2 Unit Bridging Course - Day 3

Quadratics

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A quadratic expression is an expression where it takes a standard form of:

$$ax^2 + bx + c$$

where $a$, $b$, $c$ are constants, $a \neq 0$.

For example, the following are quadratics.

- $x^2 + 4x + 4$ (in this case $a = 1$, $b = 4$, $c = 4$)
- $2x^2 - 3x + 1$ (in this case $a = 2$, $b = -3$, $c = 1$)
  Notice $-3x = +(−3)x$
- $4x^2 - 9$ (in this case $a = 4$, $b = 0$, $c = -9$)

Notice that in all of them the highest power of $x$ is 2.
Factorising a quadratic means to find 2 factors that will give you the quadratic when multiplied.

Note: not all quadratic expressions can be factorised.

For example when you factorise

\[ x^2 + 2x + 1 \]

you get

\[ (x + 1)(x + 1) \]
To factorise the quadratic, $ax^2 + bx + c$, use the following method:

1. Write down $a \times c$ and $b$ taking care to get the signs correct.
2. Write down all the factors of $a \times c$.
3. Find factors of $a \times c$, $r_1$ and $r_2$ which when added together equal $b$, ie $r_1 + r_2 = b$.
4. Replace $bx$ with $r_1x + r_2x$.
5. Factorise the first 2 terms and the last 2 terms separately.
6. Take out the common factor to finish factorising.
Example

Factorise $2x^2 - 3x + 1$

First find $a \times c$ and $b$.

$$a \times c = 2 \times 1 = 2 \text{ and } b = -3$$

The factors of $a \times c$ are

- 2 and 1
- $-2$ and $-1$

Now $-2 + (-1) = -3 = b$, therefore we will use $-2$ and $-1$. 
Replacing \(-3x\) with \(-2x - x\).

\[
2x^2 - 3x + 1 = 2x^2 - 2x - 1x + 1 \\
= 2x(x - 1) - (x - 1) \\
= (x - 1)(2x - 1)
\]

Check by expanding \((x - 1)(2x - 1)\).

\[
(x - 1)(2x - 1) = 2x^2 - x - 2x + 1 \\
= 2x^2 - 3x + 1
\]
Example 2

Example

Factorise $x^2 + 6x + 9$.

Find $a \times c$ and $b$.

$$a \times c = 1 \times 9 = 9 \text{ and } b = 6$$

The factors of $a \times c$ are

- 1 and 9
- $-1$ and $-9$
- 3 and 3
- $-3$ and $-3$

Now $3 + 3 = 6 = b$, therefore we will use 3 and 3.
Example 2 continued

Replace $6x$ with $3x + 3x$.

\[
x^2 + 6x + 9 = x^2 + 3x + 3x + 9 \\
= x(x + 3) + 3(x + 3) \\
= (x + 3)(x + 3) \\
= (x + 3)^2
\]

Check by expanding $(x + 3)^2$.

\[
(x + 3)(x + 3) = x^2 + 3x + 3x + 9 \\
= x^2 + 6x + 9
\]
Practice Questions

Factorise the following.

1. \( x^2 + 6x + 8 \)
2. \( n^2 + 3n - 4 \)
3. \( x^2 - 2x - 8 \)
4. \( x^2 - 5x + 6 \)
5. \( x^2 + 6x + 9 \)
6. \( a^2 + 5a + 4 \)
7. \( x^2 - 2x - 24 \)
8. \( 2x^2 + 3x + 1 \)
9. \( 2n^2 - 4n + 2 \)
10. \( 3x^2 - 2x - 1 \)
Answers to the practice questions.

1. \((x + 2)(x + 4)\)  
2. \((n + 4)(n - 1)\)  
3. \((x - 4)(x + 2)\)  
4. \((x - 2)(x - 3)\)  
5. \((x + 3)^2\)  
6. \((a + 4)(a + 1)\)  
7. \((x - 6)(x + 4)\)  
8. \((2x + 1)(x + 1)\)  
9. \(2(n - 1)(n - 1)\)  
10. \((3x + 1)(x - 1)\)
Notice that

\[(a + b)(a - b) = a^2 - b^2.\]

From this we can derive a special case of factorisation.

If the quadratic is a square minus a square, such that:

\[a^2 - b^2\]

then we can factorise as follows:

\[a^2 - b^2 = (a + b)(a - b).\]

This is called the difference of two squares method.
Example

Example

Factorise $9x^2 - 16$.

\[
9x^2 - 16 = (3x)^2 - 4^2 = (3x + 4)(3x - 4)
\]

Example

Factorise $4x^2 - 4$.

\[
4x^2 - 4 = 4(x^2 - 1) = 4(x + 1)(x - 1)
\]
Practice Questions

Factorise the following.

1. \(x^2 - 4\)
2. \(4n^2 - 9\)
3. \(9x^2 - 16\)
4. \(x^2 - a^2\)
5. \(a^2b^2 - 4\)
6. \(4p^2q^2 - 16\)
Answers to the practice questions.

1. $(x + 2)(x - 2)$
2. $(2n + 3)(2n - 3)$
3. $(3x + 4)(3x - 4)$
4. $(x + a)(x - a)$
5. $(ab + 2)(ab - 2)$
6. $4(pq + 2)(pq - 2)$