Solutions to Selected Exercises 7

1. iii

$$\begin{aligned} uv^{-2} \times \frac{u^{-1}}{v} &= uv^{-2} \times u^{-1}v^{-1} \\ &= uu^{-1}v^{-2}v^{-1} \\ &= v^{-3}. \end{aligned}$$

2. iii

$$ab^{-1} + bc^{-1} = \frac{a}{b} + \frac{b}{c} = \frac{2}{3} + \frac{3}{\frac{1}{2}} = \frac{2}{3} + 6 = 6\frac{2}{3}.$$

3. iv

$$\left(\frac{4}{9}\right)^{-\frac{3}{2}} = \frac{1}{(\frac{4}{9})^{\frac{3}{2}}} = \left(\frac{9}{4}\right)^{\frac{3}{2}} = \left(\left(\frac{9}{4}\right)^{\frac{1}{2}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}.$$

4. ii

$$m^{\frac{1}{3}}(m^{\frac{1}{3}} + m^{-\frac{1}{3}}) = m^{\frac{1}{3}}m^{\frac{1}{3}} + m^{\frac{1}{3}}m^{-\frac{1}{3}} = m^{\frac{2}{3}} + m^0 = m^{\frac{2}{3}} + 1.$$

5. ii

$$\frac{t^2t^{-3}}{\sqrt[3]{t}} = \frac{t^2t^{-3}}{t^{\frac{1}{3}}} = t^2t^{-3}t^{-\frac{1}{3}} = t^{2-3-\frac{1}{3}} = t^{-\frac{4}{3}}.$$

iv

$$\frac{m^{\frac{3}{2}}\sqrt[3]{m}}{m^{-3}} = \frac{m^{\frac{3}{2}}m^{\frac{1}{3}}}{\frac{1}{m^3}} = m^{\frac{3}{2}}m^{\frac{1}{3}}m^3 = m^{\frac{3}{2}+\frac{1}{3}+3} = m^{\frac{29}{6}}.$$

viii

$$\frac{1}{x^{-2}\sqrt{x^{-1}}} = \frac{1}{x^{-2}(x^{-1})^{\frac{1}{2}}} = \frac{1}{x^{-2}x^{-\frac{1}{2}}} = \frac{1}{x^{-\frac{5}{2}}} = x^{\frac{5}{2}}.$$

6. vi

$$f(x) = 1 + 2\sqrt{x} = 1 + 2x^{\frac{1}{2}} \quad \text{so} \quad f'(x) = 2\left(\frac{1}{2}\right)x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}.$$

7. iii

$$y = x^2 + \frac{2}{x^2} = x^2 + 2x^{-2} \quad \text{so} \quad \frac{dy}{dx} = 2x + 2(-2)x^{-3} = 2x - \frac{4}{x^3}.$$

8. iv

$$h(u) = 13u^2 - 5u\sqrt{u} = 13u^2 - 5uu^{\frac{1}{2}} = 13u^2 - 5u^{\frac{3}{2}}$$

$$\text{so} \quad h'(u) = 26u - \frac{15}{2}u^{\frac{1}{2}} = 26u - \frac{15}{2}\sqrt{u}.$$

vi

$$f(x) = \frac{5}{x^3} + \frac{1}{x} = 5x^{-3} + x^{-1} \quad \text{so} \quad f'(x) = -15x^{-4} - x^{-2} = -\frac{15}{x^4} - \frac{1}{x^2}.$$

9. iv

$$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} \quad \text{so} \quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} \quad \text{and} \quad f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}.$$

viii

$$y = \frac{5x^3 + 2x^{\frac{1}{2}}}{x} = 5x^2 + 2x^{-\frac{1}{2}} \quad \text{so} \quad \frac{dy}{dx} = 10x - x^{-\frac{3}{2}} \quad \text{and} \quad \frac{d^2y}{dx^2} = 10 + \frac{3}{2}x^{-\frac{5}{2}}.$$

10. i Let h be the height of the tank. The volume (V) of the tank is $V = x^2h = 32$.

$$\text{So,} \quad h = \frac{32}{x^2}.$$

ii Let the area of the sheet metal be A. Then

$$A = 4xh + x^2 = 4x\frac{32}{x^2} + x^2 = 128x^{-1} + x^2.$$

To maximise A, differentiate with respect to x and set the derivative equal to 0.

$$\frac{dA}{dx} = -128x^{-2} + 2x = -\frac{128}{x^2} + 2x = \frac{-128 + 2x^3}{x^2}$$

$$\text{so} \quad \frac{dA}{dx} = 0 \quad \text{when} \quad 2x^3 = 128 \quad \text{ie} \quad x = 4.$$

When $x < 4$, $\frac{dA}{dx} < 0$ (use $x = 1$ as a test point) and when $x > 4$ $\frac{dA}{dx} > 0$ (use $x = 5$ as a test point) so we have a minimum when $x = 4$.

When $x = 4$, $A = \frac{128}{4} + 4^2 = 32 + 16 = 48$.

So the least area of sheet metal is 48 m^2.