Two Student-centred Teaching Methods in Mathematics

Guohua Peng
Mathematical College
Sichuan University
Chengdu 610064
People's Republic of China
ghpeng@mail.sc.cninfo.net

Abstract

In this paper, I spend a little time on analyzing the outcomes of conventional teaching. Then I propose two student-centred teaching methods in mathematics: problem-centred teaching and seminar, both of which emphasize group work and develop students' skills in communication and presentation. Problem-centred teaching starts by giving students a real world problem. Students are expected to determine any new concepts or theoretical results which are needed to solve the problem. They are expected to do this by themselves, albeit under the guidance of the teacher. This process will help students realize how mathematicians think and how they tackle real world problems. The teacher will give instruction and lectures to achieve this end. The seminar is a teaching and learning model borrowed from research activity. In a seminar, students will give the lectures while the teacher will stay behind and pretend to be a listener most of the time. There are also many opportunities for group work for students in a seminar. This can help students' learning and improve their personal skills as well. These methods are hoped to stimulate the students' interest in learning mathematics. They are expected to be effective in teaching mathematics at university level in China.

Introduction

Generally Chinese mathematics teachers at university level agree that the teaching and learning of mathematics must:

1. extend the students' ways of thinking;
2. develop the students' abilities in problem handling and solving; and
3. provide applicable mathematics knowledge, expertise and skills for future needs.

We want our students to do mathematics, not to listen to and watch mathematics being done by the teacher. We want our students to be excited about doing mathematics. We want them to understand mathematics, pass their classes, and stay in school. We want students to solve problems that they recognize as relevant to their lives. To achieve this end, good teaching practice should involve the students being active in the process and make them respect and, enjoy mathematics, recognizing its usefulness and appreciating the subject in its own right[1].

I think the key strategy of mathematics teaching should focus on keeping the students' interests on mathematics. If the students are interested in learning mathematics, then the teacher's task becomes easier. But the traditional teaching methods pay more attention on teachers. We may call it teacher-centred teaching. It decreased student's learning interest to some extent.

Conventionally, mathematics teachers give lectures when they teach their classes. They believe that this is a good way to transmit knowledge. We would like to give all our students a depth of mathematics understanding because we think it is a worthwhile and necessary part of their culture. In a traditional classroom setting in Chinese universities, the teacher will begin class by reviewing, then he will teach the new lesson, and finally he will give a take-home assignment. Some lectures begin by answering questions from homework or from the classroom. The student watches, listens, takes notes and then copies what the teacher does in their assignments. If the students have difficulty in doing the assignment, they have no way to get help. Most university students live in student apartments on campus and they usually do the assignment at a time when the teachers have gone home. So the students leave the questions for the tutorial, which is isolated from lectures and supervised by another teacher. The objective of the tutorial is to help the students do more exercises, because practice plays an important role in mathematics learning. During the tutorial, it's common that the teacher himself plays the leading role. The main task for the tutor is to explain the answers to the homework and demonstrate more examples to apply new theory discussed in the foregoing lectures. In some tutorials, the teacher moves around the room trying to answer everyone's questions. But some students are too shy to ask questions. So students leave the classroom without having their questions answered and unable to understand the true meaning of the mathematics theory. This conventional method is often boring for students because their only job in the classroom is to passively sit and watch the teacher work mathematics problems on the board and then copy what the teacher did. Few teachers use learning aids to teach mathematics.

The fatal shortcoming of the above method is that it diminishes the students' interest. One report showed that nearly 70 percent of students choose mathematics because of their interest in it. But after the first year, almost twenty percent of mathematics students lose that interest, and the percentage of students who maintain interest in mathematics declines in the following years. The reasons for this phenomenon are various, and include the difficulty of mathematics, the lure of other disciplines, the transfer of their interest and the students' unrealistic expectation of mathematics. But one important reason is the boring teaching method. Simple traditional methods gradually make the students feel that mathematics is pointless and has little value to them in real life. It becomes a subject they are forced to study, but one that is useless to them in real life[2]. In the traditional classroom setting discussed above, both students and teacher are often frustrated because the students' individual needs are not met. Another disadvantage of the conventional teaching method is that it provides no way for students to practice generic skills such as communication and teamwork, which are very important for every university student.

Research has shown that students studying mathematics at university in China rarely study a communication subject, and certainly not a communication subject designed specially for mathematics. Indeed many students study mathematics because they perceive that they will not have to read or write if they take this subject. There has been considerable resistance from students and mathematics staff to the studying of generic skills. Mathematics lecturers believe that there is too little time to cover the mathematics skills and do not wish to give up teaching hours, and students find it difficult to relate to the general communication skill subjects. Some teachers even think that it's the abstract nature of mathematics that makes it what it is, so they perceive mathematics in an abstract way and teach it in a purely logical way. On the other hand, most teachers think that mathematics needs individual work rather than group work, so there is no need to emphasize teamwork.

But the reality in China is that no more than 10 percent of mathematics graduates will have the opportunity to be a mathematician. Most of the graduates from mathematics departments will choose a job which is outside the mathematics field but which may have a connection to mathematics. In recent years, more and more enterprises welcome graduates from mathematics, because they believe that a person with mathematical ability can think logically and, with a little extra non-mathematical knowledge, has the potential to solve realistic technical problems. So, on one hand, it seems the aim of mathematics teaching should be to foster the students' mathematical abilities. On the other hand, a great many employers are complaining about the graduate's lack of communication skills and team collaborative spirit.

Meanwhile mathematics is changing[3]. Some new themes should be added to the traditional teaching model. Mathematics teachers should try alternative methods in an effort to better meet the needs of their students.

Student-centred teaching is a teaching model in which the students' opinions, backgrounds and goals are acknowledged and incorporated within the learning environment. It is in sharp contrast to the traditional model in which the teacher makes most, if not all, of the decisions regarding content and materials. It also differs in degree from the student-based model of teaching, whereby the students themselves have large responsibility for course content, materials selection and direction. Good approaches to teaching have acknowledged the importance of individual students and the value of their separate life experiences, and take account of their differing backgrounds within the learning environment. Making the learners more responsible for course information, content and direction will help encourage a positive group atmosphere and produce more interesting and relevant materials. It will also provide good feedback and reduced preparation time for the teacher. In effect both students and teachers will benefit.

The teachers' role depends on the extent to which student-centred learning is incorporated in the classroom. Generally, the teacher can be an active participant in the group, a helper and resource, and a class monitor of progress. Whatever the role, the purpose is to narrow the traditional gap between teacher and student, although the teacher retains primary responsibility within the classroom.

Student-centred teaching may increase student involvement by drawing them into the learning process. It can also assist group solidarity by helping each student participate actively. Based on the Chinese university students' characteristics, I propose the following two forms of student-centred teaching methods: problem centred teaching and seminar. They are expected to be effective, especially in improving the students' communication and collaborative skills.

Problem centred teaching

Problem centred teaching is a teaching method, which is based on the belief that students can perceive the world, as do scientists. If the students are exposed to a real world circumstance, the motivation to solve the problem will force them to invent new concepts and new theory, which is what the course should cover. This can be achieved if they are given proper guidance from the teacher.

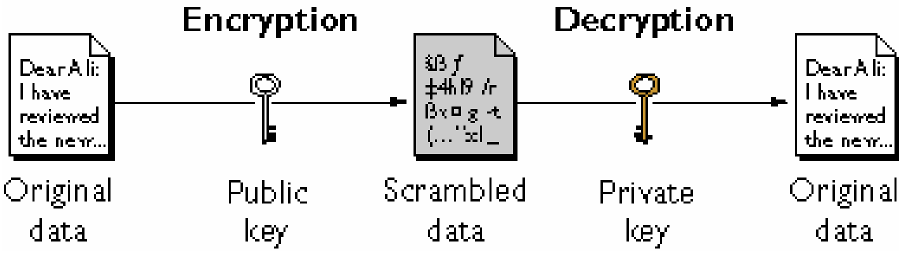
Problem centred teaching is connected with problem centred learning, which involves learning through tackling relevant problems. In tackling the problems students are motivated to acquire knowledge by actively seeking it out, as opposed to learning through exposition. Problem centred learning is a curriculum approach which helps students frame experience through a series of problem solving activities, and where the process of learning unfolds through the application of knowledge and skills to the solution of real world problems[4]. Problem centred teaching starts with a simple real world problem. The teaching strategy concerns with one or more solutions of the problem. First the students will be asked to draw a mathematical model from the real world problem. The following step is to try to find an answer to the original problem. This is different from problem solving, since the problem will not be solved without some new knowledge. The aim of problem centred teaching is not to show how to apply the knowledge but to show how to 'invent' knowledge. During the process of answer seeking, the teacher will constantly give some explanations and hints. Finally the students will draw the conclusion by themselves. Of course the students will find what they have learnt is applicable.

Problem centred teaching is a kind of interactive teaching. Interactive teaching means that the teachers try to instruct the students in discovering mathematical facts by themselves. The important aspect of problem centred teaching is that the students will see the place of mathematics in the real world and really experience the usefulness of mathematics. This can help the students to keep their positive motivation in learning mathematics. Along the way, the student will see how mathematicians think of the problem and how they finally solve this problem. If they come across a real world problem in the future, this experience will help them to find a breakthrough point to handle it and with appropriate technology, they should have the ability to solve it.

Consider applying problem centred teaching to teaching cryptography. Cryptography is the branch of mathematics that provides the techniques, which enable confidential information to be transmitted over public networks. It is the art and science of secure transmission of information. Obviously it has a strongly applicable background. Cryptography is always a course for the third or fourth year students in our university. Most of the students are interested in information technology. In general, cryptography contains many concepts and terminology such as message, encryption, plaintext, cipher text, bit, character, string, encode, decode, cipher, encipher, decipher, block cipher, transposition cipher, secret key, public key, etc. Some of them are easily understood. For example, message, cipher, encipher and decipher are terms which are commonly used in spy films. We can therefore begin the course with the following problem, without taking extra time to explain the terms.

> Encipher the following message so that anyone else has no way to decipher it in ten minutes. However you must declare how you have enciphered this message:
> *Alan Turing began his career in mathematics at King's College, Cambridge University in 1931. It was here that his tendencies to recreate previous discoveries began to emerge. Turing seemed to have little interest in using the work of previous scientists; he would typically spend time recreating their work instead. Upon graduation, Turing was made a fellow of King's College, and then moved on to Princeton University. It was during this time that he explored what was later called the 'Turing Machine'. Turing helped pioneer the concept of the digital computer. The Turing Machine that he envisioned is essentially the same as today's multi-purpose computers. He described a machine that would read a series of ones and zeros from a tape. These ones and zeros described the steps that needed to be done to solve a particular problem or perform a certain task. The Turing Machine would read each of the steps and perform them in sequence, resulting in the proper answer.*

Before the students get down to business, I'll explain that what they are doing is an analogue of the situation of public communication such as e-commercial. What they must do is to keep the information secure. If they encipher the message 'powerfully', an eavesdropper will have no way to find the original message (plaintext) even if he intercepts the enciphered message during the transmission process and has some cryptanalytic methods and apparatus. In other words, partially knowing the secret doesn't mean knowing the secret. The situation is the same with remote public communication such as email. For example, an employee of an international enterprise is on duty in Africa, but the headquarters are in Beijing. He wants to send some important data from Africa, which cannot be known by certain other people, especially business rivals. The easy and quick way is to use the Internet, by email or by logging into the web site of the enterprise and leaving the message. If he sends the message without encrypting, an information-seeker or hacker can easily intercept it and read the secret message. But if the message is enciphered, it is transmitted as a mess, which cannot be deciphered unless additional information on the encryption method has been obtained. This 'additional information' can deduce the 'secret key', which is used to decipher in public key cryptography. If the enterprise has a secure public key cryptosystem, then all staff can send encrypted messages at any time from any place where the Internet is available. The enterprise may reveal a staff member's 'public key', which is much the same as a telephone number. Meanwhile every staff member has a secret key, which must be kept secure and not revealed. The staff members use public keys to communicate between themselves. Only the person who possesses the secret key can read the message. Authentication uses the public key and the staff can use the secret key to make a digital signature as well. In short, only those people who have the public key and the secret key at the same time can read the secret message. This is the same thing as closing a door using one key, while a different key is needed to open the door. The secret key (used to open the door) cannot be deduced from the public key (used to lock the door) without additional information. Of course we must make sure that the key used to open the door is kept securely. The person who is permitted to enter the door possesses a secret key. Everybody knows the technique used to open the door, but only those people who have a secret key can actually open the door. Since the two keys are different, we can even leave the key used to close the door (public key) by the door. Email works in the same way. Everyone can send an email to me if he knows my email address, but not everyone can read my personal email. This may be explained in the following diagram.



To fulfill the above task, the students will be allocated into four groups. All members in the group are asked to give a presentation in turn to declare the encryption methods of the whole group. Each group will be asked to decipher the enciphered messages from other groups. That is, they are code-designers as well as code-breakers. These activities are expected to help them find the relation between encryption and decryption, and consequently this will help them find a secure method to encipher their message.

The key point in solving the difficult encryption task is to find a function $y = f_k(x)$ with the following properties:

1. It's easy to compute y from x. (This makes the encryption process simple and easy.)
2. It's difficult to get x if you only know y. That is, it's difficult to find the inverse function of $f_k(x)$. (This makes the code-breaking process difficult without some additional information.)
3. If you have some knowledge about $y = f_k(x)$ or k, then you can compute x easily from y. (This means the legitimate recipient can easily read out the original message x, because he has a secret key.)

This kind of function is called a *trap-door one-way function*. It's not easy for a learner to find a concrete trap-door one-way function. So the teacher will give a series of lectures in between to ensure that the student will gradually draw out the answer. For example, the students will be instructed to think about the fact that it's easy to compute the product of two numbers but it's difficult to decompose a bigger integer into the product of two relatively small divisors. But if you know one of the divisors of the bigger number, you can easily find the quotient. At this stage, the students are encouraged to use this example to construct a trap-door one-way function, which may be utilized to encrypt.

The lectures will be arranged according to the students' perception of the real world problem, and focus on how to describe the problem mathematically and how to manipulate it mathematically. The students are expected to understand the standard concepts and terminology in cryptography and the mechanism of cryptology during this process. Eventually they should have a good sense of public key cryptosystem so that they can finish their assignment using a computer program to implement their encryption and decryption process.

Problem centred teaching may be used to discuss one or more topics in a course, but it may not be easily used all through a course. I would say problem centred teaching might not be widely used in mathematics courses as well. Since mathematics is always arranged in a logical way, it is very difficult to find a real world problem with which to start a modern mathematics course. On the other hand, it's impossible to find a realistic analogy for many mathematical concepts. How can we find $\sqrt{-1}$ and a 5-dimensional space in the real world? The relationship between mathematics and reality has always been both intricate and intriguing as much complicated as interesting to deal with, and maybe we will never be able to analyze it completely and thoroughly. We might say that it is a relationship of hate and love. As to didactics, this relationship makes the teaching of mathematics quite difficult. Indeed, students are often confused about situations in which it is possible to have recourse to reality and situations where it is not.

There are two motives for developing mathematical theories. One is the real world problem. One is mathematics itself. We may say mathematical theory developed from a vertical and a horizontal direction respectively. The horizontal mathematical theories are always described in a logical route, and contribute nothing to the real world at the time they are developed. However, they may be found to have real world applications some time in the future. If a course is developed in such a horizontal way, then problem centred teaching will not be applied. From this point of view, I think problem centred teaching works for those courses, which are easily seen to have applications, like cryptography, operations research, financial mathematics, biological mathematics, game theory, decision theory. If we modify this method to start with a simple mathematical problem, it may also work for algebraic number theory. Algebraic number theory was originally established to solve Fermat's Last Theorem, a problem that can be understood by a high school student.

Seminar: student teaching

In college and graduate schools, small groups of advanced students engaged in original research or intensive study under the guidance of a professor meet regularly with the professor to discuss their reports and findings. This kind of learning form is called Seminar, which is very popular and effective at postgraduate level. Personally I think the seminar is a good way to teach students generic skills. It can be applied to mathematics teaching at undergraduate level.

Instead of the teacher giving the lecture, the students will act as the teacher. The teacher will be a listener as well as a supervisor. So most of the time it will be a student who stands before the blackboard. The teacher will sit in the classroom and he will also give a talk if it is necessary.

In general the students are placed in groups at the beginning of the course. Before they are assigned a topic from the text, they need enough time to become comfortable working with each other. If they don't know each other quite well, the teacher will give them some easy teamwork to warm up. If all members in the classroom know each other quite well, there is nothing to worry about.

The students will need to know the aims and outcomes of this new system in advance. The students will need to realize the roles they are going to play. In general, the teacher should give a general description of the course, including its background and its correlation with other courses.

The following are some suggestions for a seminar at university level:
1. Students are placed in groups of 4-6 students per group. An even number of students in each group would be preferable.
2. Every group must have a leader.
3. Every group is assigned a topic from the text to teach to the class.
4. Every group has enough time to read, plan, discuss, and prepare their lesson.
5. Every member in the group must do some part of the work.
6. Every member in the group will give a lecture in turn.
7. Every group meets with the instructor prior to teaching.
8. Every group must prepare a handout before their lectures.
9. Every student must give an oral summary to the teacher before teaching.

To ensure that all students are really involved in the program, the assessment may be taken into consideration. For example, the students are informed that other groups will assess each presentation as a group-mark, which contributes to their final personal mark. Furthermore, the students in each group are divided in pairs so that one student is asked to prepare the handout while the other prepares the presentation.

The teacher plays an important role in the seminar. He will keep in touch with the students and help the students to use the library and find references. He will also help the students communicate with each other. Of course he will supervise all through the seminar, and give additional lectures if necessary.

The seminar will help the students improve their generic skills in the following areas: demonstration of understanding of material; ability to explain a topic; creativity in presentation; group unity; communication skill; and writing.

It is hoped that the Seminar will have the following positive results:
1. an increase in mathematics communication among students;
2. an improvement in student ability to read mathematics;
3. an increase in the understanding of mathematics by the students;
4. a greater use of mathematics vocabulary by students;
5. an increased student interest in the class because of the variety in 'teachers' throughout the course;
6. a positive student attitude toward the class and mathematics; and
7. a more 'student-centred' classroom.

However, there are difficulties in making the Seminar successful.
1. How will the teacher know when a group member does not do his part? What will the teacher do about it?
2. How can the teacher help students who are really shy to relax about giving a presentation?
3. How does the teacher manage the daily schedule to allow for student tasks and follow-up exercises?
4. How can the teacher carry out the workload involved in meeting with each group outside of class?
5. How will the teacher assess student learning?
6. How can the teacher motivate those students who have given their presentation to remain involved?

Conclusion

Apart from the two teaching methods discussed above, I think computer technology and concept mapping may be used to aid teaching. Recent developments in computer technology are providing support for new learning environments. These developments make the accessing of structured information sources much easier. The wide range of delivery and access methods accommodate a myriad of learning styles so that teachers can create learning environments to meet different students' needs. Computers enable instructors to make teaching and learning more efficient, more applicable to real world problems, and more accessible to students with different backgrounds.

We use computers in the process of teaching and learning, not only to show the applications aspect of mathematics but also to show the role the computer can play in mathematics. Computer aided teaching methods include program-aided teaching and web-aided teaching.

Program-aided teaching is used when we want to show the computation aspect or geometric aspect of mathematical theories. It's always difficult to explain abstract theory during a lecture.

The experience of mathematics students entails more and more contact with sophisticated pieces of computation and the visual image of mathematical concepts. With the help of software like *Matlab* or *Mathematica*, students find that some heavy computation can be done by computer. And the computer can also show moving 3D images. This will make mathematics 'visible'.

Computer aided teaching is already used in China, but mainly for the third and fourth year students. We hope students may have the opportunity to make use of mathematics software in first year. For example, we could use *Matlab* to study systems of linear equations or integrals. I believe this can not only help the students understand those abstract theories but can also help them maintain interest in learning mathematics.

Another aspect of computer aided teaching is connected with the Internet. But this is different from the commonly known web-based teaching. We can search the Internet for useful mathematics sites, and they can be used as teaching resources[5].

In the past the pattern for teaching mathematics at university has almost exclusively been as follows: theoretical classes with the professor introducing and developing a particular topic and a workshop where students have to solve exercises and problems. It is our goal to change this pattern.

Learning mathematics is not only learning rules, statements and definitions, or demonstrating theorems using them in the solution of problems. Learning mathematics is also solving a challenging problem, trying different strategies and finding a shorter and simpler way to come to an exact conclusion. Mathematics at the university is deeply rooted in the building up of knowledge. The student must understand that he has to study and learn how to do it[6]. The solution of mathematical problems supplies students with techniques, which can be used in different areas and can even be applied to everyday problems. Mathematical thinking is logical and strict, intuitive and creative, dynamic and changing. Students who are not studying to become mathematicians have to be taken into account and motivated. There are many ways to achieve this end.

The learning outcomes depend on an interaction between the characteristics of the student, the teaching style and methods of the teacher, and the policies and practices of the department and institution[7]. There is no doubt the teacher and teaching methods play an important role in the process of learning. I believe that the more we think about teaching the more the students will draw from it.

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