INSTRUCTIONS

• All questions are to be answered.
• Use a separate answer book for section A and section B.
• All answers should include explanations in terms of physical principles.

DATA

Density of fresh water at 20 °C and 1 atm \( \rho = 1.000 \times 10^3 \text{ kg.m}^{-3} \)
Density of sea water at 20 °C and 1 atm \( \rho = 1.024 \times 10^3 \text{ kg.m}^{-3} \)
Atmospheric air pressure \( P = 1.013 \times 10^5 \text{ Pa} \)
Free fall acceleration at earth's surface \( g = 9.80 \text{ m.s}^{-2} \)
Speed of light in vacuum \( c = 3.00 \times 10^8 \text{ m.s}^{-1} \)
A large beaker full of water is placed on weighing scales and the weight of beaker plus water is noted.

(a) A rock suspended from a cord is then lowered into the water, and held fully immersed but not touching the bottom. (The water which overflows is drained on to the floor away from the weighing scales.) Explain briefly, why the reading on the weighing scales does not change when the rock is lowered into the water.

(b) The rock is now lowered so that it rests on the bottom of the beaker. Will the reading on the weighing scales increase, decrease, or stay the same? Explain your answer.

(5 marks)
Solution

(a) The downward force on the weighing scales due to the water is transmitted to the scales by the pressure at the bottom of the beaker, acting over the cross-sectional area \( A \).

The pressure is:

\[
p = p_0 + \rho g d
\]

We can ignore \( p_0 \) because the scales are immersed in air. The value of \( p \) is independent of whether there is a rock suspended in the water. Neither \( \rho \) nor \( d \) changes when the rock is present. The value of \( d \) is the same because the water level is the same – the excess has overflowed. The rock is not touching the beaker so there are no contact forces.

Alternate Solution Using Forces

Forces acting upon the rock are as follows:

\[
\begin{align*}
T &\rightarrow \\
F_b &\uparrow \\
F_g &\downarrow
\end{align*}
\]

where

\[
F_g = m_{\text{rock}} g = \rho g V_{\text{rock}} \\
F_b = \rho_{\text{water}} g V_{\text{rock}} \quad \text{(by Archimedes’ Principle)}
\]

and

\( T \) is the tension in the string.

There is no net force on the rock and so
\[ T = F_g - F_b \]

It is the buoyancy force which is transmitted through the water and for which there is a reaction force (and reading) from the scales. The tension in the string provides the difference between the gravity force on the rock and the buoyancy force.

Note that the buoyancy force is just the weight of the displaced water and is exactly that of an equivalent volume of water. So the presence of the rock makes no difference to the force on the scales. This only occurs when the displaced water is lost to the weighing scales.

(2 marks)

(b)

The rock is denser than water, so its buoyancy force is less than its weight. It is kept in equilibrium by an upward contact force from the beaker. In turn it exerts a downward contact force on the beaker.

This is in addition to the force due to the pressure of the water, which is still present.

Hence the reading on the scale increases.

(1 mark for correct answer, 2 marks for justification)
A cabbage of mass 1.0 kg is attached to a weighing scale and is fired as a projectile as shown in the diagram above. Answer the following questions about the reading on the weighing scale at various stages of the motion of the projectile composed of the cabbage and weighing scale.

What is the reading on the weighing scale:
(a) as the projectile is rising;
(b) at the top of the motion of the projectile;
(c) as the projectile is falling?

In each case, briefly explain the reasons for the answers that you have given.

(5 marks)
Solution
(a)  
Reading on scale is zero  
(1 mark)

(b)  
Reading on scale is zero  
(1 mark)

(c)  
Reading on scale is zero  
(1 mark)

- the cabbage and the scale are both under free fall
- the cabbage and scale are really falling with $g = 9.8 \text{ m.s}^{-2}$ throughout the motion
- relative to one another they have the same acceleration

so they do not exert a contact force or tension or pull force on one another

(2 marks if a coherent explanation based around the above)
Christopher and Jayne were sitting on laboratory chairs, facing each other, initially at rest on a horizontal floor. Christopher had a larger mass than Jayne. They pushed against each other. (Assume that the chairs can move across the floor without friction)

(a) While they were pushing each other, who experienced the greater magnitude of pushing force? Briefly explain your answer.

(b) At some time \( t \) after they finished pushing each other, who will have travelled further from their initial position? Briefly explain your answer.

(c) After they finished pushing each other, whose momentum was larger in magnitude? Briefly explain your answer.

(5 marks)
Solution

(a) The magnitudes of the pushing forces they experience are the same because Newton's $3^{\text{rd}}$ law says that "for every action there is an equal and opposite reaction". As they are pushing against each other their "pushing" forces are an "action-reaction pair".  

(1 mark)

(b) The magnitudes of their forces were equal, however $F = ma$ and $m$ is larger for Christopher so his acceleration must be smaller. Therefore, he reaches a smaller velocity after time $t$ than does Jayne and travels a smaller distance.

Because momentum is conserved, their final momentum must be equal but opposite (see part c). However, $m$ is larger for Christopher so his velocity must be smaller.  

(1 mark)

His final velocity must be smaller than Jayne's. Hence in a given amount of time, Jayne will travel further as she has a larger velocity.  

(1 mark)

(c) Treating Christopher and Jayne as a "system", the pushing forces between them are internal forces therefore there are no net external forces on them and so momentum is conserved.  

(1 mark)

Initial total momentum = zero = final momentum. Therefore their final momenta must be equal in magnitude and opposite in direction.  

(1 mark)

Note: 1 mark for each point mentioned or implied
FND_Q04

Question 4

(a) A beetle sits near the edge of a stationary horizontal turntable. Draw a free-body diagram showing the forces acting on the beetle. What is the net force?

(b) Later the turntable is spinning at a constant angular velocity around a vertical axis. Draw the free-body diagram for the forces acting on the beetle. Make sure that the direction towards the centre of the turntable is clear.

(c) A centripetal force is acting on the beetle. What is providing this centripetal force?

(d) The turntable begins to spin faster. Which direction should the beetle move to minimise the chance of slipping? Briefly explain your answer with reference to appropriate physical principles.

(5 marks)
Solution

(a) The free-body diagram for the stationary beetle is

\[ N \]
\[ \quad \vdots \quad W \]

\( N \) is the normal force, \( W \) is the weight force. The net force on the beetle is zero (1 mark)

(b) Now the turntable is spinning, the free-body diagram is

\[ N \]
\[ \quad \vdots \quad f \quad \text{centre of turntable} \]
\[ \quad \vdots \quad W \]

(1 mark)

(c) The centripetal force is provided by the friction \( f \) between the beetle’s feet and the turntable. (1 mark)

(d) The centripetal force on the beetle while moving in circular motion is given by:

\[ F_c = m r \omega^2 \]

If the turntable is spinning faster, a greater force between the beetle and the turntable is needed to provide the centripetal force and prevent the beetle from sliding outwards.
Friction provides this force; once the force required is greater than the maximum force that can be provided by static friction, the beetle will slip.

As $\omega$ increases, $F_c$ increases and so to keep $F_c$ less than the maximum frictional force, the beetle should decrease $r$, i.e. by moving towards the centre of the turntable.

(1 mark for direction, 1 mark for reasoning)
The picture above shows a metal plate after it has been sprinkled with sand grains and forced to vibrate.

(a) Explain why the sand grains are present in some regions but not in others.
(b) Explain how to obtain a different pattern of lines by vibrating the same metal plate.

(5 marks)
Solution

(a) The plate has a set of natural modes of vibration. The particular modes in which the plate vibrates depends on the method by which the plate is excited.

When the plate is forced to vibrate, standing waves are set up due to waves travelling throughout the plate. At certain locations, called antinodes, the waves interfere constructively resulting in the largest movement of the plate. At other locations, called nodes, the standing waves interfere destructively resulting in nodal lines where the plate does not move. Sand is removed from locations in which there is movement of the plate but remains on the nodal lines at which there is no movement. The pattern of nodal lines can be therefore seen from the sand.  

(3 marks)

(b) Change the method by which the plate is forced to vibrate. For example the plate can be struck at a different position. Or sound waves of a different frequency could be used to excite the plate to vibration.

(2 marks)
FND_Q06

Question 6
(a) Explain why X-rays are more damaging to our bodies than infrared radiation.
(b) Light waves travel from a vacuum into a piece of transparent material such as glass. What changes, if any, occur in the speed, wavelength and frequency of the light in moving from the vacuum into the material? Justify your answers.

(5 marks)
Solution

(a) Electromagnetic waves such as X-rays and infrared can be modeled as a stream of particles called photons when they interact with matter. The energy $E$ of an individual photon is given by $E = hf$ where $h$ is Planck’s constant and $f$ is the frequency of the radiation. The frequency of X-rays is considerably larger than infrared radiation. This means that the individual X-rays have much more energy than infrared photons. When X-ray photons interact with our bodies they have sufficient energy to ionize atoms and molecules which interferes with the chemistry of cells and can lead to destruction of the cells and if a person is exposed to large doses than it can lead to the malfunction of organs and death. Infrared radiation is non-ionizing and therefore is not as dangerous to our bodies.

(1 mark for photon concept; 1 mark for greater energy more damaging)

(b) When light enters the glass from the vacuum, the electromagnetic field of the light interacts with the electrons and consequently the light slows down. The speed of the light is characterized by its refractive index $n$

$$ n = \frac{c}{v} $$

where,
$c$ is the speed of light in vacuum,
and,
$v$ is the speed of light in glass.

Hence $n > 1 \Rightarrow v < c$.

(1 mark)

For all waves, the frequency $f$ is the number of oscillations per second of the wave. This does not change as it moves from one medium to another. Therefore, the frequency of the light is the same in the vacuum and the glass.

(1 mark)

The speed of a wave is given by $v = f \lambda$ where $\lambda$ is the wavelength of the light. Therefore, as the speed of the wave $v$ is reduced and the frequency $f$ is the same, the wavelength $\lambda$ of the light in the glass must be smaller than in the vacuum.

(1 mark)
A geologist has a rock sample and wishes to identify it. The rock has a mass of 0.870 kg. The rock is suspended from a spring balance and a cord and fully immersed in pure water with the rock not touching the bottom. The spring balance shows a tension \( T \) in the cord of 6.62 N.

(a) Draw a diagram showing the forces on the rock while it is fully immersed in the water.

(b) Write an equation relating (in symbols) the forces acting on the rock.

(c) Find the volume of the rock. Show your reasoning.

(d) Referring to the densities listed below, determine what the rock is made from. Show your reasoning.

Densities:

- Pure water: \( 1.00 \times 10^3 \text{ kg.m}^{-3} \)
- Gypsum: \( 2.30 \times 10^3 \text{ kg.m}^{-3} \)
- Quartz: \( 2.65 \times 10^3 \text{ kg.m}^{-3} \)
- Siderite: \( 3.90 \times 10^3 \text{ kg.m}^{-3} \)
- Barite: \( 4.48 \times 10^3 \text{ kg.m}^{-3} \)
Solution

(a)

(b) For static equilibrium,
\[ \vec{w} + \vec{F}_B + \vec{T} = 0 \]
or
\[ w = F_B + T. \]

(c) Weight of rock (in air) = \( m \cdot g = (0.870)(9.80) = 8.53 \text{ N} \).
When immersed in water, scale reads \( T = 6.62 \text{ N} \).
Therefore, \( F_B = 8.53 - 6.62 = 1.91 \text{ N} \).

By Archimedes’s Principle, this is the weight of the displaced water.

Therefore the mass of the displaced water = \( w / g = 1.91 / 9.80 = 0.194 \text{ kg} \).
And the volume of the displaced water = \( m / \rho = 0.194 / 1.00 \times 10^3 = 1.94 \times 10^{-4} \text{ m}^3 \).

(d) Density of rock = \( m / V = \frac{0.870}{1.94 \times 10^{-4}} = 4.47 \times 10^3 \text{ kg.m}^{-3} \).
The mineral is therefore barite.
A tennis player hits a ball with a speed \( v \) directed horizontally from a point which is 2.50 m above the ground as shown in the diagram. The ball travels down the court and just clears a tennis net which is 0.90 m in height and 15.0 m from the point where the ball was hit. Assume that air resistance and the size of the tennis ball can be neglected.

(a) Show that the ball just passes over the net 0.57 seconds after it was hit by the tennis player. Hint: At the time that the ball passes the net it has fallen from its initial height to the height of the net.

(b) Calculate the value of the initial speed \( v \).

(c) Calculate the velocity of the ball at its position on top of the net as shown in the diagram above.

There is a target area of the court that extends a distance of 7.0 m behind the net as shown in the diagram.

(d) Calculate the position at which the ball lands and indicate if the ball lands within the target area.

(10 marks)
Solution

(a) We consider the horizontal and vertical motion of the ball separately. For the vertical motion the ball falls under gravity from rest at a height of \( h_1 = 2.50 \text{m} \) to a height of \( h_2 = 0.9 \text{m} \) in a time \( t \). Taking the downwards direction as positive we have
\[
\frac{h_1 - h_2}{g} = \frac{1}{2} \frac{1}{2} g t^2 \\
\Rightarrow t = \left[ \frac{2(h_1 - h_2)}{g} \right]^{1/2} = \sqrt{\frac{2(2-1.6)}{9.80}} = 0.57 \text{s}.
\]
(2 marks)

(b) We consider the horizontal motion. The horizontal motion is a uniform velocity \( v \). In a time \( t \) the ball travels a distance of 15.0 m so
\[
v = \frac{15.0}{0.57} = 26.3 \text{ m.s}^{-1}.
\]
(2 marks)

(c) We need to calculate the vertical speed (positive downwards) of the ball as it has fallen from rest to the height of the net. For this motion
\[
v = 0 + g t = (9.80)(0.57) = 5.59 \text{ m.s}^{-1}.
\]
The vector diagram of the components of the velocity is as follows.

We therefore have
\[
v_r^2 = (26.3)^2 + (5.59)^2 \Rightarrow v_r = 26.9 \text{ m.s}^{-1}.
\]
(1 mark)

and
\[
\tan \theta = \frac{5.59}{26.3} = 0.213 \\
\Rightarrow \theta = 12.0^\circ
\]
(1 mark)

(d) Repeat part (a) but for the ball falling the complete distance to the ground (i.e. \( h_2 = 0.0 \)). The time \( t_2 \) is given by
\[ h_1 - h_2 = 0 + \frac{1}{2}gt^2 \]

\[ \Rightarrow t_2 = \sqrt{\frac{2(h_1)}{g}} = \sqrt{\frac{(2)(2.5)}{9.80}} = 0.714 \text{ s.} \]

In this time the ball travels a horizontal distance \( d_2 \) of

\[ d_2 = vt_2 = (26.3)(0.714) = 18.8 \text{ m.} \]

This is \( 18.8 - 15.0 = 3.8 \text{ m} \) beyond the net and is therefore within the target area.
A ballistic pendulum can be used to determine the speed of a bullet fired into it. The above diagram shows an example of such a pendulum.

A bullet of mass 0.10 kg is fired horizontally at a speed $v$ into the block of wood of mass 1.0 kg, which is suspended motionless from the ceiling by a string. The distance from the ceiling to the point of impact at the centre of the block is 0.50 m. The bullet stops in the block.

(a)  Write an expression for the following quantities in terms of the initial speed of the bullet $v$ which is not yet known:
   (i) the kinetic energy of the bullet just before the impact;
   (ii) the momentum of the bullet just before the impact;
   (iii) the speed of the block (with the bullet embedded in it) just after the collision;
   (iv) the kinetic energy of the block (with the bullet embedded in it) just after the collision.

(b)  Derive an expression for the maximum height $h$ above the test position that the block (with bullet embedded in it) reaches after the collision.

Suppose that a bullet is fired into the block with a speed such that the block rises until the string is horizontal.

(c)  Calculate the value of the initial speed of the bullet.

(10 marks)
Solution
Deduct \( \frac{1}{2} \) mark one time for missing units. No units are required when no specific values have been substituted.

(a)

(i) \[ K = \frac{1}{2} m v^2 = 0.050 v^2 \text{ J} \]  

(1 mark)

(ii) \[ p = m v = 0.10 v \text{ kg.m.s}^{-1} \]  

(1 mark)

(iii) All forces are internal to the system, momentum is therefore conserved, and so

\[ p_{after} = p_{before} \]

Therefore

\[ p_{after} = (m + M) v_{after} = m v \]

\[ \Rightarrow v_{after} = \frac{m}{m + M} v \]

\[ = \frac{0.10}{1.10} = 0.0909 v \text{ m.s}^{-1} \]

(1 mark for method, 1 mark for correct result)

(iv) The kinetic energy after the collision is

\[ K = \frac{1}{2} (m + M) v_{after}^2 \]

\[ = (0.5)(1.10)(0.0909 v^2) \]

\[ = 0.004545 v^2 \text{ J} \]

(1 mark for method, 1 mark for correct result)

(b)
The block rises to a height \( h \) until all the kinetic energy is transformed into potential energy:

\[ K = (m + M) g h \]

\[ = (1.10)(9.80)h = 10.78h \text{ J} \]

(2 marks for energy transformation: numerical value not required)

(c)
If the string is horizontal, then \( h = 50 \text{ cm} = 0.50 \text{ m} \)
so

\[ 0.004545 v^2 = (10.78)(0.50) = 5.39 \]

\[ \Rightarrow v^2 = \frac{5.39}{0.004545} = 1185.8 \]

\[ \Rightarrow v = 34.4 \text{ m.s}^{-1} \]

(2 marks)
A 2.0 m long uniform pole has a mass of 2.0 kg. A bucket of mass 4.0 kg is hung a distance of 0.20 m from the left-hand end of the pole as shown in the diagram. A bucket of mass 9.0 kg is hung a distance of 0.50 m from the right-hand end of the pole.

Henry picks up the pole at a point X, which is a distance \( x \) from the left end of the pole.

(a) Write an expression in terms of distance \( x \) for the torque around point X arising from each of the following:
   (i) the weight of the bucket near the left-hand end of the pole;
   (ii) the weight of the bucket near the right-hand end of the pole;
   (iii) the weight of the pole;
   (iv) the force exerted by the person lifting the pole and buckets.

(b) Write an expression for the net torque resulting from the individual torques in part (a).

(c) Using your results, determine at what distance \( x \) should Henry pick up the pole and have it balance.

(10 marks)
Solution
(a) To calculate the net torque of the system around point X, we need to calculate the torques due to each of the forces. There are four forces operating:

\[ F_x \]
\[ W_1 \]
\[ W_p \]
\[ W_2 \]

where:
- \( x \) is the distance of point X from the left-hand end of the pole;
- \( W_1 \) is the weight of bucket 1, acting 0.2 m from the left-hand end of the pole, so the distance from point X is \( x - 0.2 \) m;
- \( W_2 \) is the weight of bucket 2, acting 1.5 m from the left-hand end of the pole, so the distance from point X is \( x - 1.5 \) m;
- \( W_p \) is the weight of the pole, acting through the centre of mass of the pole at 1.0 m from the left-hand end of the pole, so the distance from point X is \( x - 1.0 \) m;
- \( F_x \) is the pull upwards where Henry is holding the pole, acting at distance \( x \) from the left-hand end of the pole, so the distance from point X is 0.0 m.

Take anticlockwise torques to be positive.

(i) The torque due to bucket 1 around point X is
\[ \tau_1 = (x - 0.2) \times 4.0 \text{ kg} = 4 \text{ kg} \times 0.8 \text{ kg.m} \]
(ii) The torque due to bucket 2 is
\[ \tau_2 = (x - 1.5) \times 9.0 \text{ kg} = 9 \text{ kg} \times 13.5 \text{ kg.m} \]
(iii) The torque due to the pole is
\[ \tau_p = (x - 1.0) \times 2.0 \text{ kg} = 2 \text{ kg} \times 2.0 \text{ kg.m} \]
(iv) The torque, \( \tau_x \), due to the pull of the person holding the pole at position X is 0 since the force is acting through point X.

(2 marks for correct statement of problem, 1 mark for each answer)

(b) Hence the net torque of the system anticlockwise around point X is
\[ \tau = \tau_1 + \tau_2 + \tau_p + \tau_x \]
\[ = (4x - 0.8) + (9x - 13.5) + (2x - 2.0) + (0) \]
\[ = 15x - 16.3 \text{ m} \]

(c) For the pole to balance, the net torque must be zero, so
\[ 15x - 16.3 = 0 \]
\[ \Rightarrow 16.3 = 15.0 \Rightarrow x = 16.3 / 15.0 = 1.09 \text{ m} \]

(1 mark for balance condition; 1 mark for answer)
If a diver bobs up and down on the end of a diving board, the motion can be modelled as that of a mass oscillating on a spring.

A light and flexible diving board deflects by 0.225 m from the horizontal when Robert, a 68.0 kg diver, stands on its end as shown in the diagram.

(a) Calculate the effective spring constant for the diving board.

Robert then jumps a little and lands back on the end of the board, depressing it by an extra 0.105 m after which the board moves up and down so the end of the board and Robert moves as a simple harmonic oscillator.

(b) Calculate the period of the oscillation.

(c) Sketch a graph of the position of the end of the diving board against time for two complete cycles of oscillation. Take upwards as the positive vertical direction. On your graph also show the position of the undeflected board and the position of maximum deflection downwards. Show numerical values on the time and position axes.

The amplitude of the oscillation of the diving board increases if Robert drives the motion of the board with his legs at the right frequency.

(d) Calculate the frequency of the driving force exerted by Robert’s legs to give the largest amplitude of vibration. Briefly justify your answer.

(10 marks)
Solution

(a) We have an equilibrium deflection \( y_{eq} = 0.225 \text{m} \) with a mass \( m = 68.0 \text{kg} \).

At the equilibrium position, the restoring force from the spring board is equal to the gravity force and so

\[ k \times = m g. \]

This gives

\[ k = \frac{m g}{y_{eq}} = \frac{(68.0)(9.80)}{(0.225)} = 2.96 \times 10^3 \text{N.m}^{-1}. \]

(b) \[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{68.0}{2.96 \times 10^3}} = 0.952 \text{ s} \]

(c)
(d)

Natural frequency of vibration \( f_o = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{T} = 1.05 \text{ Hz} \). \hspace{1cm} (1 \text{ mark})

When driving frequency matches natural frequency of vibration \( \rightarrow \) resonance \( \rightarrow \) large amplitude vibration. \hspace{1cm} (2 \text{ marks})
A 1.50 m long wire is held fixed at both ends under a tension of 0.154 N and vibrates at 366 Hz. There are nodes at 0.500 m from each end of the wire.

(a) Sketch the shape of the wire for this mode of vibration, clearly showing the nodes and antinodes. Which harmonic is this?

(b) Calculate the wire’s fundamental frequency.

(c) Calculate the speed of the travelling wave along the wire.

(d) Calculate the linear density of the wire.

(e) If the tension in the wire was doubled, what changes would there be in the speed of the travelling wave, and the frequency and wavelength of the fundamental mode?

(10 marks)
Solution

(a)
One possible shape of the standing wave

![Standing wave diagram]

A series of snapshots of the wave

![Snapshot series diagram]

It is the third harmonic

(b)
For the 3\textsuperscript{rd} harmonic \( f_3 = 366 \text{ Hz} = 3 f_1 \)
Therefore the fundamental (1\textsuperscript{st} harmonic) is \( f_1 = 366 / 3 = 122 \text{ Hz} \)

(1 mark)

(c)
For \( L = 1.50 \text{ m} \) the wavelength of the wave must be \( \lambda = 1.00 \text{ m} \).
The velocity is given by

\[ v = f_3 \lambda_3 = (366)(1.00) \text{ m} = 366 \text{ m.s}^{-1} \]
or equivalently

\[ v = f_1 \lambda_1 = (122)(3.00) \text{ m} = 366 \text{ m.s}^{-1} \]

(2 marks)

(d)

\[ v = \sqrt{\frac{F_L}{\mu}} \]

\[ \mu = \frac{F_L}{v^2} = \frac{0.154}{366^2} \text{ kg.m}^{-1} = 1.15 \times 10^{-6} \text{ kg.m}^{-1} \]

(2 marks)

(e)

\[ v = \sqrt{\frac{F_L}{\mu}} \]

If tension \( F_L \) doubles and the linear density \( \mu \) remains constant then:
• the speed \( v \) increases by a factor of \( \sqrt{2} \)
• the wavelength is determined by boundary conditions and so there is no change.
• the frequency increases by a factor of \( \sqrt{2} \) (because \( v = f \lambda \) and the speed increases by a factor of \( \sqrt{2} \) and the wavelength remains the same).

(3 marks)