CHAPTER ONE

AN INTRODUCTION TO THIS INVESTIGATION

1.1 PROJECT OUTLINE

Quantum mechanics is an area of immense importance to modern technologies and industries, covering a diverse range of applications from semiconductors and lasers to advances in nuclear medicine. Quantum mechanics is also a subject that most students have traditionally found both difficult and abstract. However the delivery of such material has changed little since it was developed early this century. The difficulties encountered by students appear to stem from the highly mathematical formalism in which the subject is shrouded and is further complicated by ongoing debates concerning how this very formalism should be interpreted.

The gulf between the apparent practical applications and the mathematical formalism provides an even greater challenge to academic teaching staff, who have limited time to cover the vast amount of material currently prescribed by undergraduate curricula. Quantum mechanics is an area that has not, until recently, attracted much pedagogical research. Thus, while it is obvious that quantum mechanics is an important subject, the difficulties involved would suggest that a project that tries first to identify conceptual difficulties and then to address them, is of great potential use. The purpose of this investigation is to isolate concepts, to identify learning difficulties encountered by students and thence to design an instrument to quantify these difficulties.

1.2 WHAT IS QUANTUM MECHANICS?

The evolution of this subject can be viewed in three stages: (1885-1912) a period in which a variety of experiments and explanations mounted that lacked unification; (1913-1922) which centred on the creation and development of Niels H.D. Bohr’s quantum theory; and finally (1923-1927) the period of development and formalisation of what is officially called quantum mechanics.

---

1 It is difficult to determine the commencement date of this period. The date 1885 has been chosen as it was the year the first experiments on atomic line spectra were conducted by Balmer.
During the period 1885 to 1912 a large number of experimental facts which could not be explained on the basis of existing theory were accumulated: the discovery of ordered series in atomic spectra by Johann J. Balmer, Theodore Lyman, Johannes Ryberg and Friedrich Paschen; the studies of blackbody radiation by Wilhelm Wein, John W.S. Lord Rayleigh and Sir James H. Jeans and its theoretical description by Max K.E.L. Planck; Albert Einstein’s contributions in the quantisation of energy in black body radiation, the photoelectric effect, the specific heat of solids; and Sir Ernest Rutherford’s planetary model of the atom.

The next stage began with the 1913 paper “On the Constitution of Atoms and Molecules” by Bohr which described the planetary model of a hydrogen atom based upon the quantisation of energy and angular momentum of the electron. Bohr’s theory provided an explanation to spectral phenomena and permitted the calculation of Rydberg’s constant. Bohr’s “simplistic” theory brought together many ideas and concepts that guided both experimentalists and theoreticians.

Experiments by James Franck and Gustav L. Hertz in 1914, concerning the measurement of electron energy spent on exciting mercury atoms, was direct experimental evidence for the fact that an atom may change its energy only discretely. In 1916 Arnold Sommerfeld and Peter Debye came to the conclusion that the angular momentum components in the direction of the magnetic field are quantised, thus introducing the concept of the quantisation in space. This received confirmation in experiments conducted by Otto Stern and Walther Gerlach in 1922 on splitting of atomic beams in non-uniform magnetic fields.

The Bohr models continued to develop in the period between 1913 and the early 1920s. Work by Wilson and Sommerfeld allowed some of the ad hoc aspects of the theory (the insistence on circular orbits, for example) to be abandoned. Despite this, however, the model was inherently problematic and the internal contradictions associated with the very idea of quantisation and of discrete quantum jumps became progressively more apparent through the early decades of the century. In 1923 Bohr formally introduced the correspondence principle in his article “On the Quantum Theory of Line Spectra”. According to this principle, the laws of quantum physics must turn into the laws of classical physics for large values of quantum

---

2 The correspondence principle as articulated by B.H. Bransden and C.J. Joachain (1989) in Introduction to Quantum Mechanics is as follows. ... quantum theory results must tend asymptotically to those obtained from classical physics in the limit of large quantum numbers.
numbers of a system. Thus, despite the apparent incommensurability of the classical and quantum theories, the former has been of great importance in the discovery of laws in quantum mechanics.

The birth of Quantum Mechanics proper was marked by a series of experiments, the unification of ideas and concepts, and the development of consistent mathematical models. In 1923 Arthur H. Compton’s X-ray scattering experiments clearly indicated the existence of particle-wave properties of radiation. During 1923-1924 Louis de Broglie suggested in his doctoral thesis that wave-particle duality should be extended to all micro-particles and in 1927 the idea of duality was confirmed in several laboratories worldwide by experiments on electron diffraction.

In 1924 Satendra Nath Bose carried out fundamental studies, which were extended by Einstein in the form of a statistical theory for photons which came to be known as Bose-Einstein statistics. In the framework of this theory, Planck’s formula for blackbody radiation at last found a complete explanation. During 1925 de Broglie introduced the idea of matter waves described by the so called wave function, and Wolfgang Pauli formulated his famous exclusion principle for electrons.

In 1926 Schrödinger in his paper “On Quantisation as an Eigenvalue Problem” used the wave concepts to introduce his well known differential equation for a wave function. Thus the calculation of finding the energy levels of a bound micro-particle was reduced to the problem of finding the eigenvalues of a particular differential equation. The same year Schrödinger published a paper demonstrating the equivalence of his method and that of Max Born, Werner Heisenberg and Pascal Jordan. While the formalisation of Schrödinger’s theory was readily accepted, the problem of the interpretation of the wave mechanics and the physical description of the concept of wave function remained the subject of heated debate for many years. Born, in 1926, proposed a probability interpretation of the wave function; matter waves were replaced by probability waves. The impossibility of interpreting the mathematical wave function as the amplitude of a certain real material field (like in 3 The Pauli Exclusion Principle states that no two electrons in the same atom can have the same quantum numbers. This Principle underpins a great deal of modern studies in chemistry, explaining, for example, the structure of the Periodic Table.
electromagnetic fields) was recognised. This in turn meant that de Broglie’s matter waves could not be interpreted as classical waves of any sort.

Finally in 1927 Heisenberg introduced his **uncertainty principle** and showed how the concepts of energy, momentum and position could be included in the wave description of the micro-particle. The appearance of these relations marked the final break of quantum mechanics from classical determinism and established quantum mechanics a statistical theory.

There is no one answer to the question “What is Quantum Mechanics?” Lamb has captured the essence of the practising physicist’s approach by providing what is, effectively, a definition of the subject’s utility:

“The only easy [answer] is that quantum mechanics is a discipline that provides a wonderful set of rules for calculating physical properties of matter.” (Lamb 1969)

For the student, the shift between the macro- and the micro-world is much more than merely a matter of terminology. Classical physics is based upon the relatively simple idea of the summation of forces and velocities. Quantum mechanics, however, is grounded in the notion of the probabilities of different events interfering with one another to result in the chance of an event occurring. The student is thus required to make the mental shift between classical mechanics, centred around the concepts of billiard ball collisions and an idealised motion of a projectile, and those of quantum mechanics centred around the probability of events.

### 1.3 IMPORTANCE OF TEACHING QUANTUM MECHANICS

At a very basic level, quantum mechanics can be described as the mechanics (ie. the motion and properties of motion, and the properties of matter) of the “micro-world”. In this “micro-world”, to be contrasted with the world of our everyday experience, the “macro-world”, we can find answers to questions that are apparently unsolvable in classical mechanics. Why is a diamond hard? Why does the electrical conductivity of semiconductors increase with temperature? Why does a magnet lose its properties when heated? Quantum mechanics is a more detailed theory that includes classical mechanics as an extreme limiting case. In fact quantum mechanics is the theoretical foundation of all modern theories dealing with the structure and properties of matter.
As has been pointed out earlier, quantum mechanics is an area of physics that underlies many modern technologies. Modern day applications where quantum mechanics is absolutely essential are both numerous and wide-ranging, spanning such disparate fields as material science, nuclear medicine, engineering, chemistry, biochemistry, nuclear physics and the electronics industry. It becomes clear that a comprehensive training in the physical sciences is impossible without a serious study of quantum mechanics.

Yet, despite the revolution that quantum mechanics has inspired in twentieth century physics, introductory quantum mechanics has been taught in the same fashion for the past sixty-five years and there has been little pedagogical research directed towards this area. It is important to address this apparent lack of interest. It is imperative that we are able to convey to students in an efficient, effective, appropriate environment, the ideas and concepts of quantum mechanics and to monitor their conceptual development.

The apparent lack of pedagogical interest and the lack of associated published research also manifests in the current practice of evaluating a student’s understanding of the subject by only the occasional assignment and an end of year formal examination. The inadequacies associated with this assessment strategy in identifying the specific difficulties students have with the subject are obvious. Therefore a pressing need for research that can provide formative evaluation strategies during the delivery of the subject is of paramount importance.

1.4 SOME UNANSWERED QUESTIONS

There are several difficulties facing this investigation. Firstly, quantum mechanics is a highly mathematically formal discipline. The birth of quantum mechanics brought with it a suite of axioms and mathematical theorems that are perpetuated by each generation of student textbooks. There is not yet consensus about how the subject might be taught less abstractly. As well, the subject is still in a state of flux, with longstanding questions concerning the interpretation of the formalism still being discussed in the technical literature\(^4\).

\(^4\) One of the most obvious areas of current discussion concerns Bell’s Theorem. References may be found in the bibliographies of several recent articles, for example: Mermin, N.D., “Quantum Mysteries Redefined”, *American Journal of Physics*, Vol. 62 (10) pp880-887 (1994). Other areas of critical discussion include EPR Paradox and Hidden Variable theories.
The investigation follows a line of inquiry that seeks to answer two fundamental questions.

1. Is it possible to identify the most important concepts that students need to understand in order to learn quantum mechanics “successfully”?

2. What is it about the way students conceptualise the ideas of quantum mechanics which makes them particularly “difficult”?

The preliminary answers to these questions will provide the foundations for future investigations.

1.5 LAYOUT OF THESIS

The thesis consists of two parts; the first concerns itself with the educational background and the analysis of some previous survey-based studies; the second part deals with the development and implementation of a new instrument.

Part 1
- Overview of appropriate physics and educational research
- Initial investigation involving the review of previous survey based studies
- Isolation of physics concepts

Part 2
- The development and implementation of a new instrument
- Presentation of research findings
BIBLIOGRAPHY


CHAPTER ONE .................................................................................................................... 1

AN INTRODUCTION TO THIS INVESTIGATION ................................................................. 1
1.1 PROJECT OUTLINE ................................................................................................. 1
1.2 WHAT IS QUANTUM MECHANICS? ...................................................................... 1
1.3 IMPORTANCE OF TEACHING QUANTUM MECHANICS .................................... 4
1.4 SOME UNANSWERED QUESTIONS ...................................................................... 5
1.5 LAYOUT OF THESIS ............................................................................................. 6