Applications of problem-based learning in teaching *Numerical Analysis*

Liuxiao Guo
School of Science
Southern Yangtze University
Jiangsu Province
Wuxi 214000
People's Republic of China

wxglx2001@263.net

Abstract

In this paper, the author discusses how to use problem-based learning (PBL) strategies to improve the teaching of *Numerical Analysis* in undergraduate mathematics. The current situation of teaching *Numerical Analysis* in Southern Yangtze University is reviewed with suggestions for change.

Introduction

The author of this paper took part in the program *Teaching Science in English* at The University of Sydney. In the program the author has systematically studied contemporary education theories, listened to some detailed mathematics classes, and has been exposed to various teaching methods. Many education theories and teaching methods are feasible and applicable to improve teaching. Especially, the ideas of how to stimulate student-centred learning and lifelong learning impressed the author most.

'How can I get my students to think?' is a question asked by many faculties. PBL is an instructional method that challenges students to 'learn to learn', working cooperatively in groups to seek solutions to real world problems. These problems are used to engage students' curiosity and initiate learning. PBL prepares students to think critically and analytically, and to find and use appropriate learning resources.

In this paper, the author describes the current situation in Southern Yangtze University and uses the approach of PBL to improve the teaching of numerical analysis.

Current mathematics teaching in Southern Yangtze University

It is my opinion, that most mathematics courses in Southern Yangtze University are generally taught and learned in a traditional way. Teachers put emphasis on explaining concepts, theories and mathematical problem solving techniques, with little attention to practical problems. The students are required to do many exercises, but seldom do they do real-world problems. I have discussed this with my students. Many students consider that although mathematics courses are useful, they feel courses which cover topics such as advanced mathematics, linear algebra, and function of real variables are abstract, difficult and too theoretical for them and they do not know how to apply mathematics knowledge to the real world. The *Numerical Analysis* course is no exception.

Description of *Numerical Analysis* teaching

Numerical Analysis is a major basic course for second year students in the School of Information Technology. The course is conducted over 36 hours in one semester. Students' prior knowledge includes calculus, linear algebra and basic computation knowledge. This course covers such wide areas as error analysis, approximation of continuous functions, numerical solution of systems of linear equations, polynomial interpolation, numerical integration and numerical solution of ordinary differential equations. Students hand in their assignments on a regular basis and the teacher is available to answer their questions at a regular time. At the end of semester, students take a closed-book examination and their marks for the course are determined from both the examination and their records throughout the semester. The assessment of this course is out of 100, 80% for the examination and

Mentors
Rui Bin Zhang
Alexander Molev
School of Mathematics and Statistics
Faculty of Science
Lloyd Dawe
Faculty of Education and Social Work
The University of Sydney

20% from the student's record. A mark of less than 60 is deemed to be a failure and a 'make up' examination or repetition of the course is necessary to gain the credits. The numerical methods covered in this course have great application value. It is a good chance to show applications of mathematics in solving real world problems.

In my teaching, subconsciously I have used some teaching strategies to motivate students. Before teaching every chapter, I give students some questions which need corresponding numerical solutions. For example, I give the students the 'airport line' question which needs numerical solutions of linear equation systems and some non-integral or 'difficult to integrate' formulae. I am familiar with the software package *MatLab* and I give the students some basic knowledge of *MatLab* and ask them to write programs and work out the results with numerical solutions. By doing this, students understand the numerical problem more easily and improve their ability in programming. However, I now realise this is not sufficient. Because of the time limitations of this course, it is not feasible to allow students to think up the questions themselves. The teaching methods used are mainly teacher-centred: students often listen to lectures which cover almost all of the required knowledge in detail, and do too much standard homework. They are not fully aware of the relationship between the knowledge from this course and the real world. So in the future teaching, I plan to use some problem-based learning approaches, and allow the students to be exposed to real world problems.

Why we should apply PBL theory in teaching?

What is PBL?

'Problem-based learning is a curriculum design and a teaching and learning strategy which simultaneously develops higher order thinking, disciplinary knowledge bases and practical skills by placing students in the active role of practitioners (or problem-solvers) confronted with a situation (ill-structured problem) which reflects the real world' (lllinois Mathematics & Science Academy 1994).

PBL promotes the students' motivation and makes students more engaged in learning because they feel they are empowered to have an impact on the outcome of the investigation i.e., assume more responsibility for their learning. PBL offers students an obvious answer to the questions, 'Why do we need to learn this?' and 'What does what I am doing in school have to do with anything in the real world?' The methods of PBL may train the students in higher-order thinking skills. The ill-structured problem scenario calls forth critical and creative thinking by suspending the guessing game of 'What's the right answer the teacher wants me to find?'

Why I will try to apply PBL in teaching *Numerical Analysis*

1. *Numerical Analysis* itself is a practical and computation mathematics course which has been widely used in all kinds of engineering. Many numerical methods come from the real world. Compared to other to other mathematics courses, it is relatively easy to find interesting problems to use PBL in *Numerical Analysis*.
2. Many fields of science and technology and even some areas in the humanities need detailed numerical methods. My colleagues from other science departments sometimes ask me questions which require numerical methods to handle. Good resources for real problems exist.
3. Currently the teaching material for *Numerical Analysis* stresses the theory and the education methodology has not met the changing needs of society.
4. The students are responsible for their study. PBL methods let them not just follow the lecturer, but learn how to learn from textbooks and other teaching materials.
5. To increase students' interest and motivation in the *Numerical Analysis* course which is too theoretic, abstract and boring and to make students more active learners and improve students problem solving skills and lifelong learning skills.

How to identify and analyse problems in teaching *Numerical Analysis*

How to identify and analyse problems is very important. The teacher should choose the problems which are context-based using real life situations (learning occurs in those areas in which the knowledge and skills will be used) and focus on thinking skills (problem solving, data analysis, decision making, critical thinking). If the problem comes from the real world, usually, it requires integration of inter-disciplinary knowledge, skills and behaviours (learning is integrated). We also adopt problems which are self-directed, develop lifelong learning skills and can be shared in small groups.

Further more, the problem should promote 'metacognition and self-regulated learning by asking students to generate their own strategies for: problem definition, information gathering, data-analysis, hypothesis-building and testing, and comparing these strategies against, and sharing them with, other students' and mentors' strategies' (Uden and Beaumont 2006).

Two examples of applications of PBL in teaching *Numerical Analysis*

In the first two weeks of the *Numerical Analysis* course, I will give the students closed-problems such as the one shown in Problem 1. This is a rather simple, but practicable problem. I just ask students to discuss it in groups and individually design a numerical method according to the textbook.

Problem 1 (error analysis)

What is the meaning of the sign '500 ± 10 grams' on a food package? Why is it different from the sign '300 ± 5 grams'? If you buy 3 packages with the sign '500 ± 10 grams' and 5 packages with the sign '300 ± 5 grams', what possible error might you have?

To summarise the problem:

1. What is the basic concept of absolute and relative error? and

2. What is the regulation of error transfer?

The next problem is open-ended, real-world and interesting to the students. It requires integration of inter-disciplinary knowledge including a mathematics model, knowledge of population, error analysis, numerical method, etc.

Problem 2 (polynomial interpolation, numerical integration, numerical solution of ordinary differential equations)

The population problem in China is a big problem. Southern Yangtze University is in Wuxi city which is a business city in Jiangsu Province. Allow the students to first read *Population growth in 1999–2003 about Wuxi City in Jiangsu Province* found on the web site *Wuxi Population* at http://jsw.wuxi.gov.cn/. Then find an approximate function for the data, and establish a mathematical model based on the data, which can predict the population trend in Wuxi city, yielding information on total population, births, deaths, population transfer and other elements. Finally ask the students to use numerical methods to solve the model.

With limitations from the course of *Numerical Analysis*, the problem can be summarise as follows:

1. What is the usual method to gain the approximation of a function?
2. What is the basic concept of interpolation and least squares methodologies?
3. How to use numerical interpolation and least squares methodologies?
4. What is the concept of numerical solution of ordinary differential equation?
5. What is n-order Runge-Kutta method?
6. Practicing to establish the population model? (This belong in the course 'mathematics model')
7. After we have established the mathematical model with the form ordinary differential equation, how to use numerical methods to solve the differential equation?
8. In some certain conditions, which numerical method is the best to solve the differential equation (error analysis)?

The numerical analysis theory required to solve the question is mainly two areas of knowledge: 'approximation of function', and 'numerical solution of ordinary differential equation'.

And I will carry out this problem-based project as follows:

1. give students a small lecture, introducing the question in detail, analysing the situation, identifying relevant previous experience and areas of learning, identifying potential learning resources;
2. divide the students into several groups, asking them to discuss the question and suggest a method to solve the problem;
3. provide the students with have the opportunity for self directed learning, asking students to find some references (about the problem and) and to consider the problem;
4. facilitate group discussion where students can design a mathematics experiment to observe and understand the process of the numerical arithmetic;
5. encourage students to analyse the different solutions carefully, comparing them, finding more references and in groups, discussing and sharing their data and graph results;
6. direct students that if there is new information about the problem, go back to (3), otherwise go on to the process outlined in (7);
7. allow students to give their solution and analyse their results, giving a report and a written presentation; and
8. assess the students according to the process and the result of the presentation, including the integration of identified areas of learning, the processes of situation analysis and self-directed learning. This mark would account for 40% of their final mark.

These concepts can be related to one other in order to solve the problem, and the relationships are shown as the Figures 1 and 2. By using concept mapping, the students also could add or modify the mapping or draft their own ideas.

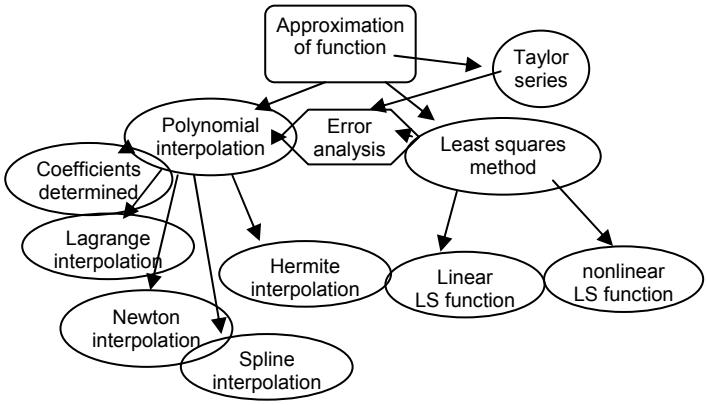

Figure 1.

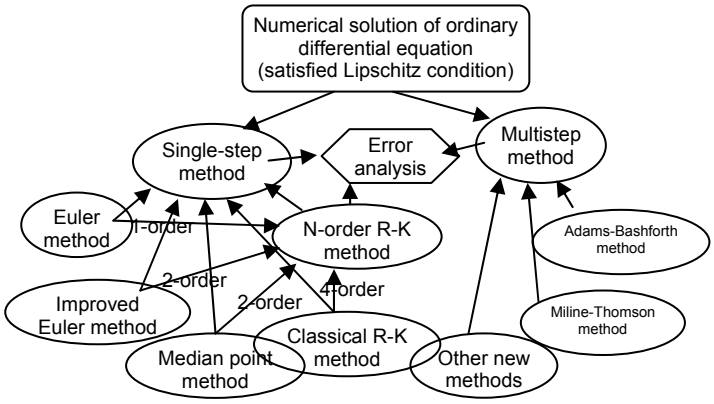

Figure 2.

In the course of *Numerical Analysis*, I plan to give simple closed problems with three or four real-world open-ended problems which would cover most of knowledge points in the textbook. The traditional assessment will be modified. I will adjust current assessment to take account of the problem-based skill components. The new assessment will consist of: homework 10%, problem project 40%, final examination 50%. This assessment will place more emphasis on real understanding and flexible problem solving ability, and student learning should be deeper.

Conclusion

The new PBL approaches will give students more motivation for learning and the students will develop independent learning skills which are important for lifelong learning. In China this is not traditional and rather new for us. Difficulties with PBL exist which we will come across. Good problems must be designed, it demands the teachers display good professional qualities and sensitivity to the relationship between the real world and abstract mathematical knowledge. Students are familiar with lectures and paper examinations. They may feel more comfortable to just follow the teachers. If students are given problems, they may feel confused and not know what to do with problems when they face them at first. (King 2006) How to assess the ability of students in the process of problem project is rather difficult. There are no standard criteria. Teachers just assess students according to performances and written presentation. (Pelaez 2002) Using PBL approaches will be a challenge but will reform the teaching. Our colleagues need to learn current education theories and cooperate with us. I still have much work to do. I would like to learn more contemporary teaching and learning methods. I have the responsibility to improve my teaching because I love my job and my students.

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