Computational Science in MATLAB
COSC1001 (Normal) & COSC1901 (Advanced)

Exam Solutions and Marking Scheme

1. Measuring acceleration due to gravity

This question assesses week 8 ("Review the application of linear algebra to physical problems", "Understand the built-in matrix manipulation functions" and "Apply matrix methods to problems in linear algebra..."), week 3 ("Write user-defined functions"), week 4 ("Understand the matrix datatype") and week 7 ("Calculate the accuracy of statistics drawn from limited populations")

(a) (3 marks)

\[0.07^2 g + 0.07b + c = 0 \quad (1)\]
\[0.53^2 g + 0.53b + c = 2 \quad (2)\]
\[0.85^2 g + 0.85b + c = 6 \quad (3)\]

\[
\begin{pmatrix}
0.07^2 & 0.07 & 1 \\
0.53^2 & 0.53 & 1 \\
0.85^2 & 0.85 & 1
\end{pmatrix}
\begin{pmatrix}
g \\
b \\
c
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
2 \\
6
\end{pmatrix}
\quad (4)
\]

(b) 3 marks

```matlab
>> A=[0.07^2 0.07 1; 0.53^2 0.53 1; 0.85^2 0.85 1];
>> y = [0; 2; 6];
>> gbc = inv(A)*y
```

(c) 2 marks

```matlab
function g = calcg(t)
    % This function computes the acceleration due to gravity, based on
    % an experiment that includes sensors placed at 0, 1 and 2 m
    A=[t(1)^2 t(1) 1; t(2)^2 t(2) 1; t(3)^2 t(3) 1];
    gbc = A\[0; 2; 6];
    g=gbc(1);
```

(d) 2 marks

```matlab
disp(['g = ' num2str(mean(g)) ' +/- ' num2str(std(g)/length(g))])
```
2. **Biased Coin** This assesses Week 5 (mostly “Generate a sequence and a matrix of random numbers” and “Simulate random processes (e.g. tossing a coin)” ) and Week 8 (“Calculate the accuracy of statistics drawn from limited populations”)

(a) (3 marks)

![Graph showing bar chart with categories 0 and 1]

(b) (4 marks)

```
clear
trials=1e5;
for n=1:trials
    results(n)=sum(round(rand(1,10)+0.1) ==1)<2;
end
disp(['Probability of less than 2 heads: ' num2str(sum(results)/trials)])
```

(c) (3 marks) Decreasing the number of trials to 1000 would mean that the probability returned would only be given as a percentage to 1 decimal place and would be highly variable.

3. **xy coordinates and example code.** This assesses weeks 1 through 4.

(a) (4 marks)

```
t = [0:0.01:2];
x = exp(-t);
y = t.*(2-t);
d2 = x.^2+y.^2;
t(d2==max(d2))
```

(b) i. (2 marks) 4 4.5
   ii. (2 marks) 4 7
   iii. (2 marks) 8 9
4. **Temperature in a Rod** This question mostly assesses week 9 ("Review eigen equations and their application to physical problems" and "Understand the built-in functions to examine eigen equations") and week 10 ("Review oscillating systems and their relation to linear algebra systems"). Technically, the heat equation is not an oscillating system, but the mathematics is nearly the same, so this is a challenging question to test the understanding of the derivations in the lecture.

(a) (5 marks)

```matlab
N=9; %An example number of segments.
v1 = 2*ones(N-1,1)
v2 = ones(N-2,1)
M=2*diag(v1) - diag(v2,-1) - diag(v2,1);
M = N^2*M; %Doesn't actually matter, but this would normalize the
%2nd derivative.
[e,v]=eig(M)
```

(b) (3 marks) The first temperature distribution will decay slowly, with \( T_1(t) = \exp(-0.014t) \) and the second temperature distribution will decay slower, with \( T_2(t) = \exp(-0.33t) \)

(c) (2 marks) After more than 10 seconds, this temperature distribution will resemble Distribution A, because the components of this distribution corresponding to all other eigenvectors will have had time to decay.