

Improving *Advanced Mathematics* teaching by adding some modern methods and ideas

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Abstract

The characteristics of *Advanced Mathematics* teaching at our university are introduced. We argue that the traditional teacher-centred teaching must be improved by incorporating modern methods and techniques. Some modern methods including problem based learning (PBL), concept mapping, 'proof without words', computer aided instruction (CAI) and some other examples are discussed.

Introduction

In the past few years, I have always strived to go my best in teaching my students, and ensuring that they learnt as much as possible. However, because I have had no formal professional training in teaching, often my teaching was based on my own experiences in learning and teaching, and it did not always work as well as I would have expected or hoped.

Through the professional development programme – *Teaching Science in English*, I have become aware of many new ideas and methods. Some of them are powerful and have been successfully used in western universities. I would like to introduce some of them into my teaching. In this paper, I would like to discuss the teaching of *Advanced Mathematics* in my university.

Advanced Mathematics teaching at our university

In our university, all first year students (except for those majoring in mathematics) are required to study *Advanced Mathematics*. It is one of the most important compulsory courses in their four years of study with a credit point value of 12. The main contents of *Advanced Mathematics* are functions, derivatives and differentials, mean value theorem and Taylor formula, integrals, geometry, series and differential equations. The content is almost equivalent to that covered in the courses, MATH 1901, 1903 and 2061 at The University of Sydney.

Advanced Mathematics is a two semester course, three 2-hour lectures per week. There are two hours set aside on each week day in a fixed location for answering students' questions, anyone can come! At the middle and end of each semester, all the students are expected to attend closed-book examinations, which account for 20% and 70% of the final mark respectively, the other 10% coming from records of exercises.

In our university, more than 3000 students are required to undertake *Advanced Mathematics*, however there are only 12 teachers involved in the teaching of the course. All classes have more than 200 students, sometimes with more than 300 students. Because of the large class sizes, teaching is very difficult and it is impossible to interact with the individuals in the class, either during or after lectures. This also forces the teachers to employ traditional methods of teaching – just give the lecture and answer the questions. The role of the teachers and the students are just like the givers and the accepters – teachers give out information and students accept it. However, the teachers in this group have done their best in their teaching, and the students study very hard. Because there have been many great achievements in mathematics competitions, the course has been established as a model course both in our university and Beijing city.

New thoughts that will be used in my course

We all know that mathematics is abstract and difficult to learn. Sometimes the traditional teaching method makes the students feel worse. For improving mathematics teaching, we should take into account the learning principles (Fox and Hackerman, 2003). The change from teacher-centred teaching approach to a student-centred one is necessary. Our goal is to combine the traditional teaching methods with new ideas and techniques to establish a teaching methodology suitable for our university. In my opinion, modern teaching methods are good, but they are also idealistic. One cannot assume that they can be used in all disciplines, anywhere and anytime. Because of the characteristics of mathematics, I think that the traditional methods should not be abandoned completely. We can borrow from these new methods, making our class more interactive and interesting, leading the students to feel that mathematics is very practical and close to real life. In this section, I will outline some methods that I think are suitable for mathematics teaching, and provide concrete examples to illustrate them.

Problem based learning (PBL)

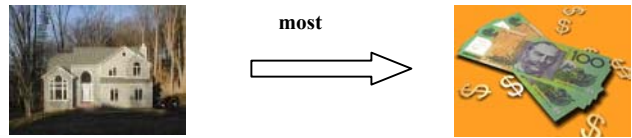
Problem based learning is a curriculum development and instructional system that simultaneously develops both problem solving strategies and disciplinary knowledge bases and skills by placing students in the active role of problem solvers confronted with an ill-structured problem that mirrors realworld problems. (Finkle and Torp, 1995)

One can see that PBL is a learner-centred educational method, which encourages students to become active learners. PBL is a specific strategy for engaging students in collaborative learning. It promotes interaction with others in the group, even outside the group. The problems given to students in this method are based on realworld problems, which can be 'messy' and have no clear answers. The goal of PBL is not to find the solution to a problem but for students to learn concepts and develop critical thinking skills. In this way, teachers move from being providers of content to facilitators of learning.

In *Advanced Mathematics*, there are many topics that are suitable for using this method. The following is one example.

Example 1

Suppose you have 100 houses for rent, how can you earn the most money? Perhaps I will just show the following picture to my students.



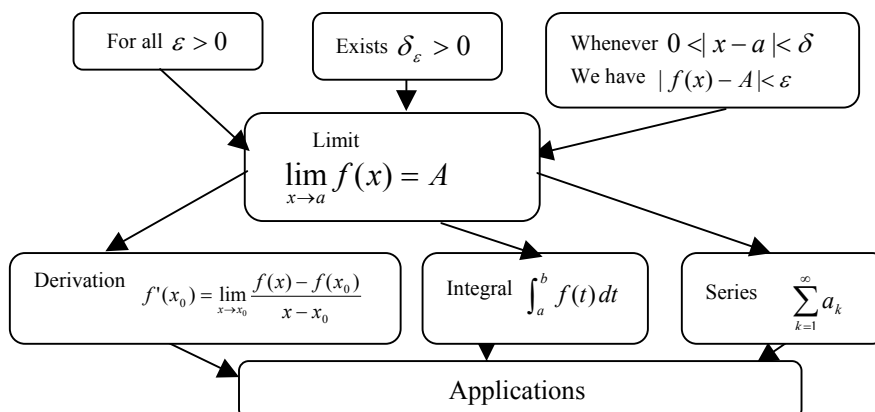
Firstly, I would divide the students into small groups each consisting of 4 or 5 people. To solve this question, the students in the group must first collect information from the property market; they should know the rental prices at present in order to establish their own price. If the price is too high, no one will rent their houses, if too cheap, they will not get the most money. They also need to know at any particular price, how many houses will not be leased, and if one house was leased, how much it may be necessary to pay for repairs. With the necessary information, the students can achieve an income function, and now it is time for mathematics. I will pose this problem at the time I introduce derivatives of the functions, and it will need at least four weeks for them to get sufficient knowledge to solve their equation. As they progress, the students will grasp derivative, differential, the relationship of derivative and the maximum (minimum), and many other important concepts.

Concept mapping

Concept mapping, which is derived from a constructivist approach to teaching and learning, is a technique used for representing knowledge graphically, where the knowledge graphs represent related concepts that are interconnected. Usually, a concept map consists of nodes and links. Nodes represent concepts within a topic, and links represent the relationship between concepts (Lanzing, 1997). It can be used to aid learning, to reconstruct knowledge, to generate ideas, and to assess understanding or diagnose misunderstanding. Concept mapping can be used in *Advanced Mathematics* classes with great effectiveness.

Example 2

The concept of limit is one of the most important topics in mathematics. One has to understand it in order to learn modern science. But this topic is very difficult to teach and is very difficult to understand. Here we can use a concept map to deal with limit.



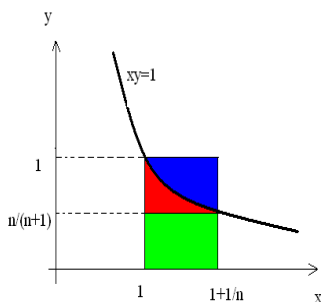
Using this scheme, we can clearly show the conditions in the definition of the limit, and we can explain to students that derivation is defined by a limit, a definite integral in fact is the limit of a Riemann sum, and the series is the limit of partial sums. All of these are the most important concepts in *Advanced Mathematics*, and are the most useful tools for their specialty learning. This will enable students to see the importance of the concept of limit, and make them study much harder!

Proof without words

Before studying in Australia, I never heard of *Proof without words*, but I was immediately attracted to the concept. Although it is not real proof in the strict sense, and there are many famous mathematicians who didn't think it is a kind of proof, but I still think it is a good way to show the students that something is right. Two examples are shown below.

Example 3

A familiar limit for e , $\lim_{n \rightarrow \infty} \ln(1 + \frac{1}{n})^n = 1$ (Nelsen, 2000)



From the geometric meaning of integral and the clear relationship among the areas in the graph, we can see

$$\frac{1}{n} \cdot \frac{n}{n+1} \leq \ln(1 + \frac{1}{n}) \leq \frac{1}{n} \cdot 1,$$

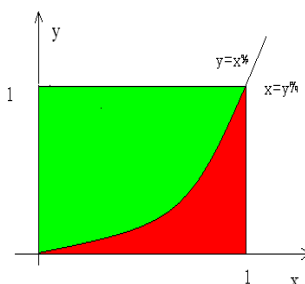
$$\text{then } \frac{n}{n+1} \leq n \cdot \ln(1 + \frac{1}{n}) \leq 1$$

and $\lim_{n \rightarrow \infty} \ln(1 + \frac{1}{n})^n = 1$. It is very easy to see the correctness of the limit.

Example 4

An Integral of a sum of reciprocal powers

$$\int_0^1 (t^{p/q} + t^{q/p}) dt = 1 \text{ (Nelsen, 2000)}$$



Also using the geometric meaning of integral, the correctness of the integral is clear. In fact, the rigorous proofs for these examples are a little difficult.

Computer aided instruction (CAI)

Although we have used computers in our classes in recent years, most of the time we use them as a substitute for the blackboard. We input all the contents into the computer and demonstrate it to the students, write almost nothing on the blackboard. But CAI is not only this, the computer can be used as a tool to strengthen our teaching, like in the example of proof without words.

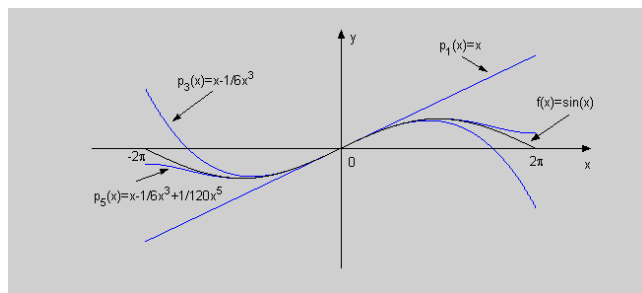
We all know that the Taylor polynomials are a very useful tool to approximate functions, and we always tell students that the larger the derivative order n , the more accurate the result will be. Why?

We can let the student consider the reason from different situations, e.g. from physical interpretation, geometric interpretation, and use a computer to demonstrate that it is right.

Although we can give students the physical or geometric interpretation, it is difficult and time consuming and does not convince the students. However, if we use a computer and some mathematic software, we can show it more directly.

Example 5

We can use polynomials of different degrees to show the approximation properties in a single graph.



Conclusion

I shall be happy if I can teach my course using these methods, and also introduce the methods to my colleagues when I am back in China. I shall be very happy if these methods can improve the undergraduate mathematics teaching at our university!

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