Experiment 6. Transmission Lines

General References:
S. Ramo, J.R. Whinnery and T van Duzer, *Fields and waves in communication electronics*, Wiley
(New York, 1994)
1 Objective

In this experiment you will investigate the propagation of pulsed and sinusoidal signals in a coaxial cable, and compare the results of your measurements with transmission line theory. The properties of a delay line, including dispersion, will also be investigated.

2 Introduction

At least two conducting paths are needed to connect one piece of electrical equipment to another. This is a consequence of the conservation of charge; the charge that enters the equipment by one path exits by the other. This is commonly achieved using a cable, in which both conducting paths are incorporated into the one structure.

Examples of cables are:

- **Coaxial cables**, in which one conductor is a hollow cylinder and the other is a wire along its axis, with the space between filled by a dielectric material, usually a solid polymer. You will have encountered coaxial cables in the laboratory as the standard way of connecting one piece of equipment to another. Coaxial cables are also commonly used to bring the signal from a TV antenna to the radiofrequency input of a TV receiver or DVD recorder.

- **Twin lead cables** which consist of two conductors of uniform cross-section and fixed separation. Ribbon cable, sometimes used to connect TV antennas to TV receivers, is an example of this kind of cable.

- **Twisted-pair cables** are a special category of twin lead cable in which two insulated wires are twisted around each other to reduce the effect of electromagnetic interference. Such cables are typically used for computer network cabling, where they have largely replaced the use of coaxial cables.

If the duration of a signal pulse (or the period of a sinusoidal signal) is much less than the propagation time along a cable, the cable behaves as a transmission line. If we ignore the resistance of the conductors, a transmission line is characterised by its capacitance per unit length, \( C' \) and inductance per unit length \( L' \).

From electromagnetic theory, the propagation speed of a signal is given by

\[
v = \frac{1}{\sqrt{L'C'}} \quad (1)
\]

The impedance presented to a signal by a long transmission line is called the characteristic impedance, and is given by

\[
Z_0 = \sqrt{\frac{L'}{C'}} \quad (2)
\]
Although the capacitance and inductance of the transmission line are distributed along its length, we can represent it by an equivalent circuit which consists of a chain of components, as shown in Fig. 6-1 where each segment represents the inductance and capacitance of unit length.

![Fig. 6-1](image)

**Fig. 6-1** : Equivalent circuit of a transmission line where each segment represents the inductance $L'$ and capacitance $C'$ per unit length.

### 2.1 Coaxial cable

If $a$ is the radius of the inner conductor and $b$ the inner radius of the outer conductor,

$$
C' = \frac{2\pi \varepsilon}{\ln(b/a)} \quad (3)
$$

$$
L' = \frac{\mu_0}{2\pi} \ln(b/a) \quad (4)
$$

where $\varepsilon$ is the permittivity of the dielectric between the conductors and $\mu = \mu_0$ has been assumed. It follows that for a coaxial cable

$$
v = \frac{1}{\sqrt{\varepsilon \mu_0}} = \frac{c}{\sqrt{\varepsilon_r}} \quad (5)
$$

$$
Z_0 = \frac{1}{2\pi} \sqrt{\left(\frac{\mu_0}{\varepsilon}\right) \ln \left(\frac{b}{a}\right)} \quad (6)
$$

where $\varepsilon_r$ is the relative permittivity of the dielectric (also called dielectric constant).

### 2.2 Twin-lead cable

If each conductor has a radius $a$, and the separation is $d$,

$$
C' = \frac{\pi \varepsilon}{\ln(d/a)} \quad (7)
$$

$$
L' = \frac{\mu_0}{\pi} \ln \left(\frac{d}{a}\right) \quad (8)
$$

where $a << d$, $\varepsilon$ is the permittivity of the dielectric between the conductors and $\mu = \mu_0$ has been assumed.
2.3 Twisted pair cable

As this is essentially a twin-lead cable that has been twisted, so that interference induced in one section of the cable are cancelled by the interference induced in an adjacent section that has been twisted through $180^\circ$, the equations for a twin-lead cable apply.

2.4 Termination of the cable

When a signal reaches the end of a cable it is reflected in a way that depends upon the terminating impedance $Z$ - the impedance which connects one conductor of the cable to the other$^1$. From transmission line theory the reflection coefficients for voltage $\rho_v$ and current $\rho_i$ are given by

$$\rho_v = -\rho_i = \frac{Z - Z_0}{Z + Z_0}$$

(9)

There are many circumstances where it is desirable that a signal not be reflected from the end of a cable, e.g. for digital information in a communication or computer circuit. As $Z_0$ is a real quantity, termination with a resistor $R$ where

$$R = Z_0$$

(10)

will ensure that there is no reflection.

If the cable is terminated by a capacitor, the reflection coefficient is given by (using complex notation):

$$\rho_v = -\rho_i = \frac{-j/\omega C - Z_0}{-j/\omega C + Z_0} = \rho_0 e^{j\phi}$$

(11)

where $-j/\omega C$ is the impedance of the capacitor, $\rho_0 = -1$ and $\phi = 2 \tan^{-1}(\omega Z_0 C)$.

2.5 Connectors

The connectors on the coaxial cables used in this experiment are standard for this size of cable; they are called BNC connectors$^2$. There are a variety of adapters available (male-to-male, female-to-female and T-pieces) which allow cables to be joined. Such connector and adapter systems are the result of careful design to ensure that they do not introduce impedance discontinuities which would cause partial reflection or attenuation of the signal.

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$^1$This could, for example, be the input impedance of another piece of equipment

$^2$BNC stands for Baby Neill-Concelman, where these two names refer to the designers of the "N" and "C" coaxial connectors respectively. For more details see http://en.wikipedia.org/wiki/BNC_connector.
2.6 Delay Line

An obvious application of a transmission line is to delay signals. For example, a signal delay of about half a microsecond is inserted in the vertical (y-axis) amplifier of an oscilloscope so that the sweep can be triggered into action before the signal reaches the vertical deflection plates. This allows the front edges of pulse waveforms to be seen. If coaxial cable is used, about 100 metres of cable would be needed, which is obviously impractical.

![Construction of the helical delay cable.](image)

Helical delay cable is one answer to this problem. The Hackethal cable used in this experiment (see Fig. 6-2) has an inner helix wound on a flexible magnetic core, which is a solid suspension of a ferrite in a suitable dielectric plastic such as polyethylene. The magnetic core increases the value of \( L' (\mu >> \mu_0) \). The result is a decrease in the phase velocity and an increase in the characteristic impedance.

Self capacitance between turns of the helix causes \( L' \) to reduce as frequency increases; this and other effects cause the phase velocity to increase with frequency making the cable dispersive\(^3\).

For a sine wave of frequency \( f \), a phase difference of \( 2\pi \) between the beginning and end of the delay line corresponds to a (phase) delay equal to one period of the sine wave, i.e. \( 1/f \). A phase difference of \( \phi \) corresponds, therefore, to a phase delay of

\[
\tau_{\text{phase}} = \frac{\phi}{2\pi f} = \frac{\phi}{\omega}
\]

(12)

For a signal consisting of a range of frequencies (e.g. a square pulse) the propagation speed of the envelope of the frequency spectrum is called the group velocity and the time of its propagation along the line is called the group delay. The latter is given by

\[
\tau_{\text{group}} = \frac{d\phi}{d\omega} = \frac{d(\phi/2\pi)}{df}
\]

(13)

\(^3\)Dispersion is a property of all transmission systems, including optical fibres, and sets limits to bit rates for transmission of digital signals.
Amplifiers and other equipment which handle a spread of frequencies are required to have a near constant group delay across their passband. This means that they need a linear change of phase with frequency over the passband. To the extent that this is not achieved, the shape of the pulse distorts due to the higher frequency components propagating at a different speed to the lower frequency components.

3 The experiment

The equipment used consists of pulse and waveform generators, various cables, adaptors and components. For measurements an oscilloscope, current probe, digital voltmeter and $LCR$ bridge are available. A short section of delay line is used in the last part of the experiment.

3.1 Signal generators

There are two signal generators: the HAMEG 8035 20 MHz Pulse Wave Generator for pulsed signals, and the HAMEG 8032 20 MHz Sine Wave generator. Both have output impedances of 50 $\Omega$.

3.2 $LCR$ bridge

The 6401 $LCR$ Databridge can measure the resistance, capacitance and inductance of a component at either 100 Hz or 1 kHz. The series option should be used for inductance measurements, the parallel option for capacitance measurements. The manufacturer’s specified accuracy is 0.25% ± 1 digit.

3.3 Current probe

The current probe is a current transformer: when clamped around a wire it gives an output that is proportional to the current in the wire. The calibration is 2 $A^{-1}$.

3.4 Directional couplers

These are three terminal components. When inserted in a coaxial line, a signal (pulse or sinewave) travelling through the coupler from the input terminal to the output terminal can be observed at the side port. A signal travelling in the opposite direction produces no output.

4 Procedure

4.1 Propagation of pulses in a coaxial cable

1. With the pulse generator set for 100 ns wide pulses at a repetition rate of 100 kHz. Use a BNC T-piece at the output of the pulse generator so you can see the pulse on channel 1 of
the oscilloscope (you will need to set channel 1 as the trigger source). Connect the end of the cable to the channel 2. Capture an image of the oscilloscope screen (see Appendix A) and print a copy for your logbook.

2. Measure the speed of propagation in the 40.5 m coaxial cable. You will find it useful to use the automatic measurement features available on the oscilloscope.

3. From your measurement of propagation speed, determine
   (i) the velocity ratio \( \text{VR} = \frac{v}{c} \) for the cable,
   (ii) the dielectric constant of the insulating medium in the coaxial cable, and compare your value (with uncertainty!) with the accepted value for polyethylene.

**Question 1:** Explain why the pulse at the end of the cable is close to twice the amplitude of the pulse from the pulse generator.

**Question 2:** In terms of frequency content, how is the reflected pulse degraded compared to the pulse from the generator? What does this tell you about the frequency characteristics of the cable?

3. Disconnect the end of the cable from the oscilloscope. Connect a short circuit to the the end of the cable and observe the effect on the reflected pulse.

4. Replace the BNC T-piece by the special connector with the exposed center conductor (with blue insulation). Attach the current probe to this adaptor, and connect its output to channel 2 of the oscilloscope.

5. Compare the reflected voltage and current pulse when the end of the cable is
   (i) open circuit
   (ii) shorted

**Question 3:** Explain in simple physical terms the changes in the voltage and current of the pulse upon reflection when the end of the cable is (i) open circuit and (ii) shorted. (It may help to consider a pulse of charge propagating along the center conductor of the cable.)

4.2 **Characteristic impedance**

1. With the aid of the special connector, measure the amplitude of the voltage and current of a pulse, and hence determine the characteristic impedance of the cable.

2. From the measured phase velocity and characteristic impedance, determine \( L' \) and \( C' \) for the cable.

3. Use the \( LCR \) bridge to measure \( L' \) and \( C' \), and compare with your directly measured value. (If we assume that the impedance of \( L' \) is much less than that of \( C' \) (i.e. \( \omega L' << 1/\omega C' \)), the equivalent circuit in Fig. 6-1 shows that the total capacitance of the cable will be measured when the end is open circuit, the total inductance when it is short circuit.)

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4Consult, for example, Kaye and Laby or the CRC Handbook.
**Question 4:** Given the dimension of the coaxial cable (outer diameter is 0.116 inches, inner diameter is 0.032 inches) calculate the characteristic impedance of the coaxial cable, using your value for the dielectric constant of the polyethylene.

**Question 5:** Confirm that the assumption $\omega L' << 1/\omega C'$ is justified.

### 4.3 Termination of the cable

1. With the aid of the 1 kΩ variable resistor, use the criterion that there is no reflection from a cable terminated by a resistor equal to its characteristic impedance to determine the characteristic impedance of the coaxial cable. How will you estimate an uncertainty for your measurement?

2. Determine the characteristic impedance of the twisted pair cable.

### 4.4 Directional couplers

In preparation for investigating the propagation of sine waves the operation of directional couplers will be examined using the pulse generator. Ensure that the end of coaxial cable is open circuit so that there is a reflected pulse.

1. Connect the directional couplers in series (using a male-male BNC adaptor) between the pulse generator and the cable so that one sees the outgoing pulse and the other sees the reflected pulse.

2. Using coaxial cables of equal length, connect the output of each directional coupler to an oscilloscope channel. Note that the directional couplers are not perfect: each coupler sees a small fraction of the ‘other’ pulse. Nevertheless, we will use them to separate sine waves travelling in the opposite directions.

### 4.5 Propagation of sinusoidal signals in a coaxial cable

1. Disconnect the pulse generator and replace it with the Hameg sinewave oscillator set to 5 MHz.

2. On the end of the 40.5 metre cable connect in turn
   
   (i) an open circuit,
   
   (ii) a short circuit, and

3. How much of the phase change observed is due to the propagation time along the cable? Calculate this from your previously measured phase velocity

4. To enable us to concentrate on the phase shift produced at the end of the cable, replace the 40.5 m cable with a BNC to BNC adaptor (which is equivalent to a piece of cable 3 cm long).
5. Measure the phase shift between the incident and reflected waves for the following loads:

(i) an open circuit,
(ii) a short circuit, and
(iii) a 1 nF capacitor.

6. From the measure phase shift calculate the capacitance of the nominal 1 nF capacitor, and compare this with the value measured using the LCR bridge.

4.6 Delay line

1. Use the LCR bridge to determine \( L' \) and \( C' \) for the delay line and show that \( Z_0 \approx 1.6 \, \text{k}\Omega \).

2. Connect the helical delay cable up as shown in Fig. 6-3, and connect a 1.6 kilohm resistor as load to eliminate reflections.

3. Feed the cable with 2\( \mu \)s wide pulses at a repetition rate of 10 kHz and observe the shapes of the pulses at the beginning and the end of the cable. Capture an image of the oscilloscope screen (see Appendix A) and print a copy for your logbook. Describe, with reference to frequency components of a square pulse, the difference between the input pulse and the delayed pulse.

\[ \text{Fig. 6-3 : Measurement of the dispersion and group delay for helical cable.} \]

**Question 6:** What can you conclude about the way phase velocity varies with frequency from the shape of the delayed pulse? From the detailed shape of the pulse, identify a “high frequency” delay and a “low frequency” delay.

**Question 7:** Which of the above delays agree with the value calculated from the measured values of \( L' \) and \( C' \)? Explain why.
4.7 Phase and group delays in the cable

The sine wave generator is now used to explore further the dispersion of the cable.

1. Replace the pulse generator with the sinewave generator, and ensure the line is terminated so that the waves travel to the load and are not reflected.

2. Set the sinewave generator to a frequency of 100kHz and look at the phase shift between the generator and the load end of the cable. It will appear to be about $\pi/4$. Check that this is sensible; ie calculate the phase shift from the cable delay time and the period of the signal. It is important when measuring phase shifts to know if integral multiples of $2\pi$ need to be added to the raw result. Confirm that this is not needed for 100 kHz.

3. Increase the frequency until the phase shift is exactly $2\pi$ and record the frequency.

4. Measure the frequencies for phase shifts of $4\pi$, $6\pi$, $8\pi$, etc until a frequency of about 10 MHz.

5. Plot the measurements and determine the group delays at the low and high ends of the frequency range (see equation 13).

Question 8: Compare the group delay measurements with the low- and high-frequency delays measured for pulse propagation in the delay line. Comment.

5 Twisted-pair cable (optional)

1. Measure the propagation speed for the cable, and with the aid of the previously measured characteristic impedance, find values for $L'$ and $C'$.

2. Comment on any discrepancies.

3. From your measurements find a value for $d/a$. Is the result reasonable?
A  Agilent DSO 1002A: saving a screen dump to a USB memory stick

1. Insert USB into the front panel slot. The DSO should instantly recognize the USB.

2. Hit the **Save/Recall** button

3. Hit **Waveform**, and select PNG file output. Also worth turning **Para(meter) Save ON** to save DSO settings

4. Hit **External**

5. Hit **New File** (simply accept the default filename)

6. Hit **Save** (this saves the screen dump as NewFile\(x\).png, and the DSO settings as NewFile\(x\).txt, where \(x\) automatically increments by 1)

**Sample output** (pulse propagation through the 40.5 m coaxial cable)

![Pulse propagation through the 40.5 m coaxial cable](image)

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