Cultivating the interest of students in higher mathematics courses

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Abstract

We present one way of structuring university study from a student's point of view. We suggest that the teacher should cultivate and stimulate students' interest in the course material, so that learning of some abstract and elusive courses will be more interesting for students. Furthermore we will stress that we can give some interesting mathematics courses for some subjects if the teacher starts the mathematics lecture with an interesting problem. We give a specific example: finding conversation of energy from heat transformation.

Introduction

University student's motivation

As university teachers we should analyze students' psychological character in a number of ways (Entwistle 1998). This enables the construction of courses which the students can follow with interest. This in turn allows us to inspire their passion to study, particularly in the case of abstract courses, such as mathematics, theoretical physics, etc. From my experience, a teacher should know some of the background of his or her students before the lecture is delivered. For example:

- the pre-requisites of the course and the level of students;
- motivation; and
- interest, such as the student's major.

Motivation is the primary force which leads to study by a university student. The stronger the motivation to one course, the more actively and (usually) efficiently the student will learn it. For university students, motivation for studying comes mainly from two sources:

1. things which **interest** the students: things the student is curious about; or things the student knows a little about and wants to know more; and
2. **responsibility** for themselves, their families and society more generally. They must gain some ability to make a contribution to their own potential, their families and society.

For first and second year students, interest and responsibility will alternatively determine action; sometimes interest is dominant, sometimes responsibility dominates. Generally speaking, the role of interest is more important than responsibility for first and second year university students. Inspired by Entwistle's (1998) figure (Figure 1) about the student characteristics, we give one structure of university study as following.

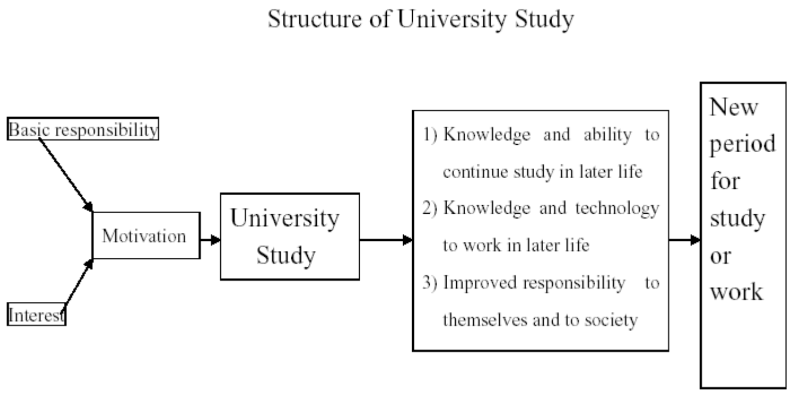


Figure 1. Entwistle's Structure of University Study

Obviously, in order to improve the learning quality and changing the passive attitude of students to the course, teacher should consider how to enhance the motivation of the student (Elton 1996; Järvelä and Niemivirta 1999). For the university teacher, it is natural to pay more attention to one aspect – interest of student – in our teaching in order to strengthen the student's motivation.

As a teacher of mathematics, it is easy to simply force students to study particular courses, or to read particular books so that the student can continue learning at university. Unfortunately, this is ineffective and likely to be boring because a first or second year student usually does not know how the knowledge will be important in the future. Also the student probably will not feel great responsibility or enthusiasm for studying the course, and the teacher may be inclined to advise the students based on their own interests rather than those of the students. A teacher has a responsibility to cultivate and stimulate students' interest in his or her course, then the students will have a stronger motivation to study the course, even where the material being taught is very abstract and elusive. We should stimulate and cultivate the student's interest through our teaching.

Interest of student

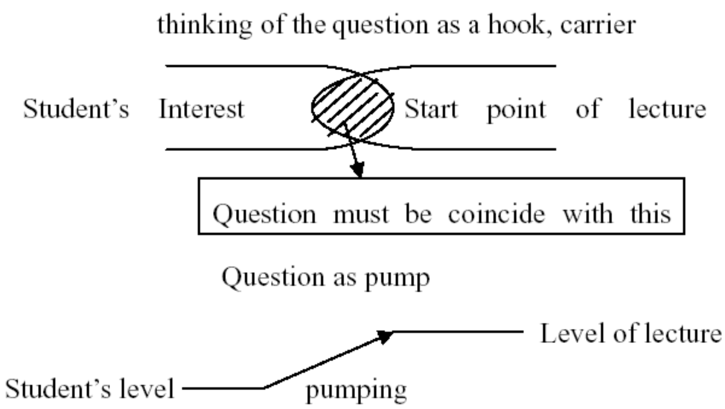
Keeping students interested in the courses is paramount. Without interest students incline to surface level learning (Entwistle 1998; Chin and Brown 2000). With interest they are maybe likely to engage in deep level learning. The purposes of stimulating the student's interest are:

1. attract students;
2. keep students active;
3. increase students' enthusiasm for your course; and
4. very importantly, if their study is interesting, and not always boring, then the student will be keen to study, and will be happy to study.

Therefore the main question is how to stimulate and cultivate the interest of students in our courses. I suggest the following ways.

1. Start your lecture with an interesting problem, which could be either theoretical or a real life problem, based on the students' background and previous experience. The explanation of the role of questions is given in the following diagram:

What is the role of questions in teaching?



i. Questions are a hook, a carrier, a pump.

ii. Direct purposes:
- to stimulate students' interest;
- to attract students and keep students active in lecture; and
- arrange the questions so that students succeed.

iii. General purposes:
- complete the teaching content by small linked steps;
- cultivate student's ability to discover and solve problems; and
- increase the enthusiasm for the course.

2. An anecdote from history about a Wolf prize winner or a Fields Medal winner, for example, I. M. Gelfand, P. D. Lax, S. S. Chern, etc.
3. Visualisation: interesting pictures are useful in geometry and topology, some interesting graphs of a solution of a PDE and other functions are also possible in some courses.
4. Include some important applications of the course.
5. Some powerful and interesting softwares: *Mathematica*, *Maple*, *Matlab*.
6. Interactive web page, our department has established one web page, including calculus, linear algebra, complex functions, mathematical physics equation. Student can find some examples, and some guides of how to learn these courses. Some teachers also present the outline of their course notes on the web page.

Based on the above analysis, the following procedure is a suggested example of how to cultivate and stimulate students' interests. For some abstract courses, we could start our lecture with one interesting question based on students' interest, then continue to analyse the question mathematically, and end by finding the answer to the question and discussing the solution, which can be summarized as shown in Figure 2. This procedure may be regarded as the application of a combination of PBL and student-centred technology (Woods 1994).

Keep student active based on an interesting problem

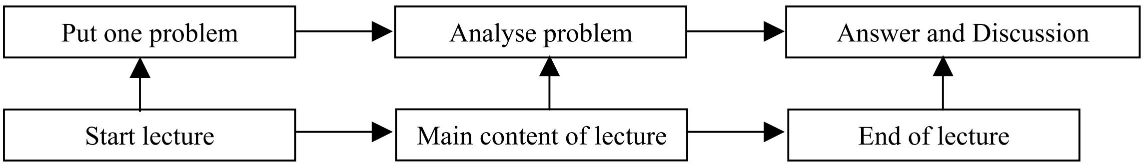


Figure 2. Combining the theories of: PBL and student-centred technology

Specific example: showing conservation of energy from the problem of heat diffusion

This example is used in the lecture of mathematical physics equations for the students from the physics, chemistry and engineering department in our university. The students have usually been taught the conservation of energy in physics courses

Problem: physical system

Consider one infinite length metal rod of uniform thickness along the x axis. Let the heat at $x = 0;\ t = 0$ be one unit, and let U=U(x,t) denote the temperature at $(x;\ t)$.

Develop a mathematical model for the above system: Ask student to reply in lecture

U is the temperature of the rod at $t > 0$

$$\begin{cases} \dfrac{\partial U}{\partial t} - \dfrac{\partial^2 U}{\partial x^2} = 0(t > 0, -\infty < x < \infty), \\ U(x,0) = \delta(x) \end{cases} \qquad (1)$$

My question to the class:

$U(x;\ t)$? U(x,t) is called the fundamental solution of heat equation.

Can we demonstrate the conservation of energy from this model?

Mathematical deduction

This is my main task in teaching, but in some ways it is boring for students. If we begin lectures using this method, this will enthuse students about the lectures because they will want to know the answer to a question they find interesting.

i. Fourier Transformation

$\hat{U}(\xi, t) = \int U(x,t) e^{-ix\xi} d\xi$, we have

$$\begin{cases} \dfrac{d\hat{U}}{dt} + |\xi|^2\ \hat{U}(\xi, t) = 0(t > 0) \\ \hat{U}(\xi, o) = 1 \end{cases} \qquad (2)$$

ii. Solve it, we have

$$\hat{U}(\xi, t) = e^{-|\xi|^2 t}, \qquad (3)$$

iii. The inverse of Fourier transformation,

$$U(x,t) = F^{-1}(\hat{U}) = \frac{1}{\sqrt{4\pi t}} e^{-|x|^2/(4t)} \qquad (4)$$

iv. An application of the fundamental solution.

So the initial question is solved directly. In order to make student understand the distribution of the temperature intuitively, it is useful to show several graphs of $U(x;\ t)$.

Discussion: now students are asked to consider the second question.

Does conservation of energy hold for this model?

What is the limit of $U(x;\ t)$ as t approach ∞? Is your result reasonable from the point of view of physics?

Can you find one physical disadvantage of this model?

By end of this lecture, most students will find the answer by themselves if they have followed my talk in the classroom. In particular, some excellent students can find some disadvantages of this mathematical model.

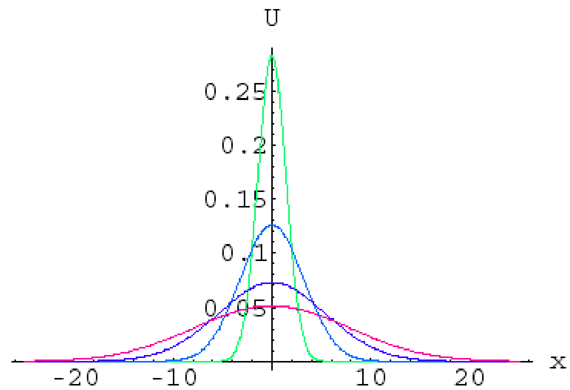


Figure 3. t =1, 5, 15, 30 in order from top

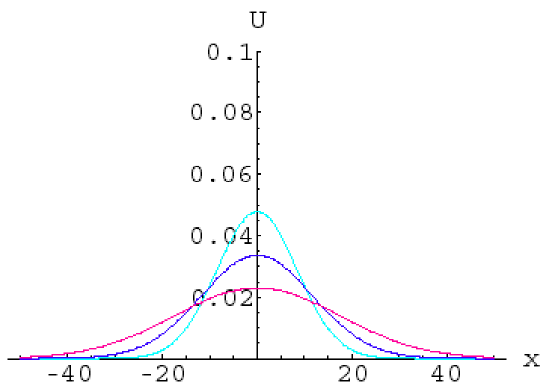


Figure 4. t =35, 70, 150 in order from top

There are two aims of this lecture: one is to establish a mathematical model from a physical system using the δ function, which has been taught to the students in previous lectures. The other aim is to teach students how to find the fundamental solution of the heat equation. In one lecture which lasts for 50 minutes, we will do mathematical deduction for about 20 minutes to solve $U(x; t)$. Before we start to deduce the mathematical formula, students will usually be attracted by the problem. Then the teaching, and the learning of students, will have one explicit final purpose which is to show the conservation of energy. After students get the answer, they are very excited and will be encouraged by their success. We can continue our course by asking them another interesting problem which is 'What are the disadvantages of this model?'. Only five minutes are needed to discuss this question.

Conclusions

It is very difficult to teach mathematical courses because they are often abstract and may be boring for students. In addition, some teachers stress mathematical rigour excessively, so that their mathematics courses are filled with abstract formulas, theorems and proofs. On the other hand, the mathematical background of students is often not very good. Most students, who are not majoring in mathematics, are afraid of mathematics courses. They think mathematics courses are boring and not applicable.

But the argument here is that this is not completely true. If we prepare our lectures from the standpoint of students, and from the consideration of how to stimulate and cultivate students' interest, we will produce better courses in most cases. We will make our teaching in mathematics courses more interesting if we pay more attention to those problems which students like, and which are related to the lectures. In this paper, basing the structure of university study on the students' point of view, we think stimulating and cultivating students' interest in lectures is often a way to make mathematics courses more interesting, which in turn keeps students active in lectures. Furthermore, a procedure is suggested: starting a lecture with one question based on students' interest; solving the question mathematically; finding the answer; and discussing the solution. A specific example is given: showing conversation of energy in heat diffusion.

We must recognize that study of any course is hard work. A student must spend time and stamina on it, and must be prepared to undertake some difficult, possibly boring courses, along the way towards becoming a mathematician and/or scientist. Their study is always motivated by interest and responsibility. As a teacher of mathematics, I have a responsibility to develop each student's interest in studying and to lead the student to feel both success and happiness at various times throughout the course. This will make the student's study active and efficient. If the two aspects (interest and responsibility) in the student's motivation are developed well, the student will be full of enthusiasm for study, even when faced with apparently boring or elusive courses.

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