

Computer aided teaching in *Linear Algebra*

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Abstract

This paper covers three issues related to the improvement of teaching and learning of *Linear Algebra* in the School of Science, Beijing Institute of Technology. First, I will consider the problems currently faced by teachers and students of this subject. Then modern advances in teaching mathematics with computers will be outlined. Finally, I will discuss my plans to change the teaching of *Linear Algebra* in order to partly solve the problems, or at least improve my lectures and increase the learning outcomes of my students. The plan contains two parts: one is to use *MATLAB* in my lectures; the other is to establish a *Linear Algebra* web site.

Introduction

I'm a teacher and currently working in Beijing Institute of Technology. I mainly teach *Linear Algebra*. As is the case for most mathematics lecturers in universities, I'm faced with several problems. These are:

1. over a period of five years, the student numbers have trebled (i.e. three times larger) but the teaching resources have not increased;
2. many students do not like mathematics but have to study it; and
3. the learning outcomes of students are poor, and some will fail the final examination. How can we solve these problems? In this paper I will talk about my plan to solve these problems in teaching *Linear Algebra*.

Modern advances in teaching mathematics with computers

As we all know, computer networks and information technology are developing quickly. Today they have become an important part of life, and are widely used to improve teaching and learning. 'Computer Aided Teaching' has become one of the most important developing directions. The main advances in teaching with computers are:

1. Web teaching and learning systems. Using computer networks and information technology many universities have established virtual education and distance learning systems, such as *WebCT*, *WeBWork*, etc. The web teaching and learning system can make many previously impossible things become realities. For example, students can login to the systems to study at any time and in any place where the Web exists; we can efficiently give instant feedback at any time of the day or night.
2. Mathematics software. Many mathematics software packages have been developed, such as *Maple*, *MATLAB* and *Mathematica*. They have very powerful functions, typically including:
 - mathematical computation;
 - algorithm development;
 - data acquisition;
 - modelling, simulation, and prototyping;
 - data analysis, exploration, and visualization;
 - scientific and engineering graphics;
 - application development; and
 - symbolic calculations.

These functions allow users to easily calculate the products of matrices, matrix inverses, and matrix eigenvalues/eigenvectors; to solve equations, differential equations and minimization problems; they also have extensive facilities for displaying, annotating and printing vectors and matrices as graphs; and provide high-level functions for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. So lecturers can use them not only as calculators but also to show students some aspects of the mathematics that are not accessible with pen and paper alone. For example, two or three-dimensional graphs or animations to illustrate theoretical concepts such as

solutions of linear equations, eigenvalues/eigenvectors, and trajectories, can be made.

My plan for change

Present situation of my class

As I have mentioned in the introduction, in the last five years, the number of students in my class has trebled (200-250 students). All of them are from engineering, computer science or economics. Most students study hard and become at least competent in *Linear Algebra*, but many students do not like the course. The reasons are:

1. it is very abstract;
2. it contains too many new concepts; and
3. they don't know what uses it has.

Nevertheless they have to study it. The learning outcomes of students are poor, and some fail the final examination. My teaching methods are still traditional: textbook; lecture notes; lectures; assignments; and question time. I noticed all the problems mentioned above many years ago, and have made some changes to try to solve them and improve the learning outcomes of students. However, the effects have not been very satisfying.

My plan for change

I am very glad to have an opportunity to participate in a professional training program *Teaching Sciences in English*, sponsored by the China Scholarship Council at the University of Sydney, first semester, 2004. At the University of Sydney I have learned many things, not only the English language, but also how to teach sciences more effectively. So when I go back to my university, the Beijing Institute of Technology, I'm going to make some changes to try to solve the problems that I mentioned above, in order to teach my courses more effectively and improve the learning outcomes of my students.

Use MATLAB to enhance my lectures Graphics

MATLAB has powerful facilities for displaying vectors and matrices as graphs, providing high-level functions for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. Using these functions I will try to make some new concepts, such as solutions of linear equations, linear independence, spanning subspaces, linear transformations, and eigenvalues/eigenvectors easier to understand and more interesting. Hopefully, this will promote deeper learning. For example, the solutions of linear equations with three variables and three equations can be displayed. It is easy to see that every equation represents a plane in three-dimensional space and a common intersection point of the three planes represents a solution of the equations. So the solution of the equations can have three different cases: unique solution (Figure 1), no solution (Figure 2) and infinite solutions (Figure 3).

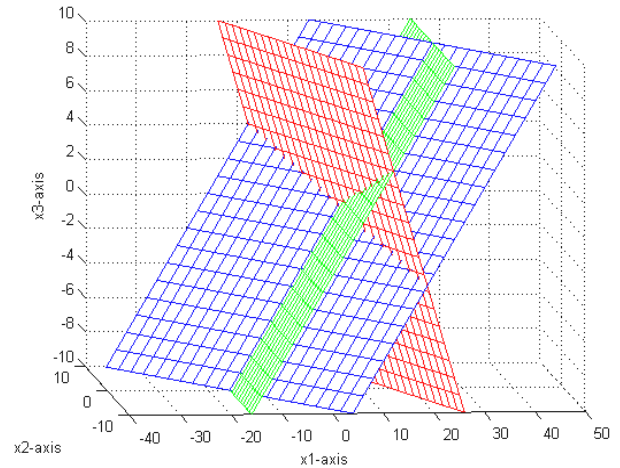


Figure 1. The three planes have a unique common intersection point

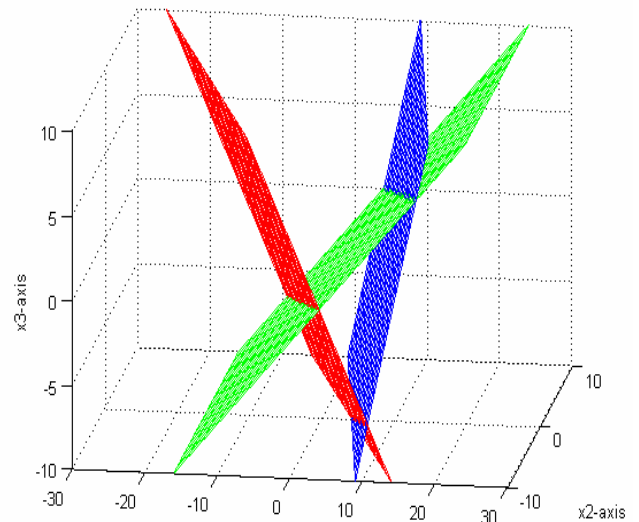


Figure 2. The three planes have no common intersection point

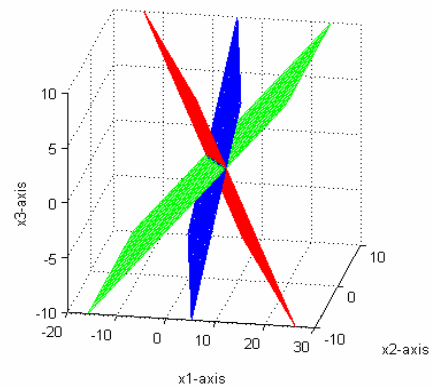


Figure 3. The three planes have infinite common intersection points

Application Laboratory

Using *MATLAB* I will do some linear algebra experiments with my students. In the 1960s, linear algebra was positioned to be the first real mathematics course in the undergraduate mathematics curriculum, in part because it

has extensive applications in many subjects, such as engineering, computer graphics and economics. In particular, all my students are from engineering, computer science or economics. So I'm going to do some linear algebra experiments and demonstrate more applications using *MATLAB*. I hope that it will make my lectures more interesting, and that more of my students will be interested in it. It will teach them how to cooperate with others and how to use mathematics to solve some problems.

Setting up a Linear Algebra web site

- Courseware. Put my courseware on the web, to allow my students to pay more attention to my lectures by listening and understanding, but not wasting time on taking notes.
- Tutorials. Currently I have no tutorials because in my university there are not enough classrooms for it. But I think that tutorials are important for my students. On the one hand, in a tutorial, students have more opportunities to ask their questions and discuss some problems with the tutor face to face; on the other hand, the tutor can know how the students are going, and how effective the lectures are. So I'm going to try to give tutorials on the web site.
- Email. Try to provide my students with a convenient way to communicate with me. Make sure that they can ask me questions and I can efficiently give them instant feedback at any time of the day or night.
- Applications. Put some application examples and exercises, especially those concerned with the students' major, on the Web. Encourage them not only to study mathematics but also to use it.
- Old Examination and Practice Examination Papers. Put old examinations, some practice examinations and their solutions on the Web. Give them more opportunities to practise. Check how they are going by themselves, and let them know what the final examination looks like.
- History. Introduce the history of *Linear Algebra*. Let them know how *Linear Algebra* got started and how it has developed to today. I hope that will help my students understand *Linear Algebra* more deeply.
- Links. Provide them with some useful links, such as some world famous university web addresses and some important mathematical association Web addresses. It will give my students a convenient way to know some of the latest developments in mathematics, particularly in *Linear Algebra*, to communicate with mathematicians worldwide and to search for more learning materials.

Conclusions

In summary, this paper has covered three issues related to the improvement of teaching and learning of *Linear Algebra* in the School of Science, Beijing Institute of Technology. I have considered the problems currently faced by teachers and students of this subject. Then modern advances in teaching mathematics with computers were outlined. Finally, I discussed my plans to reform the teaching of *Linear Algebra*. The plan contains two parts: one is to use *MATLAB* in my lectures, the other is to establish a *Linear Algebra* web site. I hope that the change can partly solve the problems, or at least improve my lectures and increase the learning outcomes of my students.

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