Relativity

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Introduction

The 1800s saw a rapid increase in scientific understanding of light and electromagnetism, culminating in the work of Maxwell who showed that light is an electromagnetic wave. Throughout this period it was assumed that light travels through a medium, the ether, like other waves. In the last part of the 19th century Michelson, Morley, and others carried out a series of critical experiments that undermined this idea and threw the foundations of physics into crisis. In 1905 Einstein resolved this crisis by accepting what electromagnetism tells us about light and modifying Newtonian physics to fit. His work yielded the theory of relativity, which has profoundly changed our most fundamental ideas of space and time. It has also yielded a variety of practical outcomes, including aspects of the Global Positioning System, and nuclear energy.

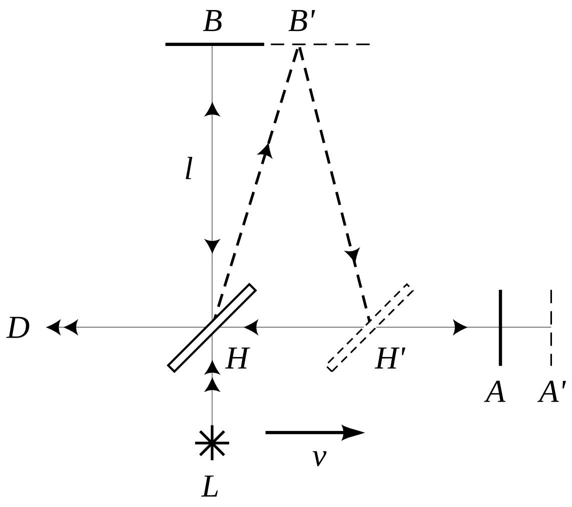
In this chapter I will discuss the role of light in the Michelson-Morley experiment and Einstein's work, and will look at some of the consequences. These include time dilation, length contraction, and the equivalence of mass and energy. My focus will be on presenting a subset of the material in the syllabus in a way that will help avoid some of the pitfalls and misconceptions that are easily fallen into, and will hopefully show how to help make some of the conceptual leaps a little easier for students. I'll also discuss some of the important philosophical issues, and some of the practical situations in which relativity is used.

The Michelson-Morley Experiment

Maxwell showed that light is an electromagnetic wave and that its speed c can be derived from electric and magnetic quantities. It was natural to suppose that, like water waves and sound waves, light waves must travel through some medium, which was dubbed the ether. Maxwell himself wrote "There can be no doubt that the interplanetary and interstellar spaces are not empty, but are occupied by a material substance or body" (the ether).

It was quickly realized that, if light moves at a fixed speed relative to the ether, we should be able to detect changes in how fast light passes us as the Earth moves through the ether in its orbit. For example, if we move toward a distant star at a velocity v, its light should pass us at $c + v$. If we move away, it should pass us at a speed $c - v$.

Michelson, as a student in Berlin in 1881, and Michelson and Morley in Cleveland in 1887, set out to measure the velocity of the Earth through the ether. They set up the experiment shown in Fig. 1. In this experiment light enters from a source L and is split into two beams by a half-silvered mirror H (which reflects half the light and transmits half). Half the light goes to mirror A and half to mirror B, each a distance l away. After reflection at these mirrors, the light returns to the half-silvered mirror and parts of both beams pass through to a detector D. Depending on how different the two path lengths HA and HB are (they won't be exactly equal in a real experiment) the two beams will interfere with differing relative phases at D. This interference will lead to bright or dark "fringes" at the detector, depending on the path difference. This apparatus is called a Michelson interferometer.



Now suppose the Earth carries the interferometer in the direction *HA* at a speed *v* through the ether. If light travels at a fixed speed relative to the ether it will take a slightly different time to travel to and from *A* than to and from *B*. Figure 1 shows the positions *A'* and *B'* of the mirrors *A* and *B* when light reflects from them, and the position *H'* of *H* when returning light reaches it on the way to the detector. We can see that the time for the light to travel along *HAH'* is

$$t_A = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2l}{c}\frac{1}{1-\frac{v^2}{c^2}}. \tag{1}$$

The time t_B to travel along *HB'H'* is just the length of this path divided by *c*. After a little algebra, this gives a time difference between the two paths of

$$t_A - t_B = t_A\left(1-\sqrt{1-\frac{v^2}{c^2}}\right). \tag{2}$$

The time difference shows up as a difference in the phase of the two beams when they arrive at the detector, because light waves complete different numbers of oscillations in different time intervals. Hence, $t_A - t_B$ will correspond to a measurable change in the interference of the two beams, depending on *v*. Michelson and Morley tried to measure this change but found a null result, despite their instrument having more than enough sensitivity to detect the predicted effect. In other words, they detected no motion of the Earth relative to the postulated ether. This result has been repeatedly confirmed since with even higher accuracy. Michelson became the first American Nobel prize-winner in physics in 1907.

The Michelson-Morley experiment showed that the velocity of the observer doesn't affect the measured speed of light. What about the velocity of the light source? One might argue that photons coming from a source that is moving toward you should pass you faster than *c*, but slower than *c* if the source is moving away from you. Today, the lack of any such effect is seen dramatically in particle accelerators, where electrons easily reach speeds of 0.99*c*: light emitted forward by the electrons passes an observer at *c*, not 1.99*c*, while light emitted backward also passes at *c*, not 0.01*c*!

Role of Measurements

Einstein took a major step in that he explicitly recognized that there is no such thing as a "miraculous" measurement that yields a universal truth without regard to how the measurement was carried out. Instead, every measurement is made using what is available in the physical world to infer properties of the phenomena in which we are interested. Hence, we cannot just assume that different observers will see the same thing and/or make the same inferences about the underlying phenomena – we must carefully analyze what they actually see and infer. This does not mean that everything is relative – all observers will agree on some things (e.g., that two objects touched one another), but may disagree on others (e.g., the time and place the objects touched).

This shift to an observationally-based perspective represented a major change in worldview and, as we will see, led to abandonment of large parts of Newtonian physics in the face of observational evidence. (Newtonian physics, of course, continues to be an excellent approximation to what is going on in regimes of low velocity and weak gravitation.)

Einstein's Postulates

The Michelson-Morley experiment, along with other 19th century discoveries, pointed to a major problem with physicists' understanding of nature. Some people tried to dismiss Michelson and Morley's result by asserting that the Earth "drags" the ether along with it, so there is no relative motion between the two. This ran into other problems and gave no insight into what was really happening.

Instead of trying to sweep the Michelson-Morley experiment under the carpet, Einstein took a different tack. He noted that electromagnetic theory yields a wave equation with only a single wave velocity *c*. He accepted the weight of evidence that this speed does not depend on the motion of the source or observer. Then he advanced his famous postulate of the constancy of the speed of light:

The speed of light is the same for all unaccelerated observers.

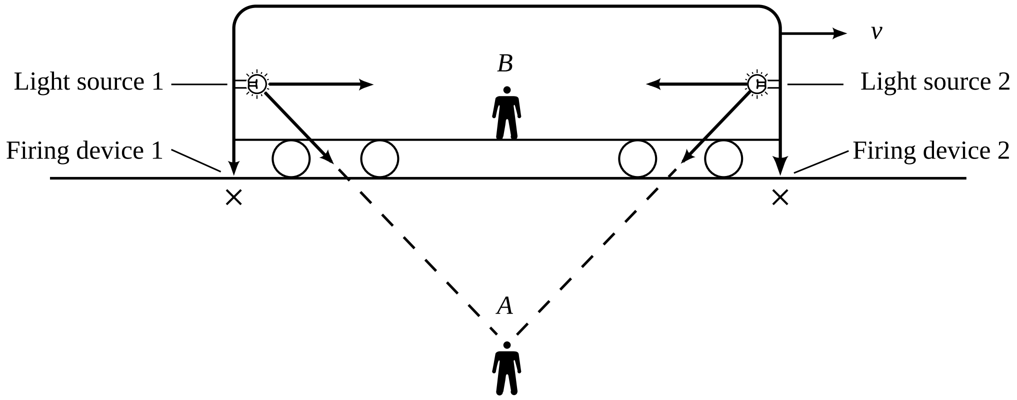
Newton had argued that position and velocity are not absolute – they are always measured relative to something else. From this he argued that all unaccelerated observers will see the same laws of Newtonian mechanics (force, energy, etc.). Einstein added electromagnetism to this idea by extending it into the Principle of Special Relativity:

All unaccelerated observers are equivalent.

In other words, all unaccelerated observers will see the same laws of physics (mechanics, electromagnetism, and everything else). Remarkably enough, all of the theory of Special Relativity follows from the constancy of c and the Principle of Special Rela tivity.

Simultaneity

In Newtonian physics it was always assumed that everyone could agree on whether or not two events happened at the same time because there was assumed to be a universal value of time. Einstein showed that simultaneity, like position and velocity, can only be judged relative to the particular frame of reference of the observer – i.e., a set of coordinates in which the observer is stationary. In particular, simultaneity depends on the observer's velocity.



Einstein proposed the thought experiment shown in Fig. 2. A train travels along a straight track with velocity relative to an observer Alice standing by the track. Bob stands in the centre of the train carriage, which has a flash lamp at each end. These lamps are both triggered at the same time in Alice's frame by switches placed one train-length apart on the track and equidistant from Alice (she's arranged this set-up by doing a few trial runs). Since the velocity of light is always c, if Alice sees the two flashes simultaneously she concludes that the lamps flashed simultaneously since they are equidistant from her. However, Bob travels away from flash 1 and toward flash 2, meeting the expanding sphere of light from 2 first. Since the two lamps are also equidistant from Bob in his frame, he concludes that lamp 2 switched on before 1. Einstein's conclusion was thus:

Simultaneity is relative.

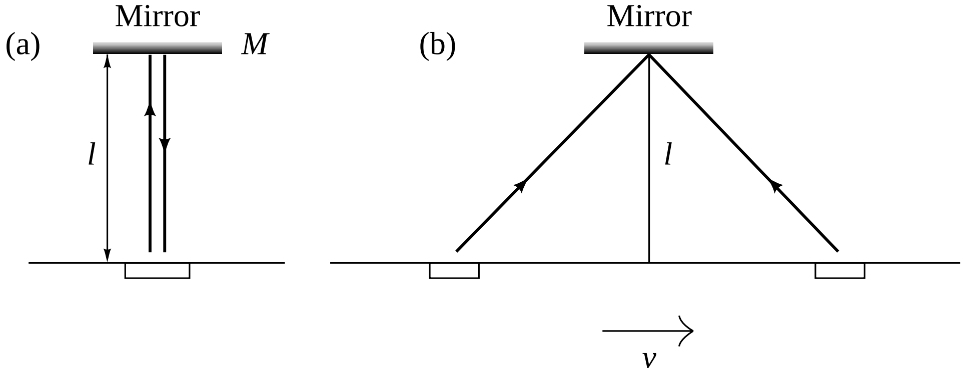
Hence, in general, observers can't even agree on whether or not two events happened at the same time! This means that there must be differences in how they measure time – a contradiction with Newton's idea of universal, absolute time.

There is an important case where all observers can always agree that two events are simultaneous – when they happen together at the same place. Then they are essentially a single event. It turns out that multiple observers also all agree on the relative time ordering of any pair of events where one can influence the other via a signal travelling between them at c or slower – so you can't end up being born before your parents meet, for example!

The relativity of simultaneity underlines the need to analyze observations and inferences very carefully – Alice and Bob end up seeing different things, and drawing different conclusions from them. However, they do agree that the lamps flashed, the detectors were triggered, etc.

Time Dilation

The relativity of simultaneity raises important questions about how we tell time. Einstein refused to assume that time is a universal quantity automatically known by all observers. Instead, he insisted that we look at how observers might really measure time. One way is the light clock shown in Fig. 3 (a). An observer Eva stationary with respect to this apparatus (i.e., in the same frame) ends out a pulse of light and notes when it returns from the mirror M a distance l away. This time interval is $t_E = 2l/c$ since the velocity of light is always c – we'll call it one "tick" of the light clock.



Now let's consider the case shown in Fig. 3 (b) where the light clock moves past another observer, Dave, at velocity toward the right. In this case the light pulse follows the path shown. The length of this path is clearly greater than that in Fig. 3 (a), so the clock will tick more slowly. A little algebra shows that the time t_D for one "tick" of the clock (as seen by Dave) is

$$\Delta t_D = \Delta t_E \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \geq \Delta t_E \ . \tag{3}$$

In other words, time passes more slowly for moving Eva than for stationary Dave! This effect is called time dilation.

An impressive demonstration of time dilation is seen when high-energy in particles from space hit the Earth's atmosphere and produce unstable particles called muons at a typical altitude of 10 km. These particles have a typical lifetime of only 2 microseconds. Even if they travelled at nearly the speed of light, they could only travel 600 m before decaying. Yet plentiful muons are seen at the Earth's surface many km below. The explanation is that time runs more slowly for fast-moving muons, so they survive long enough in the Earth's frame to hit the ground. Similar effects are seen in particle accelerators, where the lifetime of unstable particles (in the laboratory frame) is seen to increase as their velocity increases. It is also possible to measure time dilation directly by flying an atomic clock around in an aeroplane – the moving clock runs slow compared to one left on the ground.

A paradoxical thought: In Fig. 3, Eva can equally well consider herself to be stationary and Dave to be moving at a velocity $-v$. In this case, she concludes that Dave's time is running more slowly! It turns out that this is due to differences in both time and length measurement encapsulated in the Lorentz transformations of spacetime, which are (only slightly!) beyond the scope of the syllabus.

Length Contraction

Suppose Eva now sets up two light clocks at right angles to each other, both of the same length, l_E, in her frame and stationary with respect to her. One clock points along the direction of motion and the other at 90° as in Fig. 3 above. Light pulses travelling along the two clocks will return to her at the same instant $t_E = 2l_E/c$ – the two clocks tick at the same rate. Dave sees Eva to be moving past him at a velocity v. We know that he sees Eva's clock ticking at a slower rate given by (3) and that the two agree that the two pulses get back together simultaneously (since there is never disagreement about simultaneity at a single point). What does all this imply about the lengths of Eva's light clocks as seen by Dave? First, since Clock 1 is at right angles to Eva's direction of motion, its length is unaffected, so it still appears to have a length l_D to Dave. Clock 2 is affected differently. The total time for one tick (as seen by Dave) must be the same as for Clock 1 because he agrees that the light pulses leave Eva together and return together. But this time is the sum of the time t_1 for the pulse to overtake Mirror 2 and the time t_2 for it to return to Eva, as seen by Dave; i.e.

$$\Delta t = t_1 + t_2 \quad \frac{l_D}{c-v} + \frac{l_D}{c+v} = \frac{2l_D}{c}\frac{1}{1-\frac{v^2}{c^2}} \ , \tag{4}$$

where l_D is the length of light clock 2 as seen by Dave. (Note that there is no contradiction between (4) and the fact that both Eva and Dave measure the speed of light always to be c, since nothing moves faster than c in any frame.) If we use (3) and (4) we immediately find

$$l_D = l_E \sqrt{1 - \frac{v^2}{c^2}} \le l_E \ . \qquad (5)$$

In other words, lengths of moving objects contract in the direction of motion. This phenomenon is called length contraction.

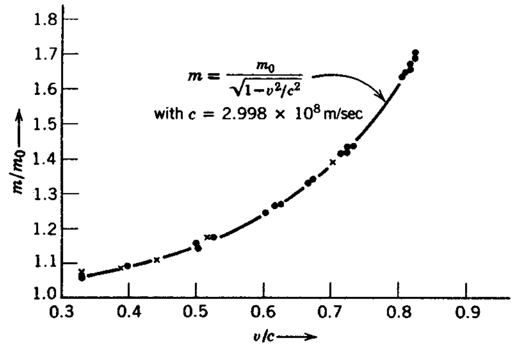
Length contraction provides an alternative, equivalent explanation of why muons produced in the upper atmosphere can reach the Earth's surface before they decay. If we look at the situation from a muon's frame, the Earth's atmosphere seems to be rushing towards the muon at a velocity near c. Hence, the atmosphere is contracted to a small enough thickness that it passes by before the muon decays.

Mass

We've seen that length and time are relative quantities that depend on the frame of reference of the observer. Einstein also showed that mass varies with velocity. The relativistic mass of a body is

$$m\ (v) = \frac{m_o}{\sqrt{1 - v^2/c^2}} \ , \qquad (6)$$

where m_0 is its rest mass (sometimes called the proper mass). Note that $m(v) \rightarrow \infty$ as $v \rightarrow c$. Figure 5 shows the theoretical variation of the mass of an electron vs. v, compared with measurements by Bucherer in 1909 and others later.



The variation of mass is very important for the design of accelerators – as the mass increases, more force is required to accelerate the particle. This force would become infinite at $v = c$, so it is impossible to accelerate a particle beyond the speed of light. In the largest accelerators, electrons have been accelerated to the point where their relativistic mass is over 10 million times their rest mass! A more homely example of a device that would not work properly without taking into account the relativistic mass variation is a TV set. Electrons fired toward the screen from the electron gun at the back of the picture tube travel at about $0.15c$ to $0.2c$, so their relativistic mass is 1% – 2% higher than their rest mass. This change must be taken into account when deflecting them to form the picture or else they will hit the wrong spot on the screen and the picture will be distorted by several mm on a large screen.

Energy

The total amount of kinetic energy that must be supplied to accelerate a body of rest mass m_0 to a velocity v can be calculated by integrating the force applied over the distance travelled. The result is

$$K = m_o c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right). \tag{7}$$

For small v we find $K = m_0 v^2/2$, exactly the result obtained by Newton. Hence, the usual kinetic energy is actually a relativistic effect!

We can write $K = E(v) - E(0)$, where

$$E(v) = m(v)c^2 \ . \tag{8}$$

This is Einstein's famous equation quantifying the equivalence of mass and energy. The energy $E(0) = m_0 c^2$ is the rest mass energy – energy that is present even when the body is at rest. If some of a body's matter can be destroyed (e.g., in a nuclear reaction), conservation of energy requires that an equivalent amount of energy must be released. This is the basis of energy generation in the Sun and nuclear reactors. The factor of c in (8) means that a lot of energy (9×10^{16} J = 90 PJ) is produced for each kg of mass destroyed.

Relativity and the Global Positioning System

I've mentioned a number of cases in which relativity must be used to obtain the right answers in physical problems, including some of technological design. One very important system that uses relativity in everyday situations is the Global Positioning System. This system is widely used to locate vehicles, ships, aircraft, hikers, geographical reference points, etc., to very high accuracy. Measurements over long periods have even directly detected continental drifts of a few cm per year!

To give high-accuracy position information, the GPS measures the distance from a hand-held receiver to satellites in a network of 24 that orbit Earth at a height of around 20 000 km (20 Mm). This is done by measuring the relative times of receipt of time-tagged signals sent out by the satellites, which also contain information on the positions of the satellites at the time of broadcast. Each satellite carries highly accurate atomic clocks, all of which are synchronized to within about 20 ns, but their rates are affected by time dilation, which tends to make moving clocks run slow, and also by gravitational effects from General Relativity, which tend to make clocks at high altitude run faster than ones on the ground. Hand-held receivers routinely incorporate these effects into their position calculations. If they didn't, the build-up in errors in just one day would lead to an 11 km error in position estimates!

Summary

In this chapter, I have tried to cover a number of topics from the HSC relativity syllabus in ways that will make them more easily digestible than they otherwise might be. I have also tried to give a number of examples of the applications of relativity in the laboratory and everyday technology, and have outlined some of the philosophical implications, especially as regards measurement and how we infer conclusions about phenomena from observations. It is worth noting that this chapter only scratches the surface. More can be found in the references below, which range from introductory to advanced.

References and Further Reading

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