Keep improving the art of teaching in *Calculus*

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Abstract

The previous work in teaching Calculus is described and some methods are introduced which are more student-centred and which make the students more active. Introducing these methods will considerably improve the art of teaching. The author will also introduce a new idea in calculus teaching. That is the idea of approach, the idea of approximation. This will help the students to get a better understanding of limit.

Introduction

Calculus is one of the most important and fundamental courses for students, especially for first year students. *Calculus* is a compulsory course at Northeastern University.

Almost all the engineering and science students are supposed to undertake this course. A teacher is supposed to teach students the concepts and theories of calculus, help them to understand the profound mathematical ideas, and improve their abilities to think logically, deeply and creatively. During this period the students' imaginative powers and computation skills should be increased. Unfortunately many students cannot achieve a deep understanding even when they have finished learning it. It seems to many students that calculus is abstract, boring and hard to learn. They are forced to learn it to pass the examination. Every teacher feels obliged to think seriously about how to teach calculus effectively. A number of researchers have studied this problem and published a great number of articles with original ideas. They have given us many good suggestions but it is far from solving the problem.

The author of this paper took part in the program *Teaching Science in English*, a collaborative program between The University of Sydney and China Scholarship Council at The University of Sydney. In this program the author has learned contemporary education theories systematically, and has been exposed to various teaching methods. These theories and teaching methods are useful, and can be applied to improve teaching. The author has learned that student-centred methods and student-active methods have become a significant trend in teaching throughout the world.

In this paper the author describes his previous work and then finds ways to improve his teaching by using the methods he has learned here. He also makes some suggestions for the future.

Review of the previous work

In the Northeastern University students have to learn calculus in one year, the course is conducted over 200 hours and includes 2-hour lectures and tutorials. In this course students have to learn analysis, differential and integral calculus, series and differential equations, vector calculus and analytic geometry. Students have to hand in their assignments on a regular basis and the teacher is available to answer their questions at a regular time. At the end of each semester, students have to take a closed-book examination and their marks for the course are determined from both the examination and their records throughout the semester. The total mark is out of 100, 80% for the examination and 20% for the record. If a student's mark is less then 60%, he fails and has to take a make up examination.

Mathematics has its specific characteristics. It is hard to understand, rich in content and especially, it is arranged in a logical order. For the majority of students, it is very difficult to learn independently, especially for the first year students.

In accordance with these specific characteristics we should regard the step-by-step method as the main method in calculus teaching. However we can combine this with other methods to get a better understanding.

Student-centred method

The author has been using a student-centred method and student active method subconsciously, always considering the students' point of view when deciding how to teach.

Since students will be exhausted during a long lecture, I break the whole into parts. I teach them a small part and then give them time to practise, and use the newly acquired knowledge to solve a minor problem. After the students have solved the problem some students will be selected to make presentations. They are asked to explain the problem they have solved on the teaching platform. Almost all the students in my classroom have the experience of performing in front of the whole class.

By judging whether or not they have learned I can decide to go over the material or move onto the next part. If they don't understand I will go over the material from a different angle to explain it again.

I can get feedback promptly; they can let me know whether they have understood or not. I sometimes explain a difficult topic time and time again until most of the students understand.

In my class students are sure to understand and learn something. Of course the understanding may be superficial, but they can get a better understanding by thinking carefully, reading reference books, and solving problems. Or I can review the material in the following days, or understanding may simply come over time. Remember that we understand something gradually with growth, development and experience.

We can do our best to help students to understand, but sometimes understanding needs time. We need to go over this concept, need to be in contact with this concept frequently and understand it gradually after having learned a concept.
So we need to create a lively and relaxed atmosphere in a classroom. So students feel they can learn with ease, and they don't hate attending, or learning.

I find that we have paid too much attention of theory, at the cost of applied problems. We have taught too much perfect mathematics, and neglect the idea of approximation. We should make some adjustments.

Improvements of teaching methods

I have found many better methods at The University of Sydney which can help students get a better understanding of some important concepts, and make students more active and teaching more student-centred. I can give some examples here.

The first one is how to explain Fourier Series. Many students find it difficult to understand the idea of Fourier Series. Although they can calculate Fourier coefficients and even expand a periodic function into a Fourier Series, they don't know what Fourier Series is about.

I tell my students that Fourier Series is simply the use of one function to approximate another function; that is using a trigonometric function to approximate a periodic function. But students find it difficult to see the relationship between the Fourier Series and the original function. Now I believe that their lack of understanding is due to the fact that they rarely plot the graphs of Fourier Series.

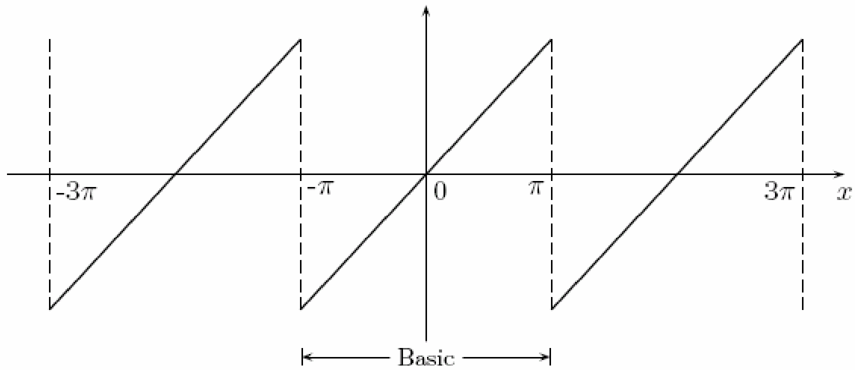
Now I will ask them to plot the graphs as it is done here.

The following graph is copied from a textbook at The University of Sydney. From this graph one can see clearly how a function is approximated by another function.

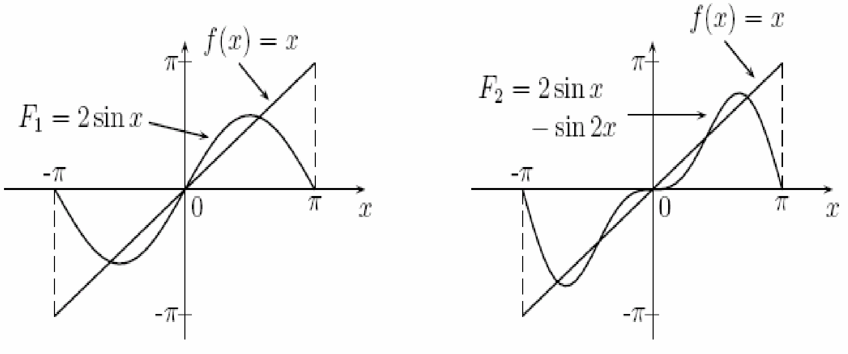
I can ask students to draw the graphs, and they will see for themselves how the Fourier Series of a function approaches that function.

Example: find the Fourier Series of:

$$f(x) = x, \quad -\pi < x < \pi \quad \text{(Basic Section)}$$
$$f(x+2\pi) = f(x), \quad \text{for all } x \quad \text{(Periodic Extension)}$$



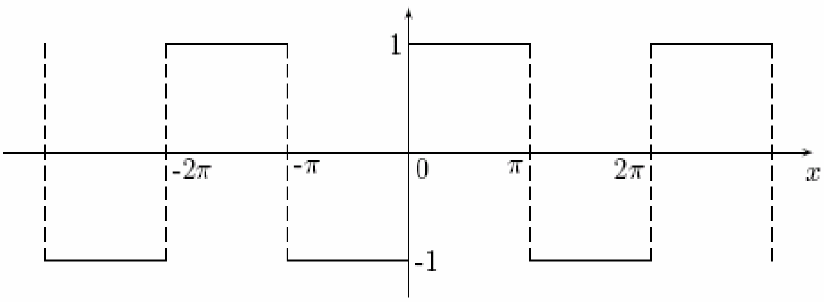
The following graphs show the first and the second partial sums.



1^{st} partial sum 2^{nd} partial sum

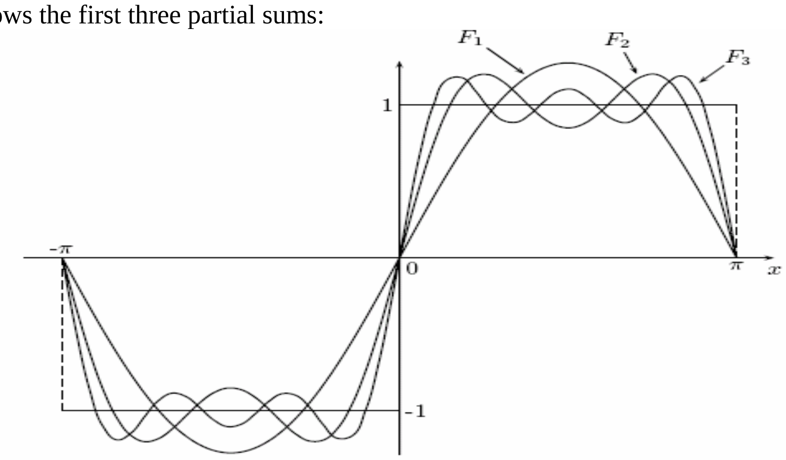
Here is another example.
Find the Fourier Series of:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases} \qquad \text{and } f(x + 2\pi) = f(x) \text{ for all } x$$



Square Wave

The following graph shows the first three partial sums:



From these graphs students can find by themselves that the more terms they choose the closer the two graphs become. They will be asked to draw better graphs and they will become more active.

The other example is the definition of the definite integral. Many students can calculate definite integrals but few students can probe deeply into the essence of definite integral. They are not fully aware that the definite integral is a kind of limit.

The following method I found in a textbook gives me a better method to describe the definite integral.

Example: how far did the car go?

A car is moving with increasing velocity, we measure the car's velocity every 2 seconds and obtain the data in the table.

Table 1. Velocity every 2 seconds

Time (sec)	0	2	4	6	8	10
Velocity (ft/sec)	20	30	38	44	48	50

We can see this clearly from the following graph:

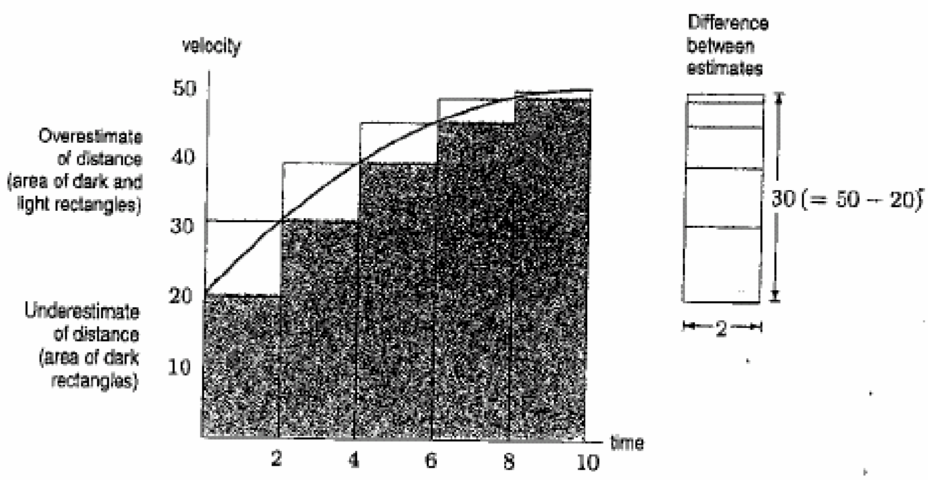


Figure 5.1: Velocity measured every 2 seconds

We don't know how to calculate the distance travelled, but we can get an estimate. We can find an estimate for the least distance travelled by the car, and for the greatest distance. By using the smallest and largest velocities over every 2 seconds we can get:

The car goes at least
(20)(2)+(30)(2)+(38)(2)+(44)(2)+(48)(2)=360 feet.

The car goes at most
(30)(2)+(38)(2)+(44)(2)+(48)(2)+(50)(2)=420 feet.

Therefore we have
360 feet <total distance travelled <420 feet.

That is the distance travelled is in the interval [360,420]. The length of this interval is 60 feet, so the greatest error is 60 feet.

This is a rough estimate. We can get a better estimate by calculating velocity of every second.

Table 2. Velocity every second

Time(sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity(ft/sec)	20	26	30	35	38	42	44	46	48	49	50

We can see this clearly from the following graph:

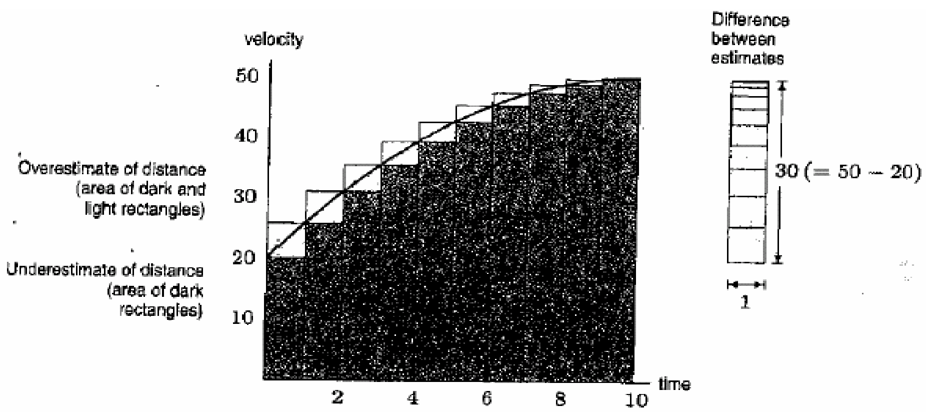


Figure 5.2: Velocity measured every second

This time we have:
The lower estimate is
(20)(1)+(26)(1)+(30)(1)+(35)(1)+(38)(1)+(42)(1)+(44)(1)+(46)(1)+(48)(1)+(49)(1)=378 feet.
The upper estimate is
(26)(1)+(30)(1)+(35)(1)+(38)(1)+(42)(1)+(44)(1)+(46)(1)+(48)(1)+(49)(1)+(50)(1)=408 feet.

378 feet <total distance travelled <408 feet.

That is, the distance travelled is in the interval [378,408], so the greatest error is 30 feet.

This is a better estimate than the previous one.

The students can be asked to make an even better estimate by using the velocity every ½ second, every ¼ second etc. They can see the total distances travelled are within smaller and smaller intervals so the errors become smaller and smaller, and finally tend to a fixed value, this fixed value is the definite integral of velocity with respect to time.

The benefit of these improvements
These improvements will make students more active and more cooperative because they will do it better if they work in groups. This will lead students to grasp the quintessence of calculus, to understand the idea of limit, the idea of approximation.

Students have learned too much perfect mathematics and find it difficult to accept approximation. They don't understand that it is possible to find an exact value by this infinite process. This is the idea of limit which is difficult to understand deeply.

Teaching is a kind of art. We should always take account of the actual situation of the students we teach. If we do so, then we will be successful.

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