2 Unit Bridging Course –Day 5

Applications of calculus II: Curve sketching

Emi Tanaka

THE UNIVERSITY OF SYDNEY

Curve sketching

The derivative is also useful when sketching functions.

If $y = f(x)$, then the value $f'(x)$ at any point will tell you whether the function is increasing, decreasing or neither at that point.

Recall:

If $f'(x) > 0$ on an interval, then the function f is increasing on the interval.

If $f'(x) < 0$ on an interval, then the function f is decreasing on the interval.

The derivative is also useful when sketching functions.

If $y = f(x)$, then the value $f'(x)$ at any point will tell you whether the function is increasing, decreasing or neither at that point.

Recall:

If $f'(x) > 0$ on an interval, then the function f is increasing on the interval.

If $f'(x) < 0$ on an interval, then the function f is decreasing on the interval.

Curve sketching

The derivative is also useful when sketching functions.

If $y = f(x)$, then the value $f'(x)$ at any point will tell you whether the function is increasing, decreasing or neither at that point.

Recall:

If $f'(x) > 0$ on an interval, then the function f is increasing on the interval.

If $f'(x) < 0$ on an interval, then the function f is decreasing on the interval.

Example

Sketch the curve $y = x^3 + 3x^2 - 9x - 8$.

First we find the stationary points.

$$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$$

Next, we let the derivative = 0 and solve the quadratic $x^2 + 2x - 3 = 0$ to get:

$$(x - 1)(x + 3) = 0$$

thus $x = 1$ or $x = -3$.

So the stationary points are $(-3, 19)$ and $(1, -13)$.

Example

Sketch the curve $y = x^3 + 3x^2 - 9x - 8$.

First we find the stationary points.

$$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$$

Next, we let the derivative = 0 and solve the quadratic $x^2 + 2x - 3 = 0$ to get:

$$(x - 1)(x + 3) = 0$$

thus $x = 1$ or $x = -3$.

So the stationary points are $(-3, 19)$ and $(1, -13)$.

Example

Sketch the curve $y = x^3 + 3x^2 - 9x - 8$.

First we find the stationary points.

$$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$$

Next, we let the derivative = 0 and solve the quadratic $x^2 + 2x - 3 = 0$ to get:

$$(x - 1)(x + 3) = 0$$

thus $x = 1$ or $x = -3$.

So the stationary points are $(-3, 19)$ and $(1, -13)$.

Example

Sketch the curve $y = x^3 + 3x^2 - 9x - 8$.

First we find the stationary points.

$$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$$

Next, we let the derivative = 0 and solve the quadratic $x^2 + 2x - 3 = 0$ to get:

$$(x - 1)(x + 3) = 0$$

thus $x = 1$ or $x = -3$.

So the stationary points are $(-3, 19)$ and $(1, -13)$.

We now examine $\dfrac{dy}{dx} = 3(x-1)(x+3)$ for the intervals

$$x < -3 \qquad -3 < x < 1 \qquad x > 1$$

$$-3 \qquad\qquad 1 \quad .$$

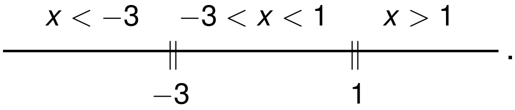
When $x < -3$, $(x-1) < 0$ and $(x+3) < 0$,

so $\frac{dy}{dx} = 3(x-1)(x+3) > 0$. Hence, the function is increasing, for $x < -3$. It is useful to draw up and complete a table as follows:

x	< -3	-3	$> -3, < 1$	1	> 1
y'	$+$ve	0	$-$ve	0	$+$ve
y	$\nearrow$	19	$\searrow$	-13	$\nearrow$

Using this table we can see the general shape of the curve.

We now examine $\dfrac{dy}{dx} = 3(x-1)(x+3)$ for the intervals



When $x < -3$, $(x-1) < 0$ and $(x+3) < 0$,

so $\frac{dy}{dx} = 3(x-1)(x+3) > 0$. Hence, the function is increasing, for $x < -3$. It is useful to draw up and complete a table as follows:

x	< -3	-3	$> -3, < 1$	1	> 1
y'	$+$ve	0	$-$ve	0	$+$ve
y	$\nearrow$	19	$\searrow$	-13	$\nearrow$

Using this table we can see the general shape of the curve.

We now examine $\dfrac{dy}{dx} = 3(x-1)(x+3)$ for the intervals

$$\begin{array}{ccc} x < -3 & -3 < x < 1 & x > 1 \end{array}$$

$$-3 \qquad\qquad 1$$

When $x < -3$, $(x-1) < 0$ and $(x+3) < 0$,

so $\frac{dy}{dx} = 3(x-1)(x+3) > 0$. Hence, the function is increasing, for $x < -3$. It is useful to draw up and complete a table as follows:

x	< -3	-3	$> -3, < 1$	1	> 1
y'	$+$ve	0	$-$ve	0	$+$ve
y	$\nearrow$	19	$\searrow$	-13	$\nearrow$

Using this table we can see the general shape of the curve.

We now examine $\dfrac{dy}{dx} = 3(x-1)(x+3)$ for the intervals

$$\begin{array}{ccc} x < -3 & -3 < x < 1 & x > 1 \end{array}$$

$$\overset{}{\underset{-3}{\|}}\qquad\qquad \underset{1}{\|} .$$

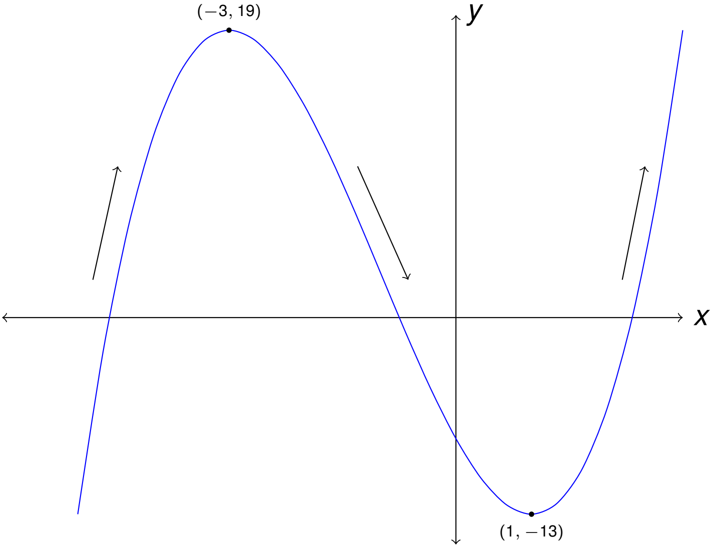
When $x < -3$, $(x-1) < 0$ and $(x+3) < 0$,

so $\frac{dy}{dx} = 3(x-1)(x+3) > 0$. Hence, the function is increasing, for $x < -3$. It is useful to draw up and complete a table as follows:

x	< -3	-3	$> -3, < 1$	1	> 1
y'	$+$ve	0	$-$ve	0	$+$ve
y	$\nearrow$	19	$\searrow$	-13	$\nearrow$

Using this table we can see the general shape of the curve.

Curve sketching



Example

Sketch the curve $y = -x^3 + 9x$ and find where it crosses the axes.

First find the stationary points,

$$\frac{dy}{dx} = -3x^2 + 9 = -3(x^2 - 3) = 0.$$

We get $x = \pm\sqrt{3}$, so the stationary points are $(-\sqrt{3}, -6\sqrt{3})$ and $(\sqrt{3}, 6\sqrt{3})$. From the derivative we get:

x	$< -\sqrt{3}$	$-\sqrt{3}$	$> -\sqrt{3}, < \sqrt{3}$	$\sqrt{3}$	$> \sqrt{3}$
y'	$-$ve	0	$+$ve	0	$-$ve
y	$\searrow$	$-6\sqrt{3}$	$\nearrow$	$6\sqrt{3}$	$\searrow$

Example

Sketch the curve $y = -x^3 + 9x$ and find where it crosses the axes.

First find the stationary points,

$$\frac{dy}{dx} = -3x^2 + 9 = -3(x^2 - 3) = 0.$$

We get $x = \pm\sqrt{3}$, so the stationary points are $(-\sqrt{3}, -6\sqrt{3})$ and $(\sqrt{3}, 6\sqrt{3})$. From the derivative we get:

x	$< -\sqrt{3}$	$-\sqrt{3}$	$> -\sqrt{3}, < \sqrt{3}$	$\sqrt{3}$	$> \sqrt{3}$
y'	$-$ve	0	$+$ve	0	$-$ve
y	$\searrow$	$-6\sqrt{3}$	$\nearrow$	$6\sqrt{3}$	$\searrow$

Example

Sketch the curve $y = -x^3 + 9x$ and find where it crosses the axes.

First find the stationary points,

$$\frac{dy}{dx} = -3x^2 + 9 = -3(x^2 - 3) = 0.$$

We get $x = \pm\sqrt{3}$, so the stationary points are $(-\sqrt{3}, -6\sqrt{3})$ and $(\sqrt{3}, 6\sqrt{3})$. From the derivative we get:

x	$< -\sqrt{3}$	$-\sqrt{3}$	$> -\sqrt{3}, < \sqrt{3}$	$\sqrt{3}$	$> \sqrt{3}$
y'	$-$ve	0	$+$ve	0	$-$ve
y	$\searrow$	$-6\sqrt{3}$	$\nearrow$	$6\sqrt{3}$	$\searrow$

Example 2

To find the y intercept we substitute $x = 0$ in the original equation to get:

$$y = -(0)^3 + 9(0) = 0.$$

So the curve crosses the y-axis at $(0, 0)$.

Now for the x intercept we substitute $y = 0$ in the original equation and solve for x.

$$-x^3 + 9x = -x(x^2 - 9) = -x(x - 3)(x + 3) = 0.$$

So we get $x = 0$, $x = 3$ or $x = -3$.
The curve crosses the x-axis at $(0, 0)$, $(3, 0)$ and $(-3, 0)$.

Note: you can check your solutions by substituting the coordinates back into the equation.

Example 2

To find the y intercept we substitute $x = 0$ in the original equation to get:

$$y = -(0)^3 + 9(0) = 0.$$

So the curve crosses the y-axis at $(0, 0)$.

Now for the x intercept we substitute $y = 0$ in the original equation and solve for x.

$$-x^3 + 9x = -x(x^2 - 9) = -x(x - 3)(x + 3) = 0.$$

So we get $x = 0$, $x = 3$ or $x = -3$.
The curve crosses the x-axis at $(0, 0)$, $(3, 0)$ and $(-3, 0)$.

Note: you can check your solutions by substituting the coordinates back into the equation.

Example 2

To find the y intercept we substitute $x = 0$ in the original equation to get:

$$y = -(0)^3 + 9(0) = 0.$$

So the curve crosses the y-axis at $(0, 0)$.

Now for the x intercept we substitute $y = 0$ in the original equation and solve for x.

$$-x^3 + 9x = -x(x^2 - 9) = -x(x - 3)(x + 3) = 0.$$

So we get $x = 0$, $x = 3$ or $x = -3$.
The curve crosses the x-axis at $(0, 0)$, $(3, 0)$ and $(-3, 0)$.

Note: you can check your solutions by substituting the coordinates back into the equation.

Example 2

To find the y intercept we substitute $x = 0$ in the original equation to get:

$$y = -(0)^3 + 9(0) = 0.$$

So the curve crosses the y-axis at $(0, 0)$.

Now for the x intercept we substitute $y = 0$ in the original equation and solve for x.

$$-x^3 + 9x = -x(x^2 - 9) = -x(x - 3)(x + 3) = 0.$$

So we get $x = 0$, $x = 3$ or $x = -3$.
The curve crosses the x-axis at $(0, 0)$, $(3, 0)$ and $(-3, 0)$.

Note: you can check your solutions by substituting the coordinates back into the equation.

THE UNIVERSITY OF SYDNEY

Example 2

To find the y intercept we substitute $x = 0$ in the original equation to get:

$$y = -(0)^3 + 9(0) = 0.$$

So the curve crosses the y-axis at $(0, 0)$.

Now for the x intercept we substitute $y = 0$ in the original equation and solve for x.

$$-x^3 + 9x = -x(x^2 - 9) = -x(x-3)(x+3) = 0.$$

So we get $x = 0$, $x = 3$ or $x = -3$.
The curve crosses the x-axis at $(0, 0)$, $(3, 0)$ and $(-3, 0)$.

Note: you can check your solutions by substituting the coordinates back into the equation.

Example 2