2 Unit Bridging Course – Day 7

Index Laws I

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THE UNIVERSITY OF SYDNEY

Indices tell us how many times to multiply a number by itself.

For example

$$2^2 = 2 \times 2$$
$$2^3 = 2 \times 2 \times 2$$
$$3^1 = 3$$
$$x^4 = x \times x \times x \times x.$$

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First index law

The first index law is just an expression of a rule which you are probably already familiar with:

First index law

For any numbers m and n, and for any number x,

$$x^m \times x^n = x^{m+n}.$$

All this says is that if we multiply a number together m times, and then a further n times, the end result is having performed $m + n$ multiplications.

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Fractional indices

As you may have seen, an index does not have to be a whole number.

Indeed, it is common to see fractional indices, i.e. expressions of the form $x^{1/2}$ or $x^{0.2}$.

If we assume the first index law still holds, $x^{1/2}$, which is usually written $\sqrt{x}$, is the number such that $x^{1/2} \times x^{1/2} = x$. In this way we can define fractional indices by stipulating that the first index law is true:

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Examples

Example

- $2^2 = 4$
- $4^{1/2} = \sqrt{4} = 2.$
- $2^3 = 8$
- $8^{1/3} = \sqrt[3]{8} = 2.$

Note: There are actually 2 square roots of 4, namely 2 and -2 (since both of these numbers square to 4!). The convention is to choose the positive number, so in this course $\sqrt{4} = 2$. The same is true for all positive numbers.

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Note on fractional indices

There are some subtleties here that we are going to ignore in this course. It is actually a difficult task to define x^a where a is a number which can't be written as a fraction (yes, such numbers exist!)

In order to define indices for these numbers, one needs to understand the mathematical discipline of real analysis – a difficult second-year university subject!

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This leads us to the second index law:

Second index law

For any numbers m and n, and for any number x,

$$(x^m)^n = x^{mn}.$$

This is really just a generalisation of the first law: multiplying a by b is the same as adding a together b times.

For example:

$(x^2)^3 = (x^2)(x^2)(x^2) = x^{2+2+2} = x^6.$

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- $(3^2)^3 = 3^6 = 729$
- $(x^2)^5 = x^{10}$

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Third index law

The third index law tells us how to divide different powers of a number:

Third index law

For any numbers m and n, and for any nonzero number x,

$$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}.$$

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Third index law

It's easy to see why this law is true when we remember the process of cancellation:

$$\frac{\overbrace{x \times x \times x \times \cdots \times x \times x}^{a \text{ times}}}{\underbrace{x \times x \times \cdots \times x \times x}_{b \text{ times}}}$$

If there are more x's on the top than the bottom, so $a > b$, we can cancel off all the x's on the bottom and just be left with x^{a-b} on the top and 1 on the bottom, which is just x^{a-b}.

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$$\frac{x^5}{x^3} = \frac{x \times x \times x \times x \times x}{x \times x \times x} = \frac{x^2}{1} = x^2.$$

After cancelling, we are left with x^2 on the top, and 1 on the bottom.

So, $\dfrac{x^5}{x^3} = x^{5-3} = x^2$.

That is,

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When we have more x's on the bottom than on the top, let's see what happens by looking at an example.

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$$\frac{x^3}{x^6} = \frac{x \times x \times x}{x \times x \times x \times x \times x \times x} = \frac{1}{x^3}.$$

This time, we are left with a 1 on the top after cancelling off some terms, and the bottom still has whatever was left over.

Of course, $\frac{1}{x^3} = x^{-3} = x^{3-6}$.

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for any number n to see that $\frac{1}{x^{b-a}} = x^{-(b-a)} = x^{a-b}$.

So in both cases,

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Important to remember

There are two other rules we need to manipulate indices.

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$$x^1 = x.$$

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