

Using student-centred teaching strategies in calculus

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Abstract

Current calculus teaching in Harbin Institute of Technology is described and student-centred teaching theory is introduced. Based on a comparison of the theory of student-centred teaching and traditional teaching methods that are teacher-centred, student-centred teaching strategies are proposed. These teaching strategies are expected to improve the quality of calculus teaching. The author will use these teaching strategies in his calculus courses when he returns to China.

Introduction

Calculus, one of the common mathematics courses, is a fundamental course for engineering and science students. Almost all engineering and science students study some kind of calculus course. The main course objectives are to give students the concepts and theories of calculus, make them understand the mathematical ideas and to develop their abilities to think logically, deeply and creatively. Also, students' intellectual and imaginative powers should be increased, and they should obtain computation skills, and obtain useful tools for future needs. But few students learn mathematical ideas thoroughly the first time they encounter them. Many students realise that calculus is abstract, and think it is boring and hard to learn. On one hand calculus is very important, on the other hand it is hard to learn, so how it should be taught and learnt becomes a big problem. A number of researchers have studied this problem and published results (Douglas 1987; Alibert 1988; Zhou 2002). These results are helpful in identifying how calculus should be taught and learnt to some extent, but they are still far from solving the problem.

The author took part in the program 'Teaching Science in English', a collaborative program between the University of Sydney and China Scholarship Council. In the University of Sydney, I have systematically learnt some contemporary theories of teaching and learning. These theories have proven to be helpful for teaching and learning in some disciplines. I think that student-centred teaching strategies can be used in calculus teaching. I plan to use these strategies in my calculus courses when I return to China. In this paper, the theory of student-centred teaching and the current style of calculus teaching in my university are introduced, and discussion is focused on how to use student-centred teaching strategies to improve the qualities of teaching calculus.

Current calculus teaching in my university

Calculus is a two semester compulsory course for most of the first year students at my university in China. The course contains 180 hours of lectures and 40 question and answer sessions in teachers' offices. The course topics include functions, sequences, limits, continuity, derivatives and differentials, integrals, differential equations, series, etc.

In my university, a calculus course is usually taught in the traditional way. Teaching is carried out in large classes which may generally have 150-350 students, teachers deliver formal lectures to transmit knowledge, and students watch, listen, take notes and receive the information passively. After classes, students should complete homework every week and can be given feedback on this. If students have questions, they can meet teachers at a fixed time every week in teachers' offices for help. At the end of each semester, students take a closed-book examination, then students gain their final marks from the examination and their homework throughout the semester. The total mark is 100, and its distribution is 80 for the examination and 20 for homework. If a student's final mark is less than 60, he or she fails the calculus course.

Traditional teaching methods are teacher-centred. The teacher plays a leading role and transfers information. 'He or she is regarded as the authoritative expert, the

main source of knowledge, and the focal point of all activity. The student is the passive recipient of the information already acquired by the teacher. The teacher selects from the discipline the information to be taught, studied, and learned.' (Committee on Undergraduate Science Education 1997)

In information transmission, the focus is on facts and skills, but not on the relationships between them. The prior knowledge of students is not considered to be important and it is assumed that students do not need to be active in the teaching-learning process (Trigwell and Prosser 1996). Research has shown the teacher-centred teaching method has some disadvantages, such as not providing an active learning environment for students, diminishing the students' interest and leading in most cases to students adopting a surface learning approach (focusing on rote memory and reproduction), and so on. But the teacher-centred teaching method also has many advantages, such as the ability to deliver a large amount of information quickly. It is an easy and safe way for teachers to teach, most students are accustomed to it, and so on. The traditional teaching method is still considered a preferable teaching method by many teachers, and a lot of success cases illustrate its effectiveness.

In the calculus course in China, the question and answer aspect of learning does not work well. Students seldom come to teachers' offices for help. The reasons may be that students are shy, or that they consider they have already learnt well, or that weak students have too many difficulties so that they do not know how to ask for help, and so on. In large classes, only a few students can get help from the question and answer sessions.

Student-centred teaching

'Student-centred teaching focuses on the student and, in particular, on the cognitive development of the student. The teacher's goal is to help students grasp the development of knowledge as a process rather than a product. The focus of classroom activities and assignments is on the student-centred process of inquiry itself, not on the products of inquiry. Students create their own conceptual or cognitive models. Content, teaching style, and methods are adapted to aid the cognitive and intellectual growth of students. Student-centred teaching combines an understanding of the way that humans process information with other factors that affect learning such as attitudes, values, beliefs, and motivation' (Committee on Undergraduate Science Education 1997). Student-centred teaching may increase student involvement by drawing them into the learning process and help students make the transition from passive listeners to active participants in their own learning. Students learn best if they are engaged in active learning, if they are forced to deal with observations and concepts before terms and facts, and if they have the sense that they are part of a community of learners in a classroom environment that is very supportive of their learning (Fraser 1986; McDermott 1991). Studies have shown that student-centred teaching leads to a strong tendency for students to adopt a deep learning approach (focusing on meaning and

understanding) which then results in good teaching and learning outcomes.

My strategies in teaching calculus

There is a variety of teaching strategies or methods. It is not straightforward to say which one is the most effective way to teach students. In most cases, a combination of different teaching strategies generally leads to good teaching outcomes. Teachers should work out which strategies are suitable for themselves, their students and their course content. There are lots of useful ideas and good methods which are part of traditional teaching strategies. Student-centred teaching strategies are also very helpful in leading to good teaching and learning outcomes. Calculus is a meaning-filled, concept-intensive course. So the aim is to combine traditional teaching strategies with student-centred teaching strategies in my calculus teaching. The course will be re-arranged. Lecture time will be reduced and tutorials will be added. The course will contain 120 hours for lectures and 60 hours for tutorials. No question and answer sessions will be needed.

Using student-centred teaching strategies in my lectures

Calculus will be taught mainly by formal lectures. It may be suitable to teach factual knowledge, such as limit calculation, differential calculation, integral calculation and so on, by traditional lectures, so I will teach these in the traditional way. There are many concepts, ideas and theories in calculus. Some of them are hard to learn and would be better taught with student-centred teaching strategies. Student-centred teaching strategies will be adopted in my lectures to help students to understand them in depth, but these strategies will not be used all the time and in every aspect of the teaching process, because time is limited and the amount of materials the students need to learn is huge.

How can student-centred teaching strategies be used in lectures? Some methods, such as presenting interesting problems, asking questions, giving students time to think, and so on, will be used to make students active and interactive in my lectures. Much attention will be paid to students since they are indicators of learning. Students should participate in the learning processes and become active learners. Students learn best when they engage in the learning processes.

Here is an example. The concept of derivative is at the core of calculus and modern mathematics and usually taught with a didactic approach. The definition of the derivative can be approached in two different ways. One is physical (as a rate of change), the other is geometrical (as a slope of curve). I will teach the concept of derivative by using student-centred teaching strategies. During my lecture, students will be asked to recall the formula of velocity of constant motion (straight line). I am sure that students can get the correct answer: the velocity is equal to the result of the distance traveled divided by the elapsed time. Then students will be asked to notice the relationship between average velocity and instantaneous velocity in constant motion. After that, students will be requested to calculate

the instantaneous velocity of changing motion (straight line). I will give students time to think about the problem and allow them to discuss it with their neighbours. In this case, the above formula does not apply directly. Students may think about studying the average velocity of changing motion and see what happens when the elapsed time gets smaller and smaller. They may naturally run into the concept of limit and then the instantaneous velocity of changing motion will be found. If students cannot discover the result for themselves, I will assist them. This concept of velocity may be extended to find the rate of change of any variable with respect to any other variable. Students can then give more examples and the class can then study the geometrical approach. Through thinking, discussing and teacher's help, students may discover that the above cases are essentially the same by eliminating their physical or geometrical meaning. They may discover for themselves that every result obtained above is the limit of average rates of change. This limit is defined as a derivative.

In the above example, students are motivated to acquire knowledge by actively participating in the learning process. Active learning will engage the students' mind in constructing meaningful knowledge of lasting benefit. Active participation by students helps them construct a better framework from which to generalise their knowledge. I believe that students will benefit a great deal from this participation. They will understand the concept of derivative better. They will realise the importance of derivative, and directly, they will develop abilities of thinking, reasoning and analysing.

Group work in tutorials

Tutorials will be added to calculus courses and carried out in small classes, each of them contains about 30 students. Tutorials should be student-centred. In tutorials, we should pay much more attention to students. Students are expected to become active learners while teachers are not the knowledge transmitters but act as learning supporters. There are many teaching methods that can be used in tutorials. Here I focus on group work. Group work is an effective way for many students to develop their conceptual frameworks and to learn problem solving skills as they try out their own ideas on other students and the instructor. The following is an example.

The definition of convergence of a sequence is a fundamental concept of mathematics analysis. The definition is given as follow:

Definition: A sequence $\{a_n\}$ converges to the number L if for each $\varepsilon > 0$ there is a corresponding positive integer N such that

$$|a_n - L| < \varepsilon, \text{ whenever } n > N.$$

The number L is called the limit of the sequence $\{a_n\}$. It is written as

$$\lim_{n \rightarrow \infty} a_n = L$$

The above definition contains symbolic language called as mathematical language. If the above definition is described by using natural language, it can be stated as follows: when

n is large enough, the number a_n can be made as close to L as one pleases. This description is easy for students to understand, but vague. The above mathematical language is precise and can be used for proof, but it is difficult for students to understand. In my university, new students usually learn this concept in their second lecture. It is the first time students learn this concept. Few students can learn it well when they first encounter it. Students can hardly understand the meaning of mathematical language. In order to help students to understand the concept of convergence and the meaning of mathematical language deeply, student-centred teaching strategies will be used. After my lecture, in which the concept of convergence is taught, a group work exercise in a tutorial will be arranged. The focus is on meaning rather than solely on facts. The class will be divided into groups (3-5 students) and some suitable tasks be assigned. Students will then be asked to work with each other in their groups and work out their results. The tasks would include the following:

1. State the definition of convergence of a sequence $\{a_n\}$.
2. Explain the meaning of ε and N in the definition. Can we fix ε ? If ε is fixed, what property does a sequence $\{a_n\}$ have? Can we restrict ε as $0 < \varepsilon < 1$? Is the N unique?
3. If we let $\varepsilon = \frac{1}{2^k}$, where k is any positive integer, in the definition, does a sequence $\{a_n\}$ converge?
4. If there is a positive integer N , for each $\varepsilon > 0$ such that

$$|a_n - L| < \varepsilon, \text{ whenever } n > N,$$
 does a sequence $\{a_n\}$ converge? Does a sequence $\{a_n\}$ have any properties?
5. Use mathematical language to describe the definition of divergence of a sequence $\{a_n\}$.

According to the actual situation, more tasks can be given if necessary. The group work will create an active and interactive learning environment for students. In their own groups, students will work with each other, share their ideas, discuss, debate and convince each other. They will be asked to write up solutions on transparencies. One student in each group will be required to describe their solutions to the class using an overhead projector. In this group work, students will be more fully engaged in the learning process. I believe that they will be active and try their best. In the list of tasks above, the last one is the hardest one. Students will run into many difficulties when they do this task. It will be a big challenge for students to do it. But it will give students major assistance in learning mathematical language. I will give them support as they attempt this. Through group work, students will not only understand the concept of convergence of a sequence more deeply, but also develop their mathematical thinking and conceptual abilities. In addition to this, students will develop some generic skills, such as communication skills, collaborative skills, practical skills, etc.

Strengthening students' learning

In order to develop abilities of lifelong learning, students should strengthen their own learning strategies. Two methods are introduced in the following.

Self-study is an important method for students' learning. Students are responsible for their learning processes independently while teachers are only the guides to students' learning. In my calculus course, there is some content, such as differentials for approximation, Newton's method for finding solutions to equations, and so on, which are suitable to learn through self-study. Through learning this content by themselves, students can solidify the new material they have learnt, enhance their learning, problem solving skills and develop their abilities to work on the questions independently. Sometimes, self-study inspires interests in the subject. Some students learn materials by themselves simply because they like the material. Students can develop abilities of lifelong learning through this method.

Writing summaries is another method for students' learning. At the end of each chapter, I will ask students to write a summary. By doing so, students can review the content, deepen their understanding and find the relationships among the various concepts.

Assessment

As we know, assessment influences students' learning. Traditional assessment is mainly determined by the final closed-book examination at the end of each semester. Some students study very hard in the last few weeks for the examination. They often learn by using a surface approach. Some of them successfully pass the examination, but forget most of the knowledge as soon as they have completed the examination. This assessment does not evaluate students properly. We should base more assessments on students' learning processes and assess their understanding at frequent intervals throughout the learning processes, meaning the final examination scores should be reduced in importance. A diverse range of assessment methods can be used during the delivery of courses. Some assessments, such as peer-assessment, self-assessment, instructor-assessment, short, in-class assignments or quizzes, final closed-examination, etc, will be used in my course. I will assess students by grading them. There will be 50 points for the final closed-book examination, 40 points for student's performance in the learning processes and 10 points for the homework.

Conclusion

It will be possible and achievable to improve the quality of teaching by using new teaching strategies, which combine traditional strategies with student-centred strategies, in my future teaching. But misusing new teaching strategies will

lead to bad teaching outcomes, so I will use these new teaching strategies carefully. There are some possible problems with such strategies, such as lack of teaching experience, difficult transition (of students from the position of knowledge receiver to active learner, teachers from knowledge transmitter to learning supporters), more expensive with the implementation, etc. Although there exist some possible problems, I believe that the new teaching strategies will have positive effects on students' learning. I will try to use these new teaching strategies in my calculus courses when I return to China and their effectiveness will be tested.

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