THE UNIVERSITY OF SYDNEY

PHYS 1001
PHYSICS 1 (REGULAR)

Solutions

JUNE 2009

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 MARKS

INSTRUCTIONS
• All questions are to be answered.
• Use a separate answer book for section A and section B.
• All answers should include explanations in terms of physical principles.

DATA
Density of fresh water \( \rho = 1.000 \times 10^3 \text{ kg.m}^{-3} \)
Free fall acceleration at earth's surface \( g = 9.80 \text{ m.s}^{-2} \)
Gravitational constant \( G = 6.67 \times 10^{-11} \text{ N.m}^2\text{.kg}^{-2} \)
Speed of light in a vacuum \( c = 3.00 \times 10^8 \text{ m.s}^{-1} \)
Speed of sound in air \( v = 344 \text{ m.s}^{-1} \)
Avogadro constant \( N_A = 6.023 \times 10^{23} \text{ mol}^{-1} \)
Universal gas constant \( R = 8.314 \text{ J.mol}^{-1}.\text{K}^{-1} \)
Boltzmann constant \( k = 1.380 \times 10^{-23} \text{ J.K}^{-1} \)
Stefan-Boltzmann constant \( \sigma = 5.67 \times 10^{-8} \text{ W.m}^{-2}.\text{K}^{-4} \)
REG_Q01

A glass dropped onto the floor from a height $h_1$ is more likely to break on a concrete floor than on carpet.

(a) Explain this observation. Illustrate your answer with sketches of force versus time graphs.

A bowl, twice the mass of the glass, is dropped from height $h_2$ but hits the floor with the same momentum as the glass.

(b) What is the velocity of impact of the bowl with the floor compared with that of the glass? Justify your answer.

(c) What is the relationship of $h_2$ (the height from which the bowl was dropped) to $h_1$ (the height from which the glass was dropped)? Explain the reasoning for your answer.

(5 marks)

Solution

(a)
Glass is acted on by gravitational force and accelerates as it drops. At the instant of impact, its momentum, $p_i = mv$. After impact the momentum of the glass is $p_f = 0$. The impulse $J = p_f - p_i$ is therefore the same for both surfaces.

If we approximate the impact as an average force $F_{av}$ acting for a time $\Delta t$, then we have

$$(F_{av} \Delta t)_{concrete} = J = (F_{av} \Delta t)_{carpet}.$$  

The duration of impact is much less for concrete than for carpet and hence the average force for concrete is much larger than the average force for carpet.

$$(F_{av})_{concrete} \gg (F_{av})_{carpet}$$

With a much larger average force the glass is more likely to break when falling onto concrete.

![Force-time graph for concrete](image1)

$J = \text{Area}, \text{ same for both situations}$

(1 mark for diagram; 1 mark for reasonable explanation)
(b) Momentum is the same and hence
\[ p_{\text{glass}} = m v_{\text{glass}} = p_{\text{bowl}} = (2m) v_{\text{bowl}} \]
\[ \Rightarrow v_{\text{bowl}} = 0.5 v_{\text{glass}} \]  
(1 mark)

(c) The potential energy of the glass, when about to be dropped, is equal to its kinetic energy when it is about to impact the floor. Hence
\[ m g h_1 = \frac{1}{2} m v^2 \]
\[ \Rightarrow v = \sqrt{2 g h_1} \]
This means that the momentum of the glass at impact is
\[ p_{\text{glass}} = m v = m \sqrt{2 g h_1} . \]
In a similar manner the momentum of the bowl is given by:
\[ p_{\text{bowl}} = (2m) v = 2m \sqrt{2 g h_2} . \]
If the momentum of the glass and the bowl are the same then
\[ p_{\text{glass}} = p_{\text{bowl}} \]
\[ \Rightarrow 2m \sqrt{2 g h_2} = m \sqrt{2 g h_1} \]
\[ \Rightarrow 4 h_2 = h_1 \]
\[ \Rightarrow h_2 = 0.25 h_1 \]  
(2 marks)
ADV_Q02=REG_Q02

An astronaut constructs an accelerometer for his rocket using two identical springs of unextended length \( L \) and spring constant \( k \), and two identical masses of mass \( m \). He hangs the springs and masses in the rocket as shown below.

Assume the rocket remains within the gravitational field of the Earth and that the acceleration due to gravity is \( g \) at all times.

(a) After launch, the rocket accelerates vertically upwards with an acceleration \( a \). Show that the length of the accelerometer (the total length of the springs, assuming the masses have negligible length) is:

\[
2L + \frac{3m(g+a)}{k}
\]

(b) Eventually the rocket runs low on fuel and the motors are shut down. What is the total length of the accelerometer after shutdown of the motors? Explain your result.

(5 marks)
**Solution**

(a) Use Newton’s second law and take upwards to be positive:

**Hookes law** states that

\[ F = -kx \]

where \( x \) is the extension of the spring beyond its natural length and \( k \) is the spring constant.

For Mass A:

\[ F_1 - F_2 - mg = ma \]

For Mass B:

\[ F_2 - mg = ma \]

\[ \Rightarrow F_2 = m(g + a) \]

Using the expression for \( F_2 \) we get

\[ F_1 = 2m(g + a) \]

Hookes law gives the extensions for each spring:

\[ F_1 = -kx_1 = 2m(g + a) \]

\[ \Rightarrow x_1 = \frac{-2m(g + a)}{k} \]

and

\[ F_2 = -kx_2 = m(g + a) \]

\[ \Rightarrow x_2 = \frac{-m(g + a)}{k} \]

Ignoring the signs which just indicate direction then the total length is given by
\[2L + \frac{m(g+a)}{k} + \frac{2m(g+a)}{k} = 2L + \frac{3m(g+a)}{k}\]  

(b)  
Once the rocket is turned off.  
\[a = -g.\]  
So the extension of the springs is zero and the length of the springs is \(2L\).  

Rocket, astronaut, and weights are now in ‘free fall’, although still going up. No force is required from the spring, so the extension is zero.  

(1 mark)
Two solid spherical balls (A and B) are released at the same time from the top of the inclined ramp. Both have the same mass $M$, but Ball A has twice the radius of Ball B. They both roll down the ramp without slipping. The dimensions of the ramp are much larger than the radius of either ball.

(a) Which ball reaches the bottom of the ramp first? Justify your answer.

*Hint:* Consider the potential, rotational, and translational kinetic energies.

In a second experiment, a solid spherical ball of radius $R$ and a solid cylinder with a circular cross section of radius $R$ are released at the same time from the top of an inclined ramp. Both have mass $M$. The dimensions of the ramp are much larger than the radius $R$ of the ball or cylinder.

Which object reaches the bottom of the ramp first if:

(b) The ramp is frictionless. Justify your answer.

(c) There is a frictional force between each object and the ramp so that they roll without slipping. Justify your answer.

*Moment of Inertia Data*

- Moment of Inertia for Solid Sphere $I = \frac{2}{5} M R^2$
- Moment of Inertia for Solid Cylinder $I = \frac{1}{2} M R^2$
Solution
(a) The potential energy lost as an object comes down the ramp is equal to the sum of the kinetic energy of translation and the kinetic energy of rotation at the bottom. If the vertical height of the ramp is $h$, the final velocity of the object (of mass $M$ and moment of inertia $I$) at the bottom of the ramp is $v$, and the rotational angular velocity of the object is $\omega$ then we have:

$$M g h = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2.$$

If the object rolls without slipping then

$$v = r \omega$$

where $r$ is the radius of the object.

Apply this to:
Object 1: solid sphere of radius $R$ rolling without slipping

$$M g h = \frac{1}{2} M v^2 + \frac{1}{2} \left( \frac{2}{5} M R^2 \right) \left( \frac{v}{R} \right)^2$$

$$\Rightarrow 2 g h = v^2 + \frac{2}{5} v^2 = 1.4 v^2$$

Object 2: solid sphere of radius $2R$ rolling without slipping

Same answer as for Object 1 because the answer does not depend on radius.

Hence the two objects will have the same velocity (at all times going down the ramp as $h$ is arbitrary). They will reach the bottom of the ramp at the same time.

(2 marks)

(b) Apply energy conservation equation to:
Object 1: solid sphere of radius $R$ on ramp with no friction

Object slides so

$$M g h = \frac{1}{2} M v^2 + 0$$

$$\Rightarrow 2 g h = v^2$$

Object 2: solid cylinder of radius $R$ on ramp without friction

Object slides and answer is the same as for Object 1 because the answer does not depend on the moment of inertia of the object.

Hence the two objects will have the same velocity (at all times going down the ramp as $h$ is arbitrary). They will reach the bottom of the ramp at the same time.

(1 mark)

(c) Apply energy conservation equation to:
Object 1: solid sphere of radius $R$ on ramp rolling without slipping
\[ M g h = \frac{1}{2} M v^2 + \frac{1}{2} \left( \frac{2}{5} MR^2 \right) \left( \frac{v}{R} \right)^2 \]

\[ \Rightarrow 2 g h = v^2 + \frac{2}{5} v^2 = 1.4 v^2 \]

\[ \Rightarrow v = \sqrt{1.43 g h} \]

**Object 2: solid cylinder of radius \( R \) on ramp rolling without slipping**

\[ M g h = \frac{1}{2} M v^2 + \frac{1}{2} \left( \frac{1}{2} MR^2 \right) \left( \frac{v}{R} \right)^2 \]

\[ \Rightarrow 2 g h = v^2 + \frac{1}{2} v^2 = 1.5 v^2 \]

\[ \Rightarrow v = \sqrt{1.33 g h} \]

Velocity for the sphere is always higher than that of the cylinder and hence it reaches the bottom first.

(2 marks)

**In each part give half of marks for correct answer and the other half for a suitable justification (which does not have to be mathematical).**
The radiation emitted from a hole in a hot furnace was measured. The intensity of the light emerging from the hole was $I = 586 \text{ W.m}^{-2}$ and the wavelength of peak spectral emittance, $I(\lambda)$ was $\lambda_{\text{peak}} = 3.57 \mu\text{m}$. You can assume that the interior of the furnace emits as a blackbody.

(a) Estimate the temperature inside the furnace.

(b) Draw a set of axes in your examination booklet that are similar to those in the diagram below. On your diagram sketch the blackbody radiation curve for this furnace and identify $\lambda_{\text{peak}}$.

The temperature in K of the furnace was doubled. As a consequence:

(c) Does $\lambda_{\text{peak}}$ decrease, stay the same, or increase?

(d) By what numerical factor does the intensity (the area under the blackbody radiation curve) change?

Solution

(a) Wien’s law:

$\lambda_{\text{peak}} T = 2.90 \times 10^{-3} \text{ m.K}$

with $\lambda_{\text{peak}} = 3.57 \mu\text{m}$

gives

$T = \frac{2.90 \times 10^{-3}}{\lambda_{\text{peak}}} = \frac{2.90 \times 10^{-3}}{3.57 \times 10^{-6}} = 812 \text{ K}.$

Students are asked to ‘estimate’ temperature so $800K$ or $810K$ or $812K$ is acceptable.

(b)
(c) \[ \lambda_{\text{peak}} \, T = 2.90 \times 10^{-3} \, \text{m.K} \].

If \( T \) increases then \( \lambda_{\text{peak}} \) decreases.

(d) Stefan-Boltzmann’s law is
\[ P_{\text{rad}} = e \sigma T^4. \]

Hence if \( T \) is doubled then \( P_{\text{rad}} \) increases by a factor of \( 2^4 = 16 \).
A swimmer is floating in deep ocean water. While she rests, she notices the motion of the ocean surface is well described by a periodic wave moving east to west. [Note: The waves are not breaking and there is no ocean current.] She bobs up and down with a total vertical displacement of 5.0 m between a wave crest and trough. She returns to the crest of a wave every 45 seconds and sees that the crests are uniformly separated by 10 m.

A general expression that describes the vertical displacement $y$ of a wave in terms of time $t$ and horizontal displacement $x$ is

$$y(x, t) = A \cos(kx + \omega t)$$

(a) Briefly describe the physical meaning of the variables $A$, $k$, and $\omega$.

(b) Use the numerical information above to calculate $A$, $k$, and $\omega$, and substitute the values into the wave equation above.

(c) At a certain time, the swimmer finds herself at the crest of a wave. What is the vertical displacement of the wave 90 seconds later and 15 metres west of the swimmer?

Solution

(a) $A$ is the amplitude of the wave. This is the maximum displacement from the equilibrium or a half of the peak to peak modulation.

$k$ is the wave number. This is $2\pi$ times the inverse of the wavelength. The wavelength is the distance between successive peaks of the wave at the same instant of time.

$\omega$ is the angular frequency of the wave. This is $2\pi$ times the frequency of the wave. It is also $2\pi$ times the inverse of the period of the wave. The period is the time interval between successive peaks of the wave at the same location.

(b) $A = \frac{5}{2} = 2.5\, \text{m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{10} = 0.63\, \text{m}^{-1}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{45} = 0.14\, \text{s}^{-1}$$

(c) A time of 90 s is exactly two periods of the wave. So at that time, the swimmer is again at a crest of the wave.

A distance of 15 m is exactly 1.5 times the wavelength. So the swimmer is at a trough of the wave. The vertical displacement is therefore given by: $y = -A = -2.5\, \text{m}$. 

(2 marks)
The diagram above shows a wire with its left end attached to a pin that is fixed to a table. The wire extends to the right over a frictionless pulley and is attached to a hanging weight of mass 2.0 kg. The linear density of the wire is $2.0 \times 10^{-2}$ kg.m$^{-1}$. The wire is made to vibrate using a magnet and electric current produced by a frequency generator (not shown in diagram and the details are not important). The wire is found to vibrate in the fundamental mode with a frequency of 40 Hz.

(a) Draw a sketch of the amplitude of the vibration along the wire.

The frequency of the generator is increased until the next highest frequency mode of vibration is observed.

(b) Draw a sketch of the amplitude of the vibration along the wire.

(c) What is the frequency of the generator for this vibration?

(d) We wish to change this system so that there is a vibration in the fundamental mode at 80 Hz. What value is required for the mass of the hanging weight if we only change this parameter?

(5 marks)

Solution

(a)

Pin  Pulley

(1 mark)
(b) 

(c) Frequency has been doubled so therefore \( f = 80\text{Hz} \).

(d) For a vibrating wire the frequency of oscillation depends on the factor \( \sqrt{\frac{T}{\mu}} \) where \( T \) is the tension in the wire and \( \mu \) is the mass per unit length. Tension in the wire is given by \( T = mg \).

Hence frequency depends on \( \sqrt{m} \) if \( \mu \) is left unchanged.

There is a vibration in the fundamental at 40Hz when a mass of 2.0kg is used. To double this frequency to 80Hz we need to increase the mass by a factor of 4. Hence a mass of 8.0kg is required.
At the ice skating rink, a small girl is skating along at a speed of $1.0 \text{ m.s}^{-1}$ when she crashes into her father who is also out on the ice skating rink. Her father, who was standing still at the time, grabs the girl and holds onto her. The girl has a mass of 20 kg and the father has a mass of 80 kg. Assume that there is negligible friction between their skates and the ice.

(a) Was this an elastic or inelastic collision? Justify your answer.

(b) What was the recoil speed of the father and daughter clinging together after the collision?

(c) What fraction of the girl’s initial kinetic energy was transformed into kinetic energy of the girl plus her father, clinging to each other after the collision?

(d) What happened to the rest of the energy?  

(10 marks)

Solution

(a) The collision was inelastic. Momentum is conserved in any collision in which there no external force acting. Energy is not conserved in a collision in which the objects stick together.  

(1½ mark)

Take the positive direction as to the right in the diagram in the following

(b) \[ p_{\text{girl}} = m_{\text{girl}} v_{\text{girl}} = (20)(1.0) = 20 \text{ kg.m.s}^{-1} \]

\[ p_{\text{father}} = (80)(0) = 0 \]

\[ p_{\text{before}} = p_{\text{girl}} + p_{\text{father}} = 20 + 0 = 20 \text{ kg.m.s}^{-1} \]

Moments after collision
\[ p_{\text{after}} = (m_{\text{girl}} + m_{\text{father}})v_{\text{recoil}} = 100v_{\text{recoil}} \]

Momentum is conserved so \( p_{\text{after}} = p_{\text{before}} \).

Hence
\[
100v_{\text{recoil}} = 20
\]
\[ \Rightarrow v_{\text{recoil}} = 0.20 \text{ m.s}^{-1} \]

(1 mark for correct answer; 1 mark for momentum before collision; 1 mark for momentum after collision; 1 mark for conservation of momentum)

(c)

**Kinetic Energy of girl before collision.**
\[ K_{\text{girl}} = \frac{1}{2} m_{\text{girl}} v_{\text{girl}}^2 = (0.5)(20)(1.0)^2 = 10 \text{ J} . \]

**Kinetic Energy of father before collision**
\[ K_{\text{father}} = \frac{1}{2} m_{\text{father}} v_{\text{father}}^2 = (0.5)(80)(0)^2 = 0 \text{ J} \]

**Total Kinetic Energy before collision**
\[ K_{\text{before}} = K_{\text{girl}} + K_{\text{father}} = 10 + 0 = 10 \text{ J} . \]

**Kinetic Energy after collision**
\[ K_{\text{after}} = \frac{1}{2} (m_{\text{girl}} + m_{\text{father}})v_{\text{recoil}}^2 = (0.5)(100)(0.2)^2 = 2.0 \text{ J} . \]

Hence 20% of the girl’s initial Kinetic Energy is transformed into Kinetic Energy of the girl+father combination after the collision.

(1 mark for correct answer; 1 mark for KE before collision; 1 mark for KE after collision)

(d)

The remaining energy is lost as heat (and perhaps sound) in the collision.

(1½ mark)
REG_Q08

A small rectangular block has a square cross-section (side length $a$) and a height $h$. The block has a mass $m$ and sits on a turntable with its centre at a distance $r$ from the axis of rotation of the turntable. The surface of the turntable has a coefficient of static friction $\mu_s$, and the turntable is spinning with angular velocity $\omega$.

(a) Copy the side view of the arrangement (in the diagram above) into your answer book. Draw and identify all the forces acting on the block, carefully giving their points of application and direction.

(b) Again copy the side view of the arrangement (in the diagram above) into your answer book. Draw and identify all the horizontal forces acting on the turntable, carefully giving their points of application and direction.

(c) Identify the forces that are Newton’s Third Law action/reaction pairs in the two diagrams.

(d) Show that the angular velocity at which the block begins to slide on the turntable is given by:

$$\omega = \sqrt{\frac{\mu_s g}{r}}$$

(10 marks)

Solution

(a)

Forces Acting on Block
(b) **Horizontal Forces on Turntable**

(c) Newton action/reaction pairs are equal in magnitude and opposite in direction. They act on different bodies. The pairs in this situation are:
- (weight force of block acting on the turntable; reaction force from turntable acting on block)
- (friction force from turntable acting on block; frictional force acting on turntable from the block)

(d) Weight force of block is given by
\[ w = -mg \]
Normal reaction force from turntable acting on the block is
\[ N = mg \]
Maximum frictional force on the block is
\[ F_{\text{fric}} = \mu_s N = \mu_s mg \]
Centripetal force acting on the block
\[ F_c = mr \omega^2 \]
When the block just begins to slide when the frictional and the centripetal force are equal and hence
\[ F_c = F_{\text{fric}} \]
\[ \Rightarrow mr \omega^2 = \mu_s mg \]
This means that
\[ \omega^2 = \frac{\mu_s g}{r} \]
\[ \Rightarrow \omega = \sqrt{\frac{\mu_s g}{r}} \]
REG_Q09

(a)  (i) State the meaning of Avogadro’s constant.

(ii) If the mass of a water molecule is $3.0 \times 10^{-26}$ kg then what is the mass of 2.0 moles of water molecules?

(b)  (i) What are the two most important differences at a microscopic level between the liquid and solid states.

(ii) What does the latent heat of fusion correspond to on the microscopic level?

An open cubic glass jar (see diagram below) is filled with 1.0 kg of water at a temperature of 300 K. The jar is placed in an oven at a temperature of 400 K. The area of each glass face is $1.0 \times 10^{-2}$ m$^2$ and the glass thickness is $2.0 \times 10^{-3}$ m.

(c) Assuming that the outer surface of the glass is at 400 K and that the jar is completely filled with water, what is the initial rate of heat flow into the water through the glass sides and the base? Ignore convection, radiation, and evaporation.

(d) By what factor would the water’s volume have increased (ignore that it would overflow the jar) when the water has reached a temperature of 350 K?

(e) What heat flow is required to the water for it all from its original temperature to become steam at 373 K?

You may need the following data.

- Specific heat of water $c = 4190 \text{ J.kg}^{-1}\text{.K}^{-1}$
- Latent heat of fusion for water $L_f = 3.3 \times 10^5 \text{ J.kg}^{-1}$
- Latent heat of vaporisation for water $L_v = 2.3 \times 10^6 \text{ J.kg}^{-1}$
- Thermal conductivity of glass $k = 1.1 \text{ W.m}^{-1}\text{.K}^{-1}$
- Linear expansion factor for water $\alpha = 1.0 \times 10^{-4} \text{ K}^{-1}$
- Density of water $\rho = 1.00 \times 10^3 \text{ kg.m}^{-3}$

(10 marks)
Solution

(a) 
(i) Avogadro’s constant is the number of particles in a mole (or molar mass) of the particles. 

\( (1 \text{ mark}) \)

(ii) Number of particles in two moles of water is \( 2N_A \)

Mass of particles in two moles of water is 
\[
2mN_A = (2)(3.0 \times 10^{-26})(6.023 \times 10^{23}) = 0.036 \text{ kg} 
\]

\( (1 \text{ mark}) \)

(b) 
(i) The liquid is disordered while the solid is strongly ordered. Molecules in a liquid are free to move around while the molecules in a solid oscillate around (slowly varying) fixed positions. 

\( (1 \text{ mark}) \)

(ii) The latent heat of fusion is the energy required to give molecules sufficient kinetic energy to break the inter-molecular bonds and attractive forces that keep molecules in relatively fixed locations in the solid state so that the molecules can move around and enter a liquid state. 

\( (1 \text{ mark}) \)

(c) 
Equation for heat flow

\[
\frac{dQ}{dt} = -k A(T_h - T_c) \frac{dQ}{dt} = -k A(T_h - T_c) \frac{dQ}{dt} = -k A(T_h - T_c) \frac{dQ}{dt} = -k A(T_h - T_c) 
\]

where

\[
\begin{array}{ccc}
T_c &=& 300 \text{ K} \\
T_h &=& 400 \text{ K} \\
k &=& 1.1 \text{ W.m}^{-1}.\text{K}^{-1} \\
A &=& (5)(1.0 \times 10^{-2}) = 5.0 \times 10^{-2} \text{ m}^2 \\
L &=& 2.0 \times 10^{-3} \text{ m} \\
\end{array}
\]

Substituting these values we get
\[
\frac{dQ}{dt} = 2750 \text{ W} \sim 2800 \text{ W} \text{ (2 sig figs).} 
\]

\( (2 \text{ marks}) \)

(d) 
Linear expansion of an object for a temperature change \( \Delta T \) is given by

\[
L(\Delta T) = L_0 \left( 1 + \alpha \Delta T \right) 
\]

Volume expansion of an object is given by

\[
V(\Delta T) = V_0 \left( 1 + 3\alpha \Delta T \right) 
\]

Hence the fractional change of volume is given by

\[
\frac{\Delta V}{V_0} = 3\alpha \Delta T 
\]

For \( \Delta T = 50 \text{ K} \) and \( \alpha = 1.0 \times 10^{-4} \text{ K}^{-1} \) we get
\[ \frac{\Delta V}{V_0} = (3)(1.0 \times 10^{-4})(50) = 0.015 = 1.5\% \] (2 marks)

(e)

\[ Q = mc \Delta T + m L_v. \]

where \( m = 1.0 \text{ kg} \), \( \Delta T = 373 - 300 = 73 \text{K} \)

and \( c = 4190 \text{ J kg}^{-1}\text{K}^{-1} \) and \( L_v = 2.3 \times 10^6 \text{ J kg}^{-1} \).

Hence

\[ Q = (1.0)(4190)(73) + (1.0)(2.3 \times 10^6) \]
\[ = 3.06 \times 10^5 + 2.3 \times 10^6 \]
\[ = 2.6 \times 10^6 \text{ J} \] (2 marks)
In the figure above, cylinder A is separated from cylinder B by an adiabatic piston which is freely movable. In cylinder A there is 0.010 mole of an ideal monatomic gas with an initial temperature 300 K and a volume of $1.0 \times 10^{-4}$ m$^3$. In cylinder B, there is 0.010 mole of the same ideal monatomic gas with the same initial temperature and the same volume as the gas in cylinder A.

Suppose that heat is allowed to flow slowly to the gas in cylinder A, and that the gas in cylinder B undergoes the thermodynamic process of adiabatic compression. Heat flows into cylinder A until finally the gas in cylinder B is compressed to a volume of $5.0 \times 10^{-5}$ m$^3$. Assume that $C_v = 12.47$ J.mol$^{-1}$.K$^{-1}$ and the ratio of heat capacities is $\gamma = 1.67$.

(a) Calculate the final temperature and pressure of the ideal monatomic gas in cylinder B after the adiabatic compression.

(b) How much work does the gas in cylinder B do during the compression? Explain the meaning of the sign of the work.

(c) What is the final temperature of the gas in the cylinder A? *(Hint: the pressure exerted by the gas in cylinder A on the piston is equal to that exerted by the gas in cylinder B on the piston.)*

(d) How much heat flows to the gas in the cylinder A during the process?

(10 marks)
Solution

(a) For the adiabatic compression:

Final Temperature

\[ T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \]

so

\[ T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}. \]

This is evaluated as

\[ = (300 \text{ K}) \left( \frac{1.0 \times 10^{-4} \text{ m}^3}{5.0 \times 10^{-5} \text{ m}^3} \right)^{0.67} = 477 \text{ K (480 K)}. \]

Final Pressure (One approach)

\[ p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \]

\[ \Rightarrow p_2 = \frac{p_1 \left( \frac{V_1}{V_2} \right)^{\gamma}}{\gamma}. \]

To get the initial pressure of the gas in cylinder B, use the ideal-gas equation:

\[ p_1 V_1 = nRT_1 \]

\[ p_1 = \frac{nRT_1}{V_1} = \frac{(0.010 \text{ mol}) (8.314 \text{ J.mol}^{-1}.\text{K}^{-1}) (300 \text{ K})}{1.0 \times 10^{-4} \text{ m}^3} \]

\[ = 2.5 \times 10^5 \text{ Pa} \]

\[ p_2 = \frac{p_1 \left( \frac{V_1}{V_2} \right)^{\gamma}}{\gamma} = 2^{1.67} (2.49 \times 10^5 \text{ Pa}) \]

\[ = 7.9 \times 10^5 \text{ Pa} \]

***Another approach****

\[ p_2 V_2 = nRT_2 \]

\[ p_2 = \frac{nRT_2}{V_2} = \frac{(0.010 \text{ mol})(8.314 \text{ J.mol}^{-1}.\text{K}^{-1})(477)}{(5.0 \times 10^{-3} \text{ m}^3)} \]

\[ = 7.9 \times 10^5 \text{ Pa} \]

(Correct formulae, 2 marks; correct final numerical results, 1 mark)

(b) For the adiabatic process:

\[ \Delta U = -W \]

\[ W = -\Delta U = -nC_v \Delta T \]

\[ W = -(0.010 \text{ mol})(12.47 \text{ J.mol}^{-1}.\text{K}^{-1})(477 \text{ K}-300 \text{ K}) = -22.1 \text{ J (22 J)}. \]

(1 mark)

The negative sign of the work means that the work is done on the gas in cylinder B by the ideal gas in cylinder A via the piston.
(c) For the ideal gas in the cylinder A according to the ideal-gas equation:
\[ p_{A2}V_{A2} = nRT_{A2} \]
we get:
\[ T_{A2} = \frac{p_{A2}V_{A2}}{nR} \].
\[ p_{A2} = p_2 = 7.9 \times 10^5 \text{ Pa} \]
\[ V_{A2} = 1.50 \times 10^{-4} \text{ m}^3 \]
\[ T_{A2} = \frac{(7.9 \times 10^5 \text{ Pa} ) (1.5 \times 10^{-4} \text{ m}^3 )}{(0.010 \text{ mol})(8.314 \text{ J.mol}^{-1} \text{K}^{-1})} = 1.432 \times 10^3 \text{ K} \]
\[ = 1.4 \times 10^3 \text{ K} \text{ (2 sig figs)} \]

(Correct formula, 1 mark; correct numerical values, 1 mark)

(d) According to the first law of thermodynamics, for the ideal gas in cylinder A:
\[ \Delta U_A = Q - W_A \]
and
\[ \Delta U_A = nC_v \Delta T \]
\[ \Delta U_A = (0.010 \text{ mol}) (12.47 \text{ J.mol}^{-1} \text{K}^{-1}) (1430 \text{ K}-300 \text{ K}) = 141.2 \text{ J} \text{ (140 J)} \]
The magnitude of the work done by the gas in cylinder A to the piston is equal to the magnitude of the work done by the piston on the gas in cylinder B, so
\[ W_A = 22.1 \text{ J} \]
\[ Q = W_A + \Delta U_A \]
\[ Q = 22.1 \text{ J} + 141.2 \text{ J} = 163.3 \text{ J} \text{ (160 J)} \]

(Correct formula and correct underlying reasoning for the work, 2 marks; correct numerical results, 1 mark)
During a physics lecture, you look up at the 15 m high ceiling to watch a giant steel ball on a vertical spring of negligible mass exhibiting simple harmonic motion. At maximum compression the ball is 5.0 m from the ceiling. At maximum extension the ball is 5.0 m from the floor. The ball has a mass of 50 kg, and you measure an oscillation period of 5.0 seconds.

(a) What is the amplitude of the ball’s vertical harmonic motion?

(b) What is the spring constant of the spring?

(c) What is the maximum acceleration experienced by the ball?

(d) Carefully sketch and label the axes of a graph showing the ball’s vertical displacement from equilibrium as a function of time. Assume that the ball is closest to the ceiling at a time \( t = 0 \) and show one full oscillation.

   (i) On your graph mark the location(s) where the ball reaches its maximum acceleration.

   (ii) Is the acceleration of the ball ever zero? If so, mark the location(s) on your graph.

Solution

(a)
Subtracting the height of the ceiling from the total distance from the ceiling and floor (15 m – 10 m) yields a total vertical displacement of 5 m. Amplitude is half of this value, 2.5 m.

   (1 mark)

(b)
Value of spring constant:
\[ \omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \]

\[ \Rightarrow k = \frac{4\pi^2 m}{T^2} \]

Substituting values we get
\[ k = \frac{4\pi^2 (50)}{5.0^2} = 78.96 = 79 \text{ N.m}^{-1} \]

(c)
Maximum acceleration:
\[ a_{\text{max}} = \frac{F_{\text{max}}}{m} = \frac{k x_{\text{max}}}{m} = \frac{(78.96)(2.5)}{50} = 3.948 = 3.9 \text{ m.s}^{-2} \]

Also equal to
\[ a_{\text{max}} = \omega^2 x_{\text{max}} \]

(d)
Question does not specify “magnitude” of acceleration so do not penalise if negative value of max acceleration is not included.

![](image.png)

(sensible graph with labels, 2 marks)

(location of max acceleration points, 1½ marks)
(location of zero acceleration point, 1½ marks)
Harry stands near a bird’s nest. The bird tries to scare him away by flying in circles around his head (see above diagram) while emitting a constant sound wave with a frequency of 1000 Hz. The bird keeps a constant distance of 1.0 m and takes 1.0 seconds to complete a full circle around Harry’s head. Assume the speed of sound in air is 344 m s\(^{-1}\).

(a) Calculate the speed of the bird.

(b) Will the frequency of the sound heard by Harry change as the bird flies around him? Briefly explain your answer qualitatively using physical terms.

Sally at some large distance away (see above diagram) also hears the bird.

(c) Explain why the frequency of the sound heard by Sally changes as the bird flies around Harry’s head.

(d) At which position(s) along the circle (A, B, C, D) is the bird when Sally hears
   
   (i) the highest frequency?
   
   (ii) the lowest frequency?

(e) Calculate the highest frequency heard by Sally.

   (10 marks)
Solution

(a) Circumference of the circle flown by the bird is
\[ C = 2\pi r = 2\pi (1.0) = 6.3\text{m.} \]
This is flown in 1.0 s and so the speed is
\[ 6.3\text{m.s}^{-1}. \]

(b) The component of the velocity of the bird (source) relative to Harry (listener) is zero so there is no doppler shift at any point of the circle and so no change in the frequency of the sound is heard.

(c) The velocity of the bird (source) relative to Sally (listener) is non zero. So the frequency is doppler-shifted. Since the bird’s velocity is constantly changing relative to Sally the doppler shift is also constantly changing.

(d) Sally will hear the highest frequency when the bird is approaching her has the largest velocity. This is at point B.

(ii) Sally will hear the lowest frequency when the bird is going away from her with the largest velocity. This is at point D.

(e)
\[ f_o = f_s \frac{v \pm v_s}{v \pm v_o} \]
Here
\[ v = 344 \text{ m.s}^{-1} \]
\[ v_s = 6.3 \text{ m.s}^{-1} \]
\[ v_o = 0 \text{ m.s}^{-1} \]
\[ f_s = 1000 \text{ Hz} \]
Hence
\[ f_o = (1000) \frac{344 + 0}{344 - 6.3} = 1019 \text{ Hz} = 1020 \text{ Hz (3 sig figures)}. \]