

## Student-centred teaching of *Numerical Analysis*

Weiwei Wang

Department of Mathematics  
School of Science  
Xidian University  
Xi'an  
Shaanxi 710071  
People's Republic of China

wwwang@mail.xidian.edu.cn

### Abstract

In this paper, the author considers some contemporary teaching and learning theories for tertiary education, as well as some excellent teaching methods observed at The University of Sydney. The theory and the practical teaching methods motivated the author to make some plans to improve her teaching, especially in the *Numerical Analysis* course. The most important thing is to motivate students and engage them in learning. Also we should help students gain some lifelong learning skills that will be of great importance in their future career. For this purpose, the author plans to incorporate student-centred teaching and learning methods in her teaching of this course. One particular change suggested is to introduce constructive learning. Many methods in this course are constructed rather than defined, and so it makes sense for students to construct these methods under the guidance of the teacher. The other suggested change is to replace the original laboratory work with group research projects to help students gain some lifelong learning skills.

### Introduction

Generally speaking, mathematics is taught and learned in a traditional way. Usually lectures are given in a 'mug to jug' manner, with little interaction and little use of modern technology. Teachers put much emphasis on explaining concepts, theories and mathematical problem-solving techniques, with little attention to practical problems. Students are required to do a great deal of exercises described in mathematical language, and are seldom given real-world problems. Overall, students are not actively involved in lectures but passively follow their teacher and are isolated from the real world in learning. Actually, traditional teaching is teacher-centred. The teacher plays a leading role in transferring information. 'The teacher is regarded as the authoritative expert, the main source of knowledge, and the focal point of all activity while students are passive recipients of information already acquired by the teacher' (Committee on Undergraduate Science Education 1997).

Many students in our department wonder why they should learn some courses, how a particular course can be used in their future work, and what they can do after graduation. Students admit they can't commit themselves to learn a subject if they are not clear as to the use of it. In fact, these students lack motivation and interest in studying mathematics so that they cannot engage in learning. Furthermore, some of our graduates say they have forced themselves to learn mathematics, but they do not retain much and they cannot flexibly use their knowledge in their work. Company managers complain that science students cannot easily adapt themselves to their work, and have little ability for team work, projects, solving real-world problems, etc. What's the problem there? Actually, this is a result of teacher-centred teaching. Research has shown that teacher-centred teaching does not provide an active learning environment for students. It diminishes students' interest and leads in most cases to students adopting a surface learning approach (focusing on rote memory and reproduction). While there is some debate on whether or not it is the universities' responsibility to give students these abilities, most people think teachers should not only transmit knowledge but also cultivate some lifelong learning skills in their students.

How to motivate students? How to develop deep learning? A psychological report (Lagowski 1990) shows that students retain 10% of what they read, 26% of what they hear, 30% of what they see, 50% of what they see and hear, 70% of what they say, 90% of what they say and do. This suggests that students should be involved in teaching and learning as much as possible. The more involved, the more active and motivated students will be. In other words, we should move from teacher-centred teaching to student-centred teaching. 'Student-centred teaching focuses on students and, in particular, on the cognitive development of students. The teacher's goal is to help students grasp the development of knowledge as a process rather

rather than a product. The focus of classroom activities and assignments is on the student-centred process of inquiry itself, not on the products of inquiry. Students create their own conceptual or cognitive models. Content, teaching style, and methods are adapted to aid the cognitive and intellectual growth of students' (Committee on Undergraduate Science Education 1997). 'Students learn best if they are engaged in active learning, if they are forced to deal with observations and concepts before terms and facts, and if they have the sense that they are part of a community of learners in a classroom environment that is very supportive of their learning' (Fraser 1986; McDermott 1991). 'Studies have shown that student-centred teaching leads to a strong tendency for students to adopt a deep learning approach (focusing on meaning and understanding) which then results in good teaching and learning outcomes. Through student-centred learning we can also teach our science students lifelong learning skills' (King 2004).

## Current teaching of numerical analysis

*Numerical Analysis* is an advanced compulsory course for third year students majoring in applied mathematics. Students prior knowledge includes calculus and linear algebra. The course covers such wide content areas as error analysis, polynomial interpolation, approximation of continuous functions in 2-norm or  $\infty$ -norm, numerical integration, numerical solutions of ordinary differential equations, solution of nonlinear equations, solution of systems of linear equations as well as eigenvalues and eigenvectors. The numerical methods covered in this course have great application value. It is definitely a good chance to show applications of mathematics in solving real-world problems. However, most textbooks stress the theory and ignore practical applications. So do teachers. It is really a great pity.

In my past teaching of this course, in order to motivate students, I introduced some applications of certain numerical methods. For example, I showed students some graphs produced by using spline interpolation and told them spline interpolation is applied in computer graphics or CAD to draw curves or curved planes. Students would have some intuition about the application of spline interpolation but that was not enough. In order to deepen students' understanding of some methods, I gave students some problems and asked them to write programs and work out numerical solutions. By doing this, students really deepened their understanding of some numerical problems and improved their ability in programming. But the problems I gave them were mathematical models rather than practical problems described in natural language. So the students were deprived of the opportunity to be exposed to real-world problems. In the future, I plan to incorporate some student-centred learning methods in my teaching of this course and provide some practical problems for students.

## Student-centred teaching of numerical analysis

### Constructive learning

Constructivism is one of the theories which support student-centred learning. This theory assumes that students construct their own knowledge on the basis of interaction with their own knowledge and communication with their teachers. Constructivist teaching models not only emphasise active and collaborative learning, but also emphasise students and teachers discovering and constructing knowledge together. It presents the students with opportunities to construct new knowledge based on their prior knowledge and understanding from previous authentic experiences. This approach encourages students to confront real-world problems which are within their everyday experience. The characteristics of constructivist teaching models include: prompting students to observe and formulate their own questions; allowing multiple interpretations and expressions of learning; encouraging students to work in groups; and the use of their peers as resources for learning.

In *Numerical Analysis*, many numerical methods are given as construction processes rather than axioms or theorems. For example: interpolation polynomials, iteration methods for solving ordinary differential equations, nonlinear equations and systems of linear equations. So it makes more sense for students to learn these methods in a constructive way. For these methods, the traditional lectures will be replaced by workshop seminars, in which students will work in groups to construct the methods rather than the teacher show them how to get the formulae. (The small class size, generally 30-40 students permits us to do that.) In order to conveniently illustrate graphs, programs or some special cases, I suggest the workshop seminars should be run in a computer laboratory. The workshop seminars can be operated as follows: at the start, the teacher motivates students by presenting an interesting real-world problem and providing a program to show how the problem can be solved by the method. Then students may engage in the topic to be discussed. Secondly, students work in groups (three or four students) to formulate the problem (make clear what has been given and what needs to be done) and construct a method to solve the problem under the guidance of the teacher. Here the teacher's responsibility is to help students on the way to get the solution. Next, one representative of each group gives an oral presentation of their answers. The teacher assesses both the presentation and the answer. The teacher should try to encourage the students to give presentations so that students may participate actively in the workshop. Finally, if necessary, the teacher explains the method.

For the theory part, I think it's better to teach and study in the traditional classroom. But there is still something to be improved. For example, after students have constructed the method, I should discuss some theoretical content such as convergence, speed of convergence, accuracy of approximation, stability. In order to help students understand these concepts, I will first ask students to test different sets of data using the program, and then give the theoretical analysis.

**Longer-term projects**

In traditional teaching and learning, students have little group work, so they lack teamwork skills. Also, the university does little in training students writing skills, so that many students cannot report their work in written form. Generally speaking, many science students can't express their ideas clearly and effectively. However society needs these kinds of abilities. Furthermore, students are happy with computer work, so I plan to replace the original computer work with some long term projects.

Students form groups themselves. The teacher provides a list of interesting practical problems from which each group can choose or a group can find a practical problem that the members are interested in. The problems should be interesting, from real life and accessible to the students. The project should be finished in three or four weeks. The groups discuss the project with the teacher and start to do it corporately. They may need to do some research on the Web or in the library. Then they formulate the problem, acquire data, choose some proper numerical methods and make a computer program to work out the numerical solution. After solving the problem, they should write a journal-like paper on their work and give a presentation in the class. During the whole process, the teacher is available to help. Both the paper and the presentation will be assessed by their peers and the teacher. The mark should cover 40% of their final mark. Some concessions could be made for weak students. For example, they could write a review paper to relate and compare different techniques for solving the same kind of problem.

**Concept mapping**

Concept mapping is an effective technique for students to integrate knowledge, to design and articulate a structure of the knowledge they have learned, and to communicate

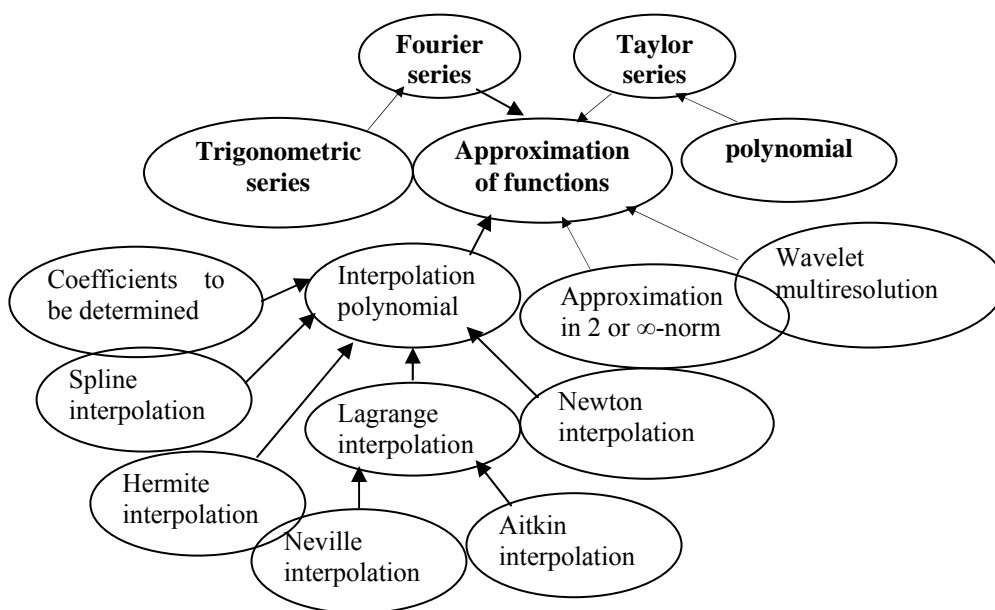
complex ideas. For a teacher, it is also helpful to explain why he or she is focusing on a particular aspect of a topic, so that students can see how particular pieces of information fit into the overall schema. It can also be used as a key plan for the lecturer in working out the best way to teach a topic. In numerical analysis, many methods are closely related. We can use concept mapping to help students to organise their conceptions into a system. For example, before students study this course, they have learnt calculus, so they may give a concept map as shown in Figure 1. When they study this course, they will learn polynomial interpolation and approximation of continuous functions in 2-norm or  $\infty$ -norm so they can incorporate these concepts into the map. Interpolation polynomials include many algorithms which can also be linked. Wavelet analysis is a newly developed approximation method which will be introduced to broaden the students' knowledge.

**Learning and teaching assessment**

Assessment has a great impact on how the students learn. If there is much emphasis on reproducing theories and facts in the assessment, the students will try to remember the facts and learn at a surface level. If the teacher emphasises real understanding and flexible problem solving skills, the student will try to learn at a deep level. So I should modify the traditional assessment and adapt it to student-centred learning. The assessment should change as shown in Table 1.

**Table 1.** Original and new assessment of numerical analysis

Original assessment	New assessment
Homework 10%	Homework 10%
Computer work 10%	Project 40%
Final examination 80%	Final examination 50%



**Figure 1.** Concept mapping of approximation of functions

## Summary

Traditional teacher-centred teaching emphasises the role of teachers but neglects the role of students. In order to cultivate students' lifelong learning skills, we should incorporate student-centred learning in teaching. Based on a consideration of contemporary teaching and learning theory, the author has suggested some student-centred learning strategies to improve the teaching and learning, especially in the course numerical analysis. Hopefully, students will gain some lifelong learning skills. But there may be some obstacles. The students have been used to the traditional teaching and learning. They may think it's the teacher's responsibility to tell them the facts, to give them the formula and to show them how to solve problems. They may be unhappy with these changes. So I should discuss the contemporary teaching and learning theory with the students to get their understanding. However, the most difficult part may come from me. As everyone knows, to change oneself is the most difficult thing. To help effect change, it will be necessary to put the electronic contemporary teaching materials collected whilst in Sydney on the university web site for consideration by my colleagues in China. I will have discussions with other participants in this program, some of whom went to the USA or to England. Anyway, I will try to change the traditional teacher-centred teaching, little by little, to student-centred teaching and learning. In the process of changing, I will find some fair methods to assess the effects of teaching and learning. I still need to get more knowledge of contemporary teaching and learning methods and try to adapt them in my teaching. Teaching is one important part of my job that I love and my students are the future constructors and servers of our country, so I must take seriously my responsibility to do well and cultivate the students to meet the needs of society and the country.

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