# 2 Unit Bridging Course – Day 7

Index Laws I

Clinton Boys





### Indices tell us how many times to multiply a number by itself.

For example

$$2^{2} = 2 \times 2$$

$$2^{3} = 2 \times 2 \times 2$$

$$3^{1} = 3$$

$$x^{4} = x \times x \times x \times x$$



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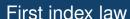




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# As you may have seen, an index does not have to be a whole number.

Indeed, it is common to see fractional indices, i.e. expressions of the form  $x^{1/2}$  or  $x^{0.2}$ .

If we assume the first index law still holds,  $x^{1/2}$ , which is usually written  $\sqrt{x}$ , is the number such that  $x^{1/2} \times x^{1/2} = x$ . In this way we can define fractional indices by stipulating that the first index law is true:

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- $-2^2=4$
- $-4^{1/2}=\sqrt{4}=2.$
- $-2^3=8$
- $-8^{1/3} = \sqrt[3]{8} = 2.$

**Note:** There are actually 2 square roots of 4, namely 2 and -2 (since both of these numbers square to 4!). The convention is to choose the positive number, so in this course  $\sqrt{4} = 2$ . The same is true for all positive numbers.



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### Note on fractional indices

There are some subtleties here that we are going to ignore in this course. It is actually a difficult task to define  $x^a$  where a is a number which can't be written as a fraction (yes, such numbers exist!)

In order to define indices for these numbers, one needs to understand the mathematical discipline of real analysis – a difficult second-year university subject!



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#### This leads us to the second index law:

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For any numbers m and n, and for any number x,

$$(x^m)^n = x^{mn}.$$

This is really just a generalisation of the first law: multiplying *a* by *b* is the same as adding *a* together *b* times.

#### For example:

$$(x^2)^3 = (x^2)(x^2)(x^2) = x^{2+2+2} = x^6.$$



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- $(2^2)^2 = 2^4 = 16$
- $-(3^2)^3=3^6=729$
- $(x^2)^5 = x^{10}$

Notice the important difference between the first index law

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The third index law tells us how to divide different powers of a number:

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For any numbers m and n, and for any nonzero number x,

$$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}.$$

It's important that *x* is nonzero, otherwise we are dividing by zero, which doesn't make sense!





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It's easy to see why this law is true when we remember the process of cancellation:

$$\underbrace{x \times x \times x \times \cdots \times x \times x}_{x \times x \times \cdots \times x \times x}$$
b times

If there are more x's on the top than the bottom, so a > b, we can cancel off all the x's on the bottom and just be left with  $x^{a-b}$  on the top and 1 on the bottom, which is just  $x^{a-b}$ .





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$$\frac{x^5}{x^3} = \frac{x \times x \times x \times x \times x}{x \times x \times x} = \frac{x^2}{1} = x^2.$$

After cancelling, we are left with  $x^2$  on the top, and 1 on the bottom

So, 
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Example

$$\frac{x^3}{x^6} = \frac{x \times x \times x}{x \times x \times x \times x \times x \times x} = \frac{1}{x^3}.$$

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Of course, 
$$\frac{1}{x^3} = x^{-3} = x^{3-6}$$
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for any number n to see that  $\frac{1}{x^{b-a}} = x^{-(b-a)} = x^{a-b}$ .

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### There are two other rules we need to manipulate indices.

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