Some ideas on how to trigger students' interest in learning calculus

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Abstract

Calculus is a fundamental course for science students. However, many of them have no interest in learning calculus and thus cannot master it because they think it is abstract and useless. This will greatly influence their further learning of other science courses, even their professional future. So, how to trigger students' interest in learning calculus is becoming more and more important. In this paper, some ideas on how to trigger students' interest in learning calculus will be presented. These ideas include: (1) adding mathematical history to trigger students' curiosity; (2) using problem-based learning (PBL) to make strong connections to the real world; and (3) using moving pictures to make concept more understandable and problems vivid. Examples are included to illustrate these ideas.

Introduction

Calculus is a fundamental course for any students who wants to study science. However, many students cannot understand the essence of calculus. One of the reasons is that they have no interest in learning calculus. Therefore, they cannot master the theory. Thus it will greatly influence their further study of science. Mathematical teachers should be aware of that and try their best to make their lectures interesting to trigger students' interest.

Students who failed to pass the final calculus examination tell me that they have no interest in learning calculus, because they think it is difficult and of little use. However, many mathematicians around me are of the opposite opionion and thought it interesting when they recall their past experiences of learning calculus. Interest is the best incentive for students to learn knowledge. Triggering students' interest is one of the key factors in learning calculus. Triggering their interest is easy to say, but difficult to do. In this paper, some ideas on how to trigger students' interest are presented. These ideas include: (1) adding mathematical history to trigger students' curiosity; (2) using PBL to make strong connections to the real world; and (3) using moving pictures to make concepts more understandable and problems more interesting. Examples are included to illustrate these ideas.

Adding mathematical history

Many students will feel drowsy when they attend a mathematical class especially during the afternoon. If teachers can tell an interesting story which is related to the teaching content, e.g., mathematical history, students would be excited and teachers can easily grasp their attention. So mathematical history is good material with which teachers can make their lectures interesting. Interest comes from curiosity or from the scene of success, e.g., understanding a difficult concept. Here I'd like to give an example to illustrate how it might be done.

The 'limit of a sequence' is a basic concept in calculus and it will be used in the later content of calculus and other mathematical courses. But many students are confused by what the limit is, even if the rigorous $\varepsilon - N$ definition is given. In fact, the mathematical concept of limit is a particularly difficult notion. One of the great difficulties in the teaching and learning of the limit concept lies not only in its richness and complexity, but also in the extent to which the cognitive aspects cannot be generated purely from the mathematical definition (Tall 1991). Remembering the definition of a limit is one thing, acquiring the fundamental conception is another. To help students to better understand the essence of the limit and its simple application, teachers can introduce the history of how people find the numerical approximation of circle rate π. Teachers can deliver it as the following steps:

1. Make it clear to the students what the circle rate π is, i.e., what is the definition of the circle rate π. Many Chinese students first meet the circle rate π when they are in primary school. At that time, they just know that π is a constant 3.14159…. I have even asked some college students what the circle rate π is and few of them can give the correct answer. It is necessary to make it clear to them.

Figure 1. Liu Hui and his *The Nine Chapters on the Mathematical Art*

2. It is important to understand the history of how people found the numerical approximation of the circle rate π. The material can include the following:
   (a) long ago, people had noticed the circle rate π;
   (b) Chinese mathematician Liu Hui and his 'Jiuzhang Suanshu' or *The Nine Chapters on the Mathematical Art*;
   (c) the work of Chinese mathematician Zu Congzhi who deriving two approximations of π, which held as the most accurate approximation for π for over nine hundred years, his best approximation was between 3.1415926 and 3.1415927, with (密率, Milü, detailed approximation) and (约率, Yuelü, rough approximation) being the other notable approximations;
   (d) Western country's work; and
   (e) modern works.
   The above (b) and (c) are the best materials of patriotic Chinese students. It is helpful to include Figure 1 in the lecture.
3. Students also can find the approximation of π using Liu Hui's mathematical art. For convenience, we consider a circle with radius $r=1$. If we use the perimeter of square to replace the circumference, the approximation of π is $a_1 = 4\sin 45^\circ$; then we use the perimeter of normal octagon to replace the circumference, the approximation of π is $a_2 = 8\sin\frac{45^\circ}{2}$.

   Generally, we use the perimeter of normal polygon with 2^{1+n} sides to replace the circumference, the approximation of π is $a_n = 2^{1+n}\sin\frac{45^\circ}{2^{n-1}}$. Clearly, the perimeter of normal polygon is more and more approach to the circle, then a_n is also closer to π as n increases. See Figure 2 for more details.
4. The limit of $\{a_n\}$ is exactly π. From the above analysis, students would know all the $a_n < \pi$ and as n goes to infinite, a_n become larger and larger, and should eventually be π, i.e., $\lim_{n\to\infty} a_n = \pi$. Table 1 can be provided to make it clearer for the students.

Table 1. Approximation of π

n	The corresponding sides of polygon	The approximation of π
1	4	2.828427125
2	8	3.061467459
3	16	3.121445152
4	32	3.136548491
5	64	3.140331157
6	128	3.141277251
7	256	3.141513801
8	512	3.14157294
9	1024	3.141587725
10	2048	3.141591422
11	4096	3.141592346

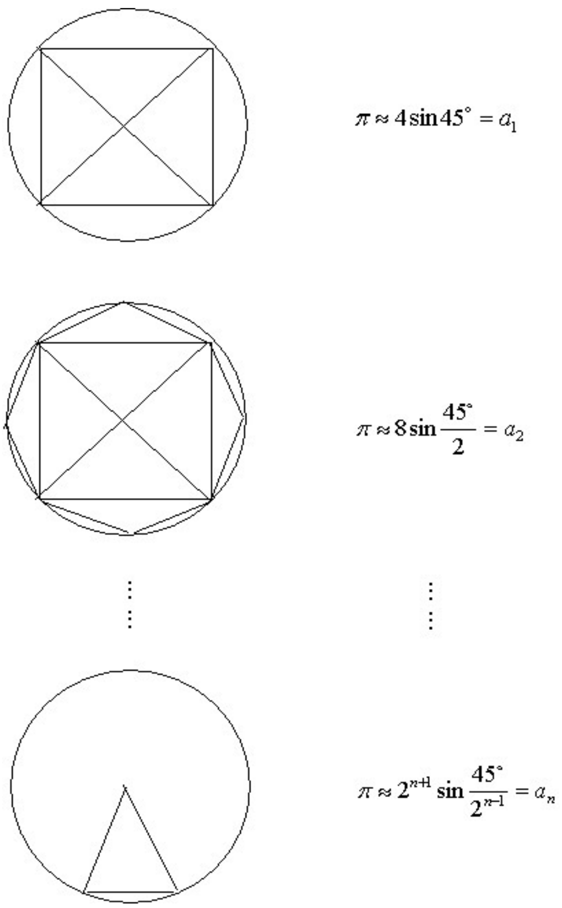

Figure 2. Using the perimeter of polygon to replace the circumference, then get the approximation of π

Some students would ask how to calculate $\sin\frac{45^\circ}{2^{n-1}}$ without computer. Teachers can tell them as follows: noticing the formulas $\sin 2\theta = 2\sin\theta\cos\theta = 2\sin\theta\sqrt{1-\sin^2\theta}$,

then $\sin\theta = \sqrt{\frac{1}{2} - \sqrt{\frac{1-\sin^2 2\theta}{4}}} = \frac{\sqrt{1-\sqrt{1-\sin^2 2\theta}}}{\sqrt{2}}$.

Hence,

$\sin 45^\circ = \frac{\sqrt{2}}{2}, \sin\frac{45^\circ}{2} = \sqrt{\sqrt{\frac{1}{8}} + \frac{1}{2}} = \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$,

$\sin\frac{45^\circ}{4} = \frac{\sqrt{1-\sqrt{\frac{2+\sqrt{2}}{4}}}}{\sqrt{2}} = \frac{\sqrt{2-\sqrt{2+\sqrt{2}}}}{2}$,The all the $\sqrt{a}$ can be calculate by hand, so do all the $\sin\frac{45^\circ}{2^{n-1}}$. In fact,

$$\sin\frac{45^\circ}{2^{n-1}} = \frac{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ddots\sqrt{2}}}}}}}{2}$$

where the numerator has n root $\sqrt{\ }$. This means students could be Liu Hui or Zu Congzhi.

5. If time permitted, another method to approach π can be introduced. Figure 3 has given the outline of the idea.

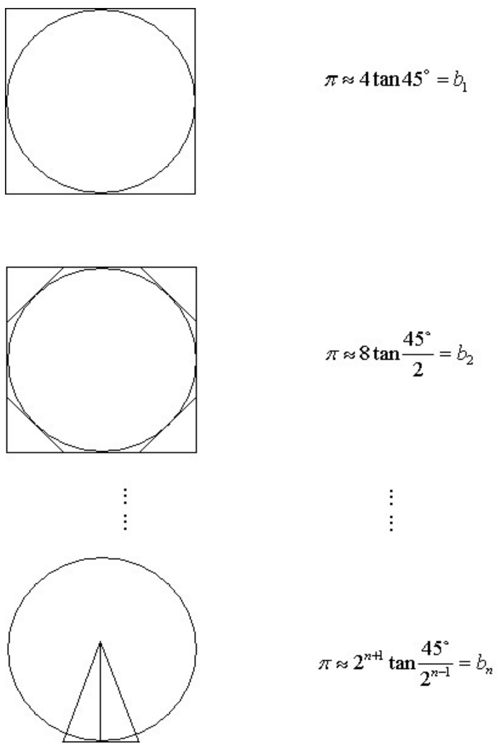

Figure 3. Another method to get the approximation of π

Similarly, $\tan\theta$ and $\tan 2\theta$ have the following relationship $\tan\theta = \frac{\sqrt{1+\tan^2 2\theta} - 1}{\tan 2\theta}$, which makes us possible to calculate $\tan\frac{45^\circ}{2^{n-1}}$ by hand.

In this example, students not only see a vivid case of limit but also receive a patriotic education.

Using PBL

PBL is a pedagogical strategy for posing significant, contextualised, real-world situations, and providing resources, guidance, and instruction to learners as they develop content knowledge and problem solving skills (Mayo, Donnelly, Nash and Schwartz 1993). In PBL, students collaborate to study the issues of a problem as they strive to create viable solutions. Unlike traditional instruction, which is often conducted in lecture format, teaching in PBL normally occurs within small discussion groups of students facilitated by a faculty tutor (Aspy, Aspy and Quimby 1993, Bridges and Hallinger 1991).

Because the amount of direct instruction is reduced in PBL, students assume greater responsibility for their own learning (Bridges and Hallinger 1991). The instructor's role becomes one of subject matter expert, resource guide, and task group consultant. This arrangement promotes group processing of information rather than an imparting of information by faculty (Vernon and Blake 1993). The instructor's role is to encourage student participation, provide appropriate information to keep students on track, avoid negative feedback, and assume the role of fellow learner (Aspy et al. 1993).

PBL involves learning through tackling relevant problems. This is distinct from learning how to solve problems. In PBL the problem may not be solvable, but nevertheless provides a rich environment for learning. This aim is to learn rather than to solve the problem. In PBL students work real-world problem cooperatively, which are very often large scale and interdisciplinary.

Interest comes from curiosity for a child, but for adults, real-world problems and practical problem can make them interesting, because these problems may be related to their personal development and hobbies. Therefore, the challenge in PBL is good material that will trigger students' interest.

However in the course of *Calculus*, not all the content can be taught by PBL. I believe that the ordinary differential equation connects more tightly with the real world and some parts of it can be taught by PBL.

Here we give a concrete case to illustrate the method of PBL, which may be applicable to the other part of *Calculus*.

Problem

The population problem is becoming more and more serious, please develop a mathematical model to mimic the trends in population and predict the population for the following 20 years. Provide some suggestions to decrease the increasing population. Can your models be applied to other problems? If 'yes', what are they?

This problem is ill-defined and interdisciplinary. To provide a solution of this problem, students should add additional assumptions and research data about the population. Teachers can guide students by using a number of steps.

1. Give some background about the serious problem of population which can be delivered in an interesting way if teachers have suitable material.
2. Divide the learners into several groups with each group having four to six people.
3. Explain to them that they may give more than one model and these models can be more and more complex.
4. If the students are really confused, provide them with hints about the factors which influence population, e.g., birth and death rate.
5. Encourage those students who can give the differential equation $\frac{dP}{dt} = kP$ to solve it using separable variable. Using this model let them know how fast the population increases as time goes on and the defect of this model.
6. Remind students that the population cannot increase infinitely because of the limited resources on the earth. Students should consider other factors.
7. Lead students to create the equation $\frac{dP}{dt} = kP(M - P)$, and solve it. Draw the different graphs of function $P(t)$ according to its different initial conditions.
8. Compare the above two models especially their model assumptions. Tell them about the Malthus' Model and Logistic Population Model. Let students discover both the advantages and the disadvantages of the two models.
9. Generalise the model to other fields such as contagion disease model, the spread of new technique, the spread of rumour, etc.

From the investigation of the problem, students can:

1. learn the definition of differential equations and how to solve the separable variable equations;
2. learn the relationship between mathematics and the real world;
3. learn that mathematics is very useful and interesting, even if it is abstract;
4. learn that problems which have different backgrounds may have the same method in order to solve them on the condition that people make their real meaning clear; and
5. develop a deeper understand of the world.

Using moving pictures

People know that describing an object using a picture is clearer than using only words. If we add not only pictures but some moving pictures when we explain a concept or a problem, it would make the concept or problem more interesting and students will find it much easier to understand.

Few students dislike watching TV or movies because there are wonderful moving pictures and sound in them. However, many students dislike listening to mathematical lectures because they feel it is boring although they know they should learn it. So why not let students study mathematics in a relaxed environment? Could teachers include movies or some kind of moving pictures in their lectures? The answer to the above question is, at least partly, yes. Making movies or files requires a lot of money and time. Yet if it can make the study of mathematics more enjoyable, then it is worth it. To save money, we can do it using a computer, which means we make digital moving pictures.

Many students say that they cannot understand some complex space curve and know nothing about the intersection between the two different space curves, even though teachers provide them with the corresponding drawings. If teachers give them a moving picture, which shows how the two space curves intersect, they would understand it better. Generally, the student would have more interest in the moving pictures than just words. So the students' interest in learning mathematics would be triggered by the moving pictures.

Conclusion

In this paper, I have introduced three methods to trigger students' interest in learning calculus. Of course, there must be other methods which are not mentioned here. If these methods can be properly applied to the problems and concepts in calculus, students develop a greater interest in calculus. Although these methods require teachers to have a wide knowledge of mathematics, pedagogy and technology etc., it is beneficial for students to learn knowledge with interest and form a good habit of lifelong learning. Certainly, these ideas can be used in other science courses.

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