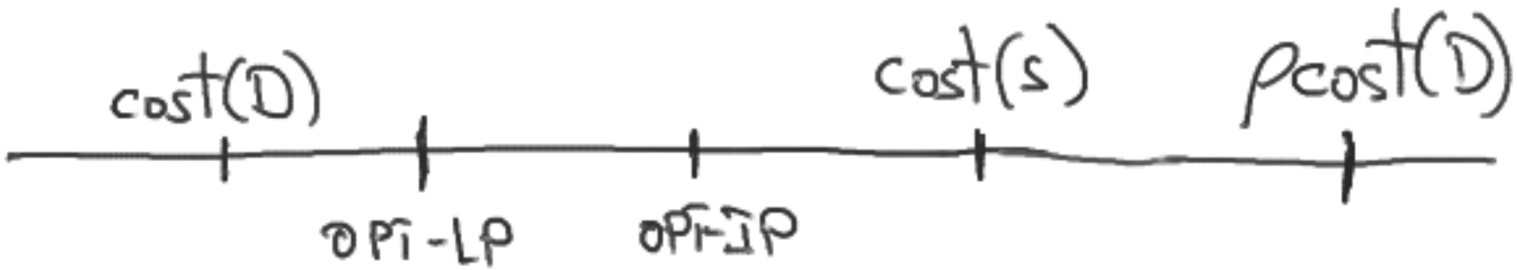
Primal IP



For some lucky problems LP = IP

Primal-dual Algorithm

- Construct a dual solution D
- Construct an integrel solution S
- Argue that $\mathrm{cost}(S) \leq \rho \, \mathrm{cost}(D)$



Local Ratio

Alternative approch, doesn't require LP

- Decompose $w = w_1 + w_2 + \dots + w_n$
- Construct a solution S
- Argue $w_k(S) \leq \rho\, w_k(A) \quad \forall A$

$\Rightarrow S$ is ρ-approximate

Primal-dual & Local-Ratio

- Equivalent paradigms
- Hundreds of papers
- Weight decomp. using "simple" models

Optimizing the LR can pay off!

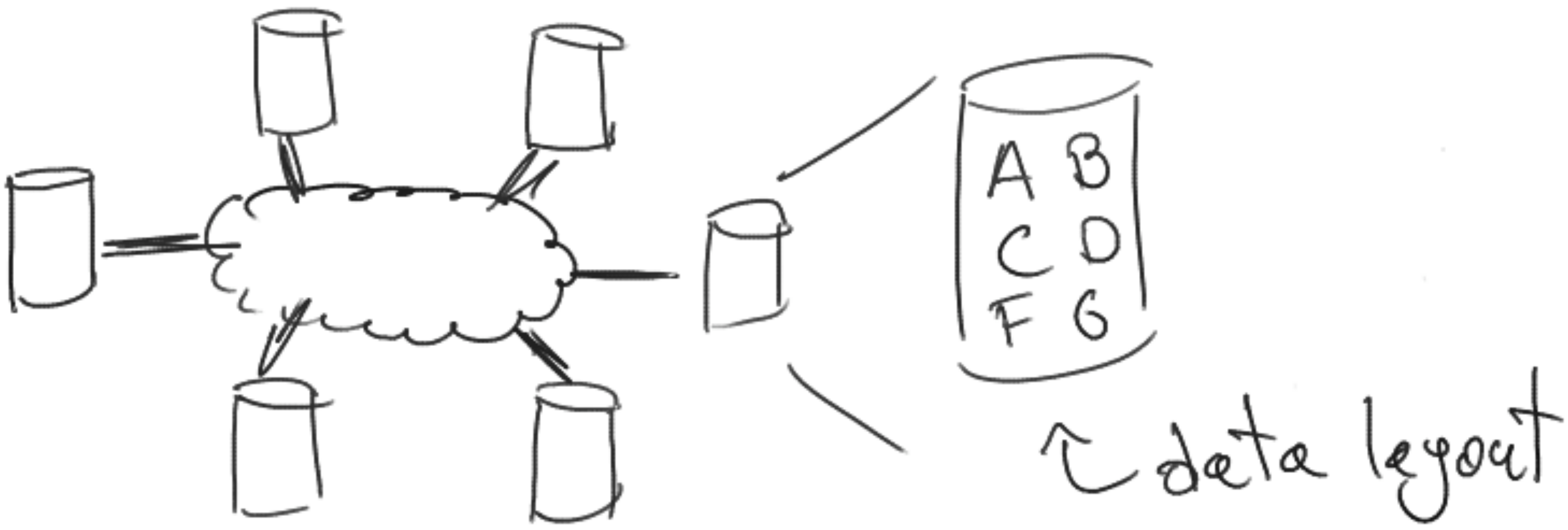
Talk Outline

✓ - Background

- Data Migration

- Adaptive Local Ratio

The Data Migration Problem



- # of requests per item
- disk load

may change over time

Data Migration

Transfer graph $G=(V,E)$ $\begin{cases} V: \text{disks} \\ E: \text{transfers} \end{cases}$

Want to schedule the transfers $\equiv$ Partition E into matchings $M_1, \dots M_k$

Objective : $\min \sum_v \omega_v C_v$

Previous Results

- NP-hard
- 3-approx (LP rounding) [K03]
- shown to be tight [GHKS]
- 3-approx (primal-dual) [GM]

Decompose $\omega = \omega_1 + \omega_2 + \cdots + \omega_n$

[$K=0$] for $v \in V$ do $\ell(v) \leftarrow \text{nil}$

[$K \geqslant 1$] // let $UN(\mu) = \{ v \in N(\mu) \mid \ell(v) = \text{nil} \}$

choose μ maximizing $\Delta = |UN(\mu)|$
and a model $\hat{\omega}$ with support in $UN(\mu)$

$\omega_K \leftarrow \min \left\{ \frac{\omega(v)}{\hat{\omega}(v)} \middle| \hat{\omega}(v) \neq 0 \right\} \hat{\omega}$ // $\omega_K \leq \omega$

$\omega \leftarrow \omega - \omega_K$

for $v \in UN(\mu) \wedge \omega(v) = 0$ do $\ell(v) \leftarrow \Delta$

Construct a solution S

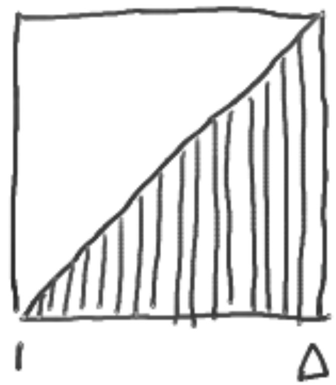
- Sort $(u,v) \in E$ by $\begin{cases} \min(\ell(u), \ell(v)) \\ \max(\ell(u), \ell(v)) \end{cases}$

- Greedily schedule E in order

Lemma [GM]: every $v \in V$ finishes by $\ell(v) + d_v - 1$

Bounding the Local Ratio

$$\hat{\omega}(v) = \begin{cases} 1 & \text{if } v \in UN(\mu) \\ 0 & \text{ow} \end{cases}$$

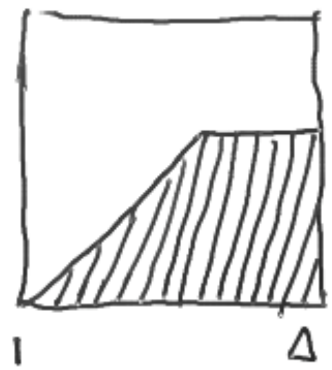


Lemma [GM]: $\hat{\omega}(S) \leq 3\,\hat{\omega}(A) \quad \forall A$

Using two models

$$\hat{\omega}(v) = \mathbb{I}\{v \in UN(\mu)\}, \text{ or}$$

$$\hat{\omega}(v) = \mathbb{I}\{\forall x \in UN(\mu) \cdot d_x \leq d_v\}$$



Lemma: $\hat{\omega}(S) \leq 2.82\, \hat{\omega}(A) \quad \forall A$

Talk Outline

✓ - Background

✓ - Data Migration

- Adaptive Local Ratio

Using the best model

Find $\hat{\omega}$ minimizing $\dfrac{UB(\hat{\omega})}{LB(\hat{\omega})}$

$$UB(\hat{\omega}) = \sum_i \hat{\omega}_i (d_i + \Delta - 1)$$

$$LB(\hat{\omega}) = \min_M \sum_{(i,j) \in M} \hat{\omega}_i \max(d_i, j)$$

Towards an LP formulation

$$\rho(d) = \min_{\substack{\text{s.t.} \\ \hat{\omega}_i \geq 0}} \frac{UB(\hat{\omega})}{LB(\hat{\omega})} = \min UB(\hat{\omega})$$

$$\text{s.t.} \quad LB(\hat{\omega}) \geq 1, \quad \hat{\omega}_i \geq 0$$

$$UB(\hat{\omega}) = \sum_i \hat{\omega}_i (d_i + \Delta - 1)$$

$$LB(\hat{\omega}) = \min_M \sum_{(i,j)\in M} \hat{\omega}_i \max(d_i, j)$$

Towards an LP formulation

$$\rho(d) = \min \sum_i \hat{\omega}_i \left(d_i + \Delta - 1\right)$$

s.t.

$$\sum_{(i,j)\in M} \hat{\omega}_i \max(d_i, j) \geq 1 \quad \forall M$$

$$\hat{\omega}_i \geq 0$$

ELIPSOID METHOD IS NOT PRACTICAL

Succinct LP formulation

$$
\begin{aligned}
\rho(d) = \quad & \min \sum_i \hat{\omega}_i \left(d_i + \Delta - 1\right) \\
\text{s.t.} \quad & \sum (y_i - z_i) \geq 1 \\
& y_i - z_j \leq \hat{\omega}_i \max(d_i, j) \\
& \hat{\omega}_i, y_i, z_i \geq 0
\end{aligned}
$$

Bounding the Local Ratio

Def: $\rho = \sup_{d} \rho(d)$

Thm: S is ρ-approximate and this is tight

Now we only need to bound ρ

Experimental Evaluation

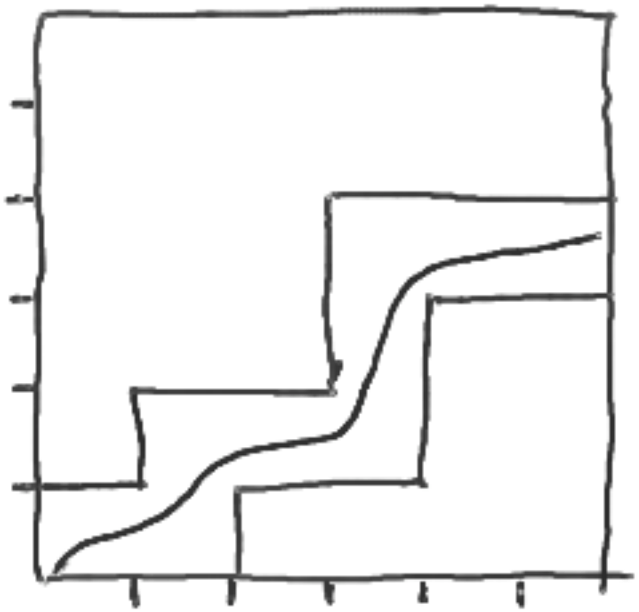
Exhaustive search for small values of Δ

$$\rho_\Delta = \max_{d : |d| = \Delta} \rho(d)$$

Δ	ρ_Δ
1	1
2	1.5
3	1.7273
4	1.9310
5	2.0115
6	2.1042
7	2.1863
8	2.2129
9	2.2589
10	2.2857

Δ	ρ_Δ
20	2.4453
30	2.5006
40	2.5275
50	2.5447
60	2.5556
70	2.5667
80	2.5728
$\vdots$	$\vdots$
∞	?

Experimental Evaluation



Let k be grid size

Error $\to 0$ as $k \to \infty$

$$\#\text{paths} = C_k \sim \frac{4^k}{k^{1.5}}$$

STILL RUNNING AFTER TWO WEEKS!

$$\rho = \sup_{d} \rho(d)$$

$$\rho_\Delta = \max \alpha$$

$$\sum_i x_{ij} \geq \alpha \qquad \forall j$$

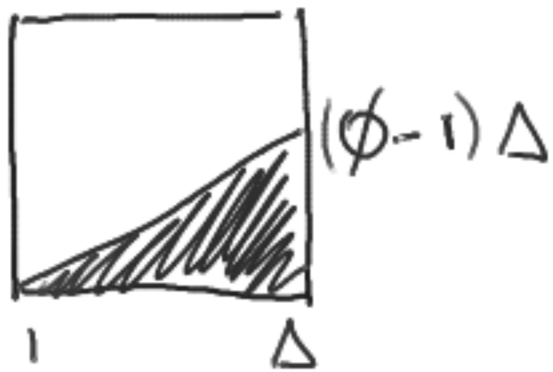
$$\sum_j x_{ij} \leq \alpha \qquad \forall i$$

$$\sum_j x_{ij} \max(d_i, j) \leq d_i + \Delta - 1 \qquad \forall i$$

$$d_i, x_{ij}, \alpha \geq 0$$

Thm: $\rho = 1 + \phi \approx 2.61$

Intuition:



- weights exploit irregularities of d
- if d is "flat", only two models are useful

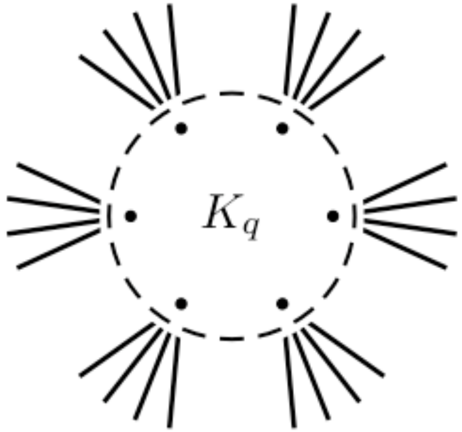
Concluding Remarks

- Generalizations
- First LP-guided local-ratio Alg
- Optimizing the local ratio can make a difference

Thanks for
your attention !

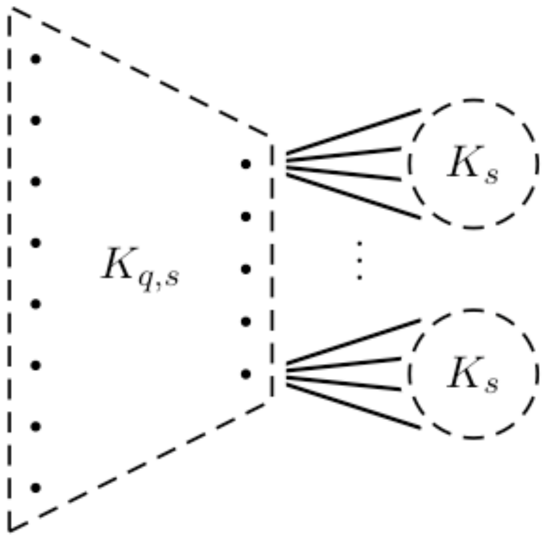
Do you really need $\ell(\cdot)$?

Forget about $\ell(\cdot)$ and just go with greedy



Sort $(u,v) \in E$ by

$$\begin{cases} \min(d_u, d_v) \\ \max(d_u, d_v) \end{cases}$$



Thanks for
your attention!