THE UNIVERSITY OF SYDNEY

PHYS 1901
PHYSICS 1A (ADVANCED)

JUNE 2009

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 MARKS

INSTRUCTIONS
• All questions are to be answered.
• Use a separate answer book for section A and section B.
• All answers should include explanations in terms of physical principles.

DATA

Density of fresh water $\rho = 1.000 \times 10^3$ kg.m$^{-3}$
Free fall acceleration at earth's surface $g = 9.80$ m.s$^{-2}$
Gravitational constant $G = 6.67 \times 10^{-11}$ N.m$^2$.kg$^{-2}$
Speed of light in a vacuum $c = 3.00 \times 10^8$ m.s$^{-1}$
Speed of sound in air $v = 344$ m.s$^{-1}$
Avogadro constant $N_A = 6.023 \times 10^{23}$ mol$^{-1}$
Universal gas constant $R = 8.314$ J.mol$^{-1}$.K$^{-1}$
Boltzmann constant $k = 1.380 \times 10^{-23}$ J.K$^{-1}$
Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W.m$^{-2}$.K$^{-4}$
SECTION A

Question 1

It has been proposed that future space stations should be rotating cylinders, providing the astronauts with artificial gravity. Consider such a cylinder of radius $R$ rotating with an angular velocity of $\omega$ in deep space, far away from any gravitational fields.

(a) Identify the forces acting on an astronaut who is rotating with the space station. By comparing to an astronaut standing on the surface of the Earth, briefly describe how the astronaut in the rotating space station experiences “weight”.

(b) If an astronaut in a rotating space station of radius $R$ is to experience the same weight as she would on Earth, show that the space station must be rotated at an angular velocity of

$$\omega = \sqrt{\frac{g}{R}}$$

Ensure that you justify your answer.

(c) Suppose an astronaut rotating with the space station releases a ball from her hand. Qualitatively describe the motion of the ball as observed by an external observer (not rotating with the space station).

(5 marks)
Question 2

An astronaut constructs an accelerometer for his rocket using two identical springs of unextended length $L$ and spring constant $k$, and two identical masses of mass $m$. He hangs the springs and masses in the rocket as shown below.

Assume the rocket remains within the gravitational field of the Earth and that the acceleration due to gravity is $g$ at all times.

(a) After launch, the rocket accelerates vertically upwards with an acceleration $a$. Show that the length of the accelerometer (the total length of the springs, assuming the masses have negligible length) is:

$$2L + \frac{3m(g+a)}{k}$$

(b) Eventually the rocket runs low on fuel and the motors are shut down. What is the total length of the accelerometer after shutdown of the motors? Explain your result.

(5 marks)
Question 3

Two solid spherical balls (A and B) are released at the same time from the top of the inclined ramp. Both have the same mass $M$, but Ball A has twice the radius of Ball B. They both roll down the ramp without slipping. The dimensions of the ramp are much larger than the radius of either ball.

(a) Which ball reaches the bottom of the ramp first? Justify your answer.

Hint: Consider the potential, rotational, and translational kinetic energies.

In a second experiment, a solid spherical ball of radius $R$ and a solid cylinder with a circular cross section of radius $R$ are released at the same time from the top of an inclined ramp. Both have mass $M$. The dimensions of the ramp are much larger than the radius $R$ of the ball or cylinder.

Which object reaches the bottom of the ramp first if:

(b) The ramp is frictionless. Justify your answer.

(c) There is a frictional force between each object and the ramp so that they roll without slipping. Justify your answer.

\begin{align*}
\text{Moment of Inertia Data} \\
\text{Moment of Inertia for Solid Sphere} & \quad I = \frac{2}{5} MR^2 \\
\text{Moment of Inertia for Solid Cylinder} & \quad I = \frac{1}{2} MR^2
\end{align*}

(5 marks)
Question 4

(a) Why does the water temperature at your favourite beach only vary by a few degrees between peak summertime and mid-winter, while the daytime air temperature can vary by as much as 30 degrees between seasons?

(b) Whilst lying on the beach on a hot day, it is common to feel a breeze blowing towards you from the sea. Explain briefly why this occurs.

(c) In what fundamental way does a Stirling engine differ from an internal combustion engine, such as that used in cars?

(d) Suppose you place an ideal Stirling engine on a block of ice. If the surrounding air temperature is 22.0 °C, calculate the maximum efficiency of the engine.

(5 marks)

Question 5

Harry stands near a bird’s nest. The bird tries to scare him away by flying in circles around his head while emitting a sound wave with a constant frequency. The bird keeps a constant distance from Harry.

Sally, standing at a large distance from Harry, also hears the bird (see the above diagram).

(a) At which position(s) of the bird (A,B,C,D) does Sally hear the highest frequency? Briefly explain your answer.

(b) At which position(s) of the bird (A,B,C,D) does Harry hear the highest frequency? Briefly explain your answer.

(c) At which position(s) of the bird (A,B,C,D) do Harry and Sally hear the same frequency? Briefly explain your answer.

(5 marks)
Question 6

Consider a double pendulum, such as the one in the corridor of the Physics building, which consists of two square plates that are free to rotate. If the system is given a large amount of energy, it may exhibit chaotic behaviour.

(a) Briefly explain what is meant by the statement that the system exhibits chaotic behaviour.

(b) After a while, when enough energy has been lost through friction, it will no longer behave chaotically. Briefly explain why this is so.

(c) If the plates are clamped together so that they move as a single object, the system will not exhibit chaotic behaviour. Briefly explain why this is so.

(5 marks)
SECTION B
(Please use another book to answer this section)

Question 7
A proton of mass $m$ moves in one dimension in a potential energy function given by

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x}$$

where $\alpha$ and $\beta$ are positive constants. The proton is released from rest at $x_0 = \frac{\alpha}{\beta}$.

In the following, ensure that you justify your answers.

(a) Show that the potential can be written as

$$U(x) = \frac{\alpha}{x_0^2} \left( \frac{x_0}{x} \right)^2 - \frac{x_0}{x}$$

(b) Sketch a graph of the potential as a function of $x$ and identify the point $x_0$.

(c) Give a qualitative description of the motion, identifying the minimum and maximum values of $x$ reached during the motion.

(d) What is the force on the proton as a function of $x$?

(e) Let the proton now be released from rest at $x_i = \frac{3\alpha}{\beta}$. Give a qualitative description of the motion; how does this differ from the motion described in (c)?

(10 marks)
Question 8

On a warm summer’s day at the South Pole, a worker is pushing boxes up a rough plank inclined at an angle $\alpha$ above the horizontal. The plank is partially covered in ice, with more ice near the bottom of the plank than near the top. He pushes the boxes at the bottom and each slides up the plank to a height that depends on how hard he pushed.

The coefficient of friction $\mu$ increases with distance $x$ up the plank such that:

$$\mu = \gamma x$$

where $\gamma$ is a positive constant. The bottom of the plank is at $x = 0$. Assume that the coefficients of static and kinetic friction are equal, so that $\mu_s = \mu_k = \mu$.

(a) Sketch a diagram showing all the forces acting on a box while on the plank.

(b) Show that as a box of mass $m$ slides from $x = 0$ to $x = A$, the work done by friction on the box is

$$W = - \frac{\gamma A^2}{2} mg \cos \alpha$$

(c) If the box has an initial velocity of $v_0$ as it starts up the plank at $x = 0$, show that the maximum distance along the plank that the mass travels, $B$, is the solution to the equation

$$\gamma g B^2 \cos \alpha + 2 g B \sin \alpha - v_0^2 = 0$$

Derive this equation, but do not solve it.

(d) Again consider the mass sliding up the slope with an initial velocity $v_0$. Show that when the box first comes to rest, it remains at rest if

$$v_0^2 \geq \frac{3 g \sin^2 \alpha}{\gamma \cos \alpha}$$

(10 marks)
Question 9

It is a hot day and the air temperature in your room is 30.0 °C so you turn on the air conditioner, which brings the temperature down to 22.0 °C. The room is sealed and contains 2500 moles of air.

(a) Calculate the change in entropy in the room. Use \( c_{air} = 29.11 \text{ J.mol}^{-1}.\text{K}^{-1} \).

(b) Using the refrigerator statement, explain how the room can continue to cool below the surrounding outside temperature (30°C) without violating the 2nd law of thermodynamics.

(c) Suppose once the temperature is constant in your room that you open a door leading to an adjacent room of identical volume that you’ve kept in a vacuum for the purpose of conducting “secret physics experiments”. Calculate the entropy change as the \( N \) molecules of air expand freely to occupy the volume of both rooms. Assume there is no change in the air temperature.

(10 marks)
Question 10

In the figure above, cylinder A is separated from cylinder B by an adiabatic piston which is freely movable. In cylinder A there is 0.010 mole of an ideal monatomic gas with an initial temperature 300 K and a volume of $1.0 \times 10^{-4} \text{ m}^3$. In cylinder B, there is 0.010 mole of the same ideal monatomic gas with the same initial temperature and the same volume as the gas in cylinder A.

Suppose that heat is allowed to flow slowly to the gas in cylinder A, and that the gas in cylinder B undergoes the thermodynamic process of adiabatic compression. Heat flows into cylinder A until finally the gas in cylinder B is compressed to a volume of $5.0 \times 10^{-5} \text{ m}^3$. Assume that $C_V = 12.47 \text{ J.mol}^{-1}.\text{K}^{-1}$ and the ratio of heat capacities is $\gamma = 1.67$.

(a) Calculate the final temperature and pressure of the ideal monatomic gas in cylinder B after the adiabatic compression.

(b) How much work does the gas in cylinder B do during the compression? Explain the meaning of the sign of the work.

(c) What is the final temperature of the gas in the cylinder A? \textit{(Hint:} the pressure exerted by the gas in cylinder A on the piston is equal to that exerted by the gas in cylinder B on the piston.)

(d) How much heat flows to the gas in the cylinder A during this process?

(10 marks)
Question 11

A heavy chain with mass $M$ and length $L$ is hanging vertically from your hand. Nothing is attached to the bottom of the chain, and the chain is not moving.

(a) The tension in the chain is not the same at all points. Is it greater near the top or near the bottom? Explain briefly.

(b) You generate a transverse pulse by jerking your hand sideways and back again. As the pulse travels down the chain, does it get faster or slower? Explain briefly.

(c) Consider a point on the chain at a distance $y$ from the bottom. Derive an expression for $v(y)$, the speed of the pulse as a function of $y$, and show that it does not depend on the mass of the chain.

(d) You now generate a transverse standing wave in the chain by shaking your hand from side to side at a frequency of 3 Hz. The standing wave has several nodes that are not equally spaced. Is the spacing between the nodes greater near the top or near the bottom. Explain briefly.

(10 marks)
Question 12

An object with mass $m$ hangs from a spring that is attached to the ceiling. The spring has mass $M$, length $L$, and spring constant $k$. Assuming the mass $M$ of the spring is negligible, the sum of the kinetic and potential energy at any moment can be written as

$$ \frac{1}{2} m v^2 + \frac{1}{2} k x^2 + mg x $$

where $x$ is measured vertically upwards from the equilibrium position of the object.

(a) By taking the time derivative of this expression, show that the object undergoes simple harmonic motion, with angular frequency

$$ \omega = \sqrt{\frac{k}{m}}. $$

*Hint:* First differentiate with respect to time (using the chain rule) and then try a solution of the form $x(t) = A \cos(\omega t) + x_0$.

(b) Now we will include the mass $M$ of the spring. At any given moment the object (and therefore, one end of the spring) is travelling at speed $v$, while the other end of the spring is fixed to the ceiling. Assuming that the spring stretches linearly, show that the total kinetic energy of the spring at this moment is

$$ \frac{1}{6} M v^2. $$

*Hint:* Divide the spring into pieces of length $dx$. Find the mass, speed, and kinetic energy of each piece and then add using integration.

(c) Determine the angular frequency of the system if the mass $M$ of the spring is included.

*Hint:* Use the same method as in part (a), but this time include the kinetic energy of the spring, as found in part (b). You may use the answer to part (b), even if you were unable to derive it.

(10 marks)

THIS IS THE END OF YOUR QUESTIONS