‘Involve me, I will understand’: How to improve students’ understanding in the Mathematics course *Ordinary Differential Equations*

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Introduction

When talking about the influence of teaching method on learning, Confucius (551-479 BC), a great teaching scientist in ancient China, said:
Tell me, I will forget; show me, I may remember; involve me, and I will understand.

Among these, the third one seems to be one of the ideas in constructivism. It means that students can understand knowledge if they participate in the process of teaching. But what is ‘understanding’, and how can I make students involved in the process of teaching in my class? This article tries to find the answers.

What does ‘understanding’ mean?

If you ask what is the goal of teaching, most teachers would probably say they want students to understand what they teach. However, ‘understanding’ can mean different things. Traditionally, ‘understanding’ means that students can reproduce content from lecture notes and fulfill the examination requirements. This type of understanding has been questioned by more and more teachers and students in recent years. Even the best students, who have achieved high marks in final examinations, often complain that they don’t know how to apply the knowledge they have learned to real situations; that is, they don’t really understand what they have learned. A student (who was found to have never attended my lecture) wrote to me in 2002:
‘Since I came to university, I have seldom attend lectures. I found it very funny that even some of our university students still care for nothing but passing examinations. Meanwhile, some of our teachers seldom pay much attention to preparing for lectures. They seldom teach students how to think. They say what the textbooks say and test what they have taught. So I am wondering what we can learn in class and what we can do in the future …’

I have to admit that this complaint correctly demonstrates deficiencies of traditional teaching and learning, which usually results in rote learning. Research shows that rote learning is only a retold understanding which eludes conceptual change, and that students lose ownership over their understanding and become alienated from it. This kind of learning can hardly last a long time because students don’t really understand what they have been taught. Now what is real ‘understanding’?

Experts suggest that a good practical test of understanding is being able to explain to someone else, or being able to adapt and to use the concepts that had been learned in a novel circumstance; in other words, to be able to solve real-life problems.

Although some students have excellent ideas about what real understanding means in real life, in practice many things may prevent students from achieving their own personal understanding of the content. For example, much assessment practice appears to reward reproducing. This in turn stimulates students to find strategies (e.g., rote-learning) to cater for the assessment. So how to make students really understand is a crucial problem for teachers. In the next section, based on my experience and on the theories of teaching that I have learned at The University of Sydney, I will suggest some strategies to be used in the course ODEs (Ordinary Differential Equations) in the future. By analysing the importance of modifying curriculum and teaching strategies we can answer the second question posed in the introduction.

Strategies to improve students' understanding in ODEs

Course Description

Ordinary Differential Equations is a compulsory subject for second year students in the department of mathematics at Zhengzhou University in the first semester. The content of the teaching includes: first order differential equations, higher order differential equations, systems of linear equations, Sturm-Liouville theory and stability theory. The teaching material is mainly based on a published textbook. Like all of the other ODEs textbooks available in our library, this book is truly a duplicate from the former Soviet edition. The guiding principle of this book seems to be 'rigorous is best'. It is so abstract that it is not only hard for students to understand, but also for teachers to teach. Furthermore, the applied examples, which are used to show how useful the ODEs is, are very few and outdated (e.g., falling body, electric circuit). So students still have no idea about how to use ODEs to solve current real-life problems. Now, it is time that we (as teachers) make changes. And something must be changed; otherwise we will inevitably lose our students someday.

In the following, I would like to talk about modifying the curriculum first and leave teaching strategies until later.

Modifying the Curriculum

To make students interested and involved in your teaching, you should have an attractive curriculum first. No one wants to learn a course if he/she feels it is not useful, interesting or accessible. I intend to adjust the curriculum of ODEs and add some topics which may attract students and hopefully benefit their thinking and understanding. These topics are discussed below.

Modelling

Mathematical modelling plays a very important role in the history of ODEs. The most successful example is the mathematical models of Newton's laws in mechanics that changed the history of science. So letting students realise the importance of modelling, and encouraging them to find some basic skills about modelling by themselves, should be considered in the course ODEs.

The mathematical modelling approach to problem solving consists of five steps:

1. ask the question;
2. select the modelling approach;
3. formulate the model;
4. solve the model; and
5. answer the question.

Obviously, this is a big challenge for students because it needs not only mathematical skills but relevant knowledge of science as well.

I will design some interesting practical problems for students and encourage them to set up their models. For example, how does an air conditioner control the temperature of a house? Although the principle that the problem uses is very simple—Newton's law of cooling—the process of the air conditioner controlling the temperature is very complicated. Students should consider when the air conditioner switches on or off, and some other important parameters such as the temperatures inside and outside as well. In the process of modelling, students should think as mathematicians or scientists. They must find appropriate ways by themselves. Now, the role of a student changes from a passive listener to an active problem solver. Accordingly, the criteria of the assessment should be reformed. We emphasise students' problem solving ability, rather than their ability to obtain an answer. Students do not necessarily obtain the answers the teacher wants. It is enough if they learn how to think, how to find an appropriate approach to solve a problem, and gain some lifelong learning skills such as communication and discussion in this process.

Numerical Methods

If you open any a contemporary textbook of ODEs used in America, you will find a new phenomenon—numerical methods are included. There are three sound reasons: one is that except for only a few ODEs, most important ODEs in natural sciences either can't be solved analytically or can't easily be solved; another one is that the numerical solutions (approximation to the analytical solutions) do work in practice because many variables are indeed discrete, e.g., population; the third one is that computers provide a powerful tool to do computations. However, textbooks of ODEs in China still neglect the function of numerical methods because we haven't written new textbooks for more than twenty years. So it is urgent to incorporate numerical methods in our course. In this way, students may not be discouraged by the scarcity of analytically solvable ODEs. They may feel that ODEs is really useful in solving real life problems. In this part, I will encourage students to use *Mathematica* to do computations and to sketch graphs of solutions. It may be an unexpected excitement for students majoring in information and computation science.

History

Many students feel that mathematics is boring. One reason may be that mathematics is quarantined from history. Students can hardly find a trace of history in our textbooks. They don't know what happened in history or who contributed what to mathematics. What they can see is only a pile of abstract symbols. So most of them think mathematics is a game only for a few mathematical geniuses, or that it is none of their business. This is, no doubt, very harmful to our students and mathematics itself. In fact, the history of mathematics is very colourful. Almost everything in ODEs has an interesting story. I find these stories are very instructive for our students. Each time when I specify something else and tell the story related to it, I find that students are very interested in who proposed this problem and why, or how mathematicians thought at that time. It doesn't take us much time to tell a story, but it is really helpful to our students. They find that mathematics was created not only by a few geniuses, but also by people who simply put in a lot of effort. Furthermore, through history students may feel the pulse of mathematics and locate the position of a subject in the whole framework of mathematics. Through history mathematics will become a little vivid, or not so boring as students think. More importantly, the events and anecdotes may motivate students.

But experience tells me that the output is not always proportional to the input. Since ODEs has existed for more than 300 years, you cannot expect students to understand all the things in a limited time. It may be necessary to remove or compress some topics, and leave those things which may be tricky and complicated for students to consider outside classtime.

Teaching strategies

As Confucius said, teaching strategies may influence the methods of learning. But among the many teaching methods (e.g., behaviourism, developmentalism and constructivism) we have learned in The University of Sydney, we cannot simply say which one is the best since one person’s meat may be another person's poison. The only standard of good teaching should be whether it fits your students and makes them really understand. Quality teaching should be aimed at promoting deep level processing of information in the mind of the students. Now, I am going to introduce some strategies in my teaching.

Concept mapping and conceptual changes

Constructivism indicates that the process of learning involves the constant construction and reconstruction of meaning and each individual’s construction is necessarily different as it starts from a different point. The construction of meaning is an ongoing and continuous process which must actively involve the learner.

There are many different kinds of differential equations in the course ODEs, such as: separable equations, first order linear equations, homogeneous equations, exact equations, Bernoulli equations. Before students learn ODEs, the only concept of an equation in their mind is an algebraic equation. They don’t know that there are other types of equations. Differential equation is such a new concept that even after learning this part, most of them are still confused by the many different methods for solving ODEs. What they are able to see is only the trees, rather than the forest. In fact, there are logical relationships among them. If you see an equation (concept) as a node, and if you link equation *A* to *B* with an arrow whenever *A* can be transformed into *B*, then you will get a map of the equations. This technique is called concept mapping.

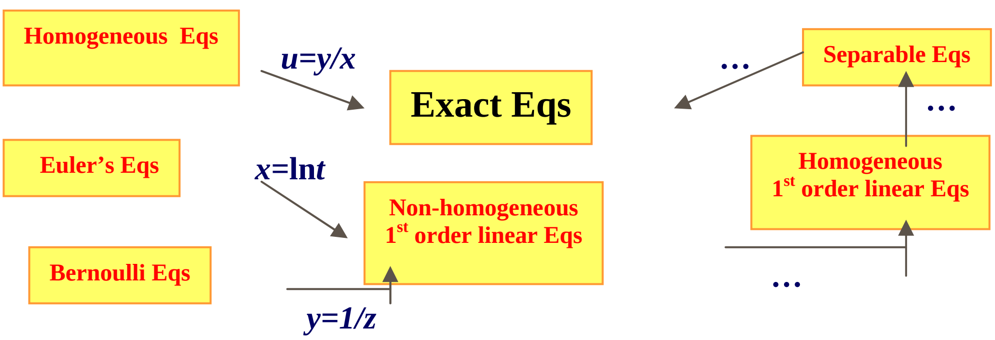
From the map, students will clearly know what it is they have understood, what it is they still do not understand and what needs further study. Instead of drawing the map myself, I will ask students to draw it. Of course, this is usually a nerve-racking problem for students. But it is really helpful for their understanding and construction of new meaning. Once students finish the concept map, they will have a clear pattern in their mind.

I will encourage students to articulate their previous knowledge and the new experiences in order to make them process the information deeply. For example, when I teach first order linear differential equations, I will give the general solution and let students find the general structure of the solution. I will then ask, ‘can you give me the structure of the general solution to the linear algebraic equations?’ By comparison, they may astonishingly find that the solutions of these two different kinds of linear equations (differential and algebraic) have the same structure. Finally, I will make them consider the structures of general solutions to other kinds of linear equations and see what will happen. Hopefully, they will eventually conclude that the general solutions to all kinds of ‘linear’ equations have the same structure, and that the ‘linearity’ is the essential factor in it. In this way, students’ views are widened and conceptions in their minds are changed.

Heuristic Teaching

Encouraging students to conjecture and discover solutions by themselves can really make students think and also change their conceptions. The following is a very sound case I often use.

Before I begin to introduce systems of linear equations $\mathbf{Y}'=A\mathbf{Y}$ where $\mathbf{Y}$ is a vector and A is a matrix, I always make students recall the solution $y=c\exp(ax)$ to the first order linear equation $y'=ay$ where y, a and c are scalars, which they have learned previously. Then I ask them to guess the form of solutions to the first equation, since it has a similar form. They may say it should be $\mathbf{Y}=C\exp(Ax)$. But what does $\exp(Ax)$ mean here? Is C still a scalar? I will give them a few minutes to think. Quite often, they don't have any ideas. Now I will give them some hints such as the expansion of $\exp(ax)$ to induce them to get the expansion of $\exp(Ax)$......In this way, students’ conceptions will change. It leads students to a deep level of learning.



Incorporating Computer and OHT

Teaching instruments are able to affect learning, too. Basically there are three teaching instruments being used at present—'chalk and board', computer and overhead transparencies (OHT). Although it has been suggested that 'chalk and board' should be discarded, mathematicians are still fond of it because mathematics has its own characteristics.

One of the characteristics is its 'logical' nature. Mathematical learning is a process of reasoning. Students should use their previous knowledge and present experience to persuade themselves to believe that what they are listening to is true. It needs time. So the teaching should be at a slow pace. 'Chalk and board' is able to meet that demand. More importantly, during the reasoning process you may often have better ideas than those you have prepared. It is very easy to try it, and very flexible to add or remove something from the board. Of course you may make a mistake, but sometimes even such a mistake can be instructive for students to understand. However, 'chalk and board' does have some disadvantages. For example, you cannot replay what you have erased. That is why I plan to use the computer and OHT as assistants.

'Abstract' is another characteristic of mathematics. Some concepts are very hard for students to imagine (e.g., trajectories to an equation), and this hinders students' understanding. So how to make abstract concepts able to be visualized is a serious problem for us. Very fortunately, many software packages have been developed. With these packages, students are able to make plots in two and three dimensions to illustrate theoretical concepts. Animations can be used to illustrate particular concepts such as trajectory plots and directional fields. I will encourage students to exploit the animating and computing power to achieve a deep understanding of differential equations. *Mathematica* is an example of the type of sophisticated software I intend to apply in my class.

OHT, to some extent, possesses some advantages of both chalk and board and the computer. Even though it is often prepared beforehand, we are still able to write on it if we leave some space. Furthermore, it can be replayed. So effectively using computer and OHT in your class is a wise decision.

Conclusion

In the above, I introduced some teaching strategies aiming at improving students' understanding. Some of them have been proved to be effective in my class (e.g., history and concept map). Some others may meet challenges and barriers. The greatest one may be resistance from students—they will be willing to change the learning strategies they have been used to applying? The other one is how to allocate time to each part if I add some new topics. But students will be the beneficiaries if you pay enough attention to teaching.

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References

Biggs, J. (1999) *Teaching for quality learning at university*. SRHE and Open University press, Buckingham.

Carter, C. (2000) *Higher Education for the Future*. Basil Blackwell, Oxford.

Entwistle, N. (1998) Improving Teaching Through Research on Student Learning. In J. Forest (Ed.) *University Teaching International Perspectives*. New York; Garland Publishing Inc., 73-112.

King, M. (2004) Courses notes: *Teaching Science in English*. The University of Sydney.