Amme 3500 : System Dynamics and Control

Block Diagrams

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Block Diagrams

• As we saw in the introductory lecture, a subsystem can be represented with an input, an output and a transfer function

\[ U(s) \rightarrow \frac{H(s)}{Y(s)} \rightarrow Y(s) \]

Input: control surfaces (flap, aileron), wind gust
Aircraft output: pitch, yaw, roll


Course Outline

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Examples of subsystems

- Automobile control:

  ![Automobile control diagram]

  - Desired course of travel
  - Actual course of travel

  - Sensing
  - Measurement
  - Steering Mechanism
  - Automobile

Examples of Subsystems

- Antenna control

  ![Antenna control diagram]

  - Control Systems Engineering

Mathematical Modelling

- In the time domain, the input-output relationship is usually expressed in terms of a differential equation

  \[ \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \cdots + b_0 u(t) \]

  - \((a_{n-1}, \ldots, a_0, b_m, \ldots b_0)\) are the system’s parameters, \(n \geq m\)
  - The system is LTI (Linear Time Invariant) iff the parameters are time-invariant
  - \(n\) is the order of the system

Mathematical Modelling

- In the Laplace domain, the input-output relationship is usually expressed in terms of an algebraic equation in terms of \(s\)

  \[ s^n Y(s) + a_{n-1} s^{n-1} Y(s) + \cdots + a_0 Y(s) = b_m s^m U(s) + \cdots + b_0 U(s) \]
Cascaded systems

- In time, a cascaded system requires a convolution
  \[ u(t) * h(t) = \int u(\tau) h(t - \tau) \, d\tau \]
- In the Laplace domain, this is simply a product
  \[ Y(s) = U(s)H(s)H_1(s) \]

A General Control System

- Many control systems can be characterised by these components/subsystems

Components of a Block Diagram

- A block diagram is made up of signals, systems, summing junctions and pickoff points

Simplifying Block Diagrams

- The objectives of a block diagram are
  - To provide an understanding of the signal flows in the system
  - To enable modelling of complex systems from simple building blocks
  - To allow us to generate the overall system transfer function
- A few rules allow us to simplify complex block diagrams into familiar forms
Typical Block Diagram Elements

• Cascaded Systems

\[ R(s) \rightarrow G_1(s) \rightarrow G_2(s) \rightarrow G_3(s) \rightarrow C(s) = G_3(s)G_2(s)G_1(s)R(s) \]

Typical Block Diagram Elements

• Parallel Systems

\[ R(s) \rightarrow G_1(s) \rightarrow G_2(s) \rightarrow C(s) = G_1(s) + G_2(s) \]

Typical Block Diagram Elements

• Feedback Form

\[ E(s) = R(s) - H(s)C(s) \]

\[ C(s) = G(s)E(s) \]

\[ \frac{C(s)}{G(s)} = R(s) - H(s)C(s) \]

\[ G(s)R(s) = 1 + H(s)G(s) \]

\[ \frac{C(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)} \]

Moving Past A Summation

a) To the left past a summing junction

b) To the right past a summing junction
Moving Past Pick-Off Points

a) To the left past a summing junction

\[
\begin{align*}
\frac{R(s)}{G(s)} & = \frac{R(s)}{G(s)} \\
\frac{R(s)G(s)}{G(s)} & = \frac{R(s)}{G(s)} \\
\frac{R(s)G(s)}{G(s)} & = \frac{R(s)}{G(s)}
\end{align*}
\]

b) To the right past a summing junction

\[
\begin{align*}
\frac{R(s)}{G(s)} & = \frac{R(s)}{G(s)} \\
\frac{R(s)G(s)}{G(s)} & = \frac{R(s)}{G(s)} \\
\frac{R(s)G(s)}{G(s)} & = \frac{R(s)}{G(s)}
\end{align*}
\]

Example I: Simplifying Block Diagrams

• Consider this block diagram
• We wish to find the transfer function between the input \( R(s) \) and \( C(s) \)

Example I: Simplifying Block Diagrams

a) Collapse the summing junctions

b) Form equivalent cascaded system in the forward path and equivalent parallel system in the feedback path

c) Form equivalent feedback system and multiply by cascaded \( G_1(s) \)

Example II: Simplifying Block Diagrams

• Form the equivalent system in the forward path
• Move \( 1/s \) to the left of the pickoff point
• Combine the parallel feedback paths
• Compute \( C(s)/R(s) \) using feedback formula
Example III: Shuttle Pitch Control

- Manipulating block diagrams is important but how do they relate to the real world?
- Here is an example of a real system that incorporates feedback to control the pitch of the vehicle (amongst other things).

Example III: Shuttle Pitch Control

- The control mechanisms available include the body flap, elevons and engines.
- Measurements are made by the vehicle’s inertial unit, gyros and accelerometers.

Example III: Shuttle Pitch Control

- A simplified model of a pitch controller is shown for the space shuttle.
- Assuming other inputs are zero, can we find the transfer function from commanded to actual pitch?

Example III: Shuttle Pitch Control

- Combine $G_1$ and $G_2$.
- Push $K_1$ to the right past the summing junction.

Example III: Shuttle Pitch Control

- Push $K_1 K_2$ to the right past the summing junction
- Hence $T(s) = \frac{K_1 K_3 G_1(s) G_2(s)}{1 + K_1 K_3 G_1(s) G_2(s) \left(1 + \frac{s^2}{K_3 K_1}\right)}$

Closed Loop Feedback

- We have suggested that many control systems take on a familiar feedback form
- Intuition tells us that feedback is useful – imagine driving your car with your eyes closed
- Let us examine why this is the case from a mathematical perspective

Single-Loop Feedback System

- Error Signal $e(t) = d(t) - f(t) = d(t) - K_f(t)$
- The goal of the Controller $K(s)$ is:
  - To produce a control signal $u(t)$ that drives the ‘error’ $e(t)$ to zero
Controller Objectives

- Controller cannot drive error to zero instantaneously as the plant $G(s)$ has dynamics
- Clearly a ‘large’ control signal will move the plant more quickly
- The gain of the controller should be large so that even small values of $e(t)$ will produce large values of $u(t)$
- However, large values of gain will cause instability

Feedforward & Feedback

- Driving a Formula-1 car
- Feedforward control: pre-emptive/predictable action: prediction of car’s direction of travel (assuming the model is accurate enough)
- Feedback control: corrective action since the model is not perfect + noise

Why Use Feedback?

- **Perfect feed-forward controller: $G_n$**
  
  \[
  d(t) + l(t) = \frac{1}{G_n} G y(t)
  \]

  - Output and error:
    \[
    y = \frac{G}{G_n} d + Gl
    \]
    \[
    e = d - y = \left(1 - \frac{G}{G_n}\right) d - Gl
    \]
  - Only zero when:
    \[
    \begin{align*}
    G_n &= G & \text{(Perfect Knowledge)} \\
    l &= 0 & \text{(No load or disturbance)}
    \end{align*}
    \]
Closed Loop Equations

\[ e = d - GKe - Gl \]

Collect terms:

\[ (1 + KG)e = d - Gl \]

Or:

\[ e = \frac{1}{1 + KG} d - \frac{G}{1 + KG} l \]

Demand to Error Transfer Function

Load to Error Transfer Function

Equivalent Open Loop Block Diagram

Similarly

\[ y = \frac{GK}{1 + KG} d - \frac{G}{1 + KG} l \]

Demand to Output Transfer Function

Load to Output Transfer Function

Equivalent Open Loop Block Diagram

Rejecting Loads and Disturbances

\[ y = \frac{G}{1 + KG} l \]

If \( K \) is big: \( KG >> 1 \)

Load to Output Transfer Function

If \( K \) is big:

\[ y \approx \frac{G}{KG} l = \frac{1}{K} l \approx 0 \]

Perfect Disturbance Rejection

Independent of knowing \( G \)

Regardless of \( l \)

Tracking Performance

\[ y = \frac{GK}{1 + KG} d \]

Demand to Output Transfer Function

If \( K \) is big:

\[ KG >> 1 \]

\[ y \approx \frac{GK}{KG} d = d \]

Perfect Tracking of Demand

Independent of knowing \( G \)

Regardless of \( l \)
Two Key Problems

\[ u = K(d - y) \rightarrow \text{Large } K \text{ requires large actuator power } u \]

Power: \( u = K(d - y) \rightarrow \text{Large } K \text{ requires large actuator power } u \)

Noise: \( u = K(d - y + n) \rightarrow \text{Large } K \text{ amplifies sensor noise} \)

In practise a Compromise \( K \) is required

Conclusions

- We have examined methods for computing the transfer function by reducing block diagrams to simple form
- We have also presented arguments for using feedback in control systems
- Next week, we will look at more closely on feedback control

Further Reading

- Nise – Sections 5.1-5.3
- Franklin & Powell – Section 3.2