SYNOPSIS

This manual contains a detailed description of a computer program which enables the user to analyse the stability of an earth slope using Bishop’s simplified method for circular failure surfaces or Morgenstern and Price’s analysis for non-circular surfaces.

The program, XSLOPE for Windows, is written in Microsoft Visual Basic v5.0 with the supporting dynamic link library, VBXSLOPE.DLL, written in Lahey Fortran 90 v4.5. The program runs on Windows 95/98/NT.

The program has been developed in the Centre for Geotechnical Research (CGR) within the School of Civil and Mining Engineering, University of Sydney.

Nigel P. Balaam

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DISCLAIMER

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Although considerable care has been taken to ensure the accuracy of this software and its manual, the author and the University of Sydney accept no responsibility for the accuracy of the results obtained from their use.

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1. INTRODUCTION

1.1 WHAT IS XSLOPE?

XSLOPE for Windows is a computer program that enables the user to compute the stability of an earth slope using Bishop's (1955) simplified method for circular failure surfaces or Morgenstern and Price's (1965, 1967) analysis for non-circular failure surfaces. The slope may be divided into a number of soil layers with different soil properties. XSLOPE has been developed and modified over the last twenty years with the first DOS version made commercially available in 1982.

In the Bishop analysis a circular surface of rupture is assumed and then the equilibrium of the sliding mass of soil is considered by dividing this mass into a number of slices (Figures 1 and 2). This process is repeated for a large number of circles and the minimum factor of safety determined. Failure by clockwise or anti-clockwise rotation can be considered. The effort in locating the critical circle is considerably reduced by employing the option to automatically generate the centres and radii for a specified number of circles. The algorithm used by XSLOPE provides a very good estimate of the location of the critical centre for a wide variety of problems. This initial estimate can then be used to size and locate a rectangular grid of circle centres.

The Morgenstern and Price analysis is fully interactive, i.e. the non-circular failure surface is defined and modified by using the mouse. As the surface is updated the analysis is performed and the results continually updated and tabulated on the screen.
Pore pressures within each soil layer can be calculated by a number of different methods, viz. from the depth below a piezometric surface, by using a pore pressure coefficient $r_u$, from a user specified grid of pore pressures, or from a grid of pore pressures generated by program FESEEP. This flexibility provides a mechanism for modelling all possible pore pressure variations.

Soil layers can be assigned non-uniform strength properties, viz. cohesion and friction angle values can be specified in the horizontal and vertical directions. This then provides the mechanism for analysing soils that exhibit anisotropic strength. Additionally, for each soil layer the values of cohesion and friction angle can vary linearly below a specified elevation and therefore soils with linearly increasing anisotropic strength can be modelled.

External normal and shear tractions can be applied to segments along the surface of the slope. The effect of an earthquake is modelled by applying a set of horizontal and vertical forces at the centroid of each slice. These forces are calculated using the horizontal and vertical seismic coefficients which are assumed to vary with depth.

**XSLOPE for Windows** replaces the DOS version that was made commercially available in 1982. It is a fully interactive program that provides the user with options to assemble and edit the data, perform analysis, inspect the output file and graphically check the data and plot the
results of the analysis. An interactive analysis option is also available that allows the user to interactively analyse circles as the mouse is moved.

1.2 ABOUT THIS MANUAL

You can use this manual as a tutorial to learn how to use XSLOPE and for reference after you have become familiar with the program.

This manual contains the following major sections:

- Introduction - describes what XSLOPE does and how to use this manual.
- Getting Started - describes how to install the program and how to get additional help.
- Program XSLOPE - this section describes the program, shows how to initiate the program and traces a typical program session.
- Program Options - in this section detailed notes are provided for the options available.
- FESEEP - this section provides documentation on how to generate a grid of pore pressures suitable for use with program XSLOPE. The pore pressures are calculated by program FESEEP for Windows which performs a finite element analysis of steady state seepage.
- Examples - illustrates typical problems that can be solved with XSLOPE, the input data and output generated by XSLOPE are included.
- Specification of the Input Data - this appendix describes the data required for the successful execution of program XSLOPE.
- Method of Analysis - this appendix sets out the governing equations for Bishop's simplified and the Morgenstern and Price methods.
2. GETTING STARTED

This section will help you install the XSLOPE program and describe the files that are included in this package.

2.1 SYSTEM REQUIREMENTS

XSLOPE for Windows has been developed for use on a 100% IBM PC compatible computer running Windows 95/98/NT. The minimum (and optimum) screen resolution is 800x600. The optimum color setting for the monitor is 256 colors.

The program has been developed using Microsoft Visual Basic v5.0. The supporting dynamic link library, VBXSLOPE.DLL, has been generated using Lahey Fortran 90 v4.5.

2.2 INSTALLING XSLOPE

You install XSLOPE for Windows on your computer using the Setup program which is on the distribution CD. The Setup program decompresses and installs the files in the appropriate directories.

A series of files are copied to the application directory, that by default is directory “xslope”. These files are listed below.

i) Program

XSLOPE.EXE Stability Analysis of Earth Slopes.

ii) Example problems

*Analysis of a submerged slope*

Ex1Submerged.D00 Data file.
Ex1Submerged.P00 Plotting file.
Ex1Submerged.S00 Solution file.
Analysis of an embankment with steady seepage

Ex2Seepage.D00 Data file.
Ex2Seepage.P00 Plotting file.
Ex2Seepage.S00 Solution file.
Ex2Seepage.POR Pore pressure grid file.

2.3 PROGRAM SUPPORT

If you are having trouble installing XSLOPE for Windows on your computer, or if you have a problem that you are encountering when running XSLOPE, call Nigel Balaam on either: 02-9351-2193 (Office) or 02-9351-3923 (Centre for Geotechnical Research, University of Sydney) or FAX 02-9351-3343. Mark the faxes for the attention of Nigel Balaam.

Email: N.Balaam@civil.usyd.edu.au
3. PROGRAM XSLOPE

3.1 DESCRIPTION OF PROGRAM

XSLOPE for Windows computes the stability of an earth slope using Bishop's (1955) simplified method for circular failure surfaces or Morgenstern and Price's (1965, 1967) analysis for non-circular failure surfaces. The slope may be divided into a number of soil layers with different soil properties. In the Bishop analysis a circular surface of rupture is assumed and then the equilibrium of the sliding mass of soil is considered by dividing this mass into a number of slices (Figures 1 and 2). This process is repeated for a large number of circles and the minimum factor of safety determined. Failure by clockwise or anti-clockwise rotation can be considered. The effort in locating the critical circle is considerably reduced by employing the option to automatically generate the centres and radii for a specified number of circles. The algorithm used by XSLOPE provides a very good estimate of the location of the critical centre for a wide variety of problems. This initial estimate can then be used to size and locate a rectangular grid of circle centres.

The Morgenstern and Price analysis is fully interactive, i.e. the non-circular failure surface is defined and modified by using the mouse. As the surface is updated the analysis is performed and the results continually updated and tabulated on the screen.

Pore pressures within each soil layer can be calculated by a number of different methods, viz. from the depth below a piezometric surface, by using a pore pressure coefficient \( r_u \), from a user specified grid of pore pressures, or from a grid of pore pressures generated by program FESEEP. This flexibility provides a mechanism for modelling all possible pore pressure variations.

External normal and shear tractions can be applied to segments along the surface of the slope. The effect of an earthquake is modelled by applying a set of horizontal and vertical forces at the centroid of each slice. These forces are calculated using the horizontal and vertical seismic coefficients which are assumed to vary with depth.

Several features of the program are worth highlighting:

- The circles are sorted into ascending order of factor of safety before being written to the solution file. The sorting routine used is very efficient and was published by Houlsby and Sloan (1984).

- The centres and radii for a user-specified number of circles can be generated automatically. The algorithm provides a good initial estimate of the location of the critical centre. This initial estimate can subsequently be used to determine the location and size of a user-specified grid of circles. Using this approach the critical circle can be determined very efficiently.
XSLOPE - DESCRIPTION

- A comprehensive summary table may be generated for a circle, which lists for each slice the values used in computing the factor of safety. This provides the last essential step in design, in which the values of weight, normal force, pore pressure etc. for each slice in the critical circle are checked before reporting this circle's factor of safety.

- The methods that are available for defining the pore pressures provide sufficient flexibility so that any pore pressure variation throughout the slope can be adequately modelled. One of these methods involves the use of the pore pressures computed in a finite element steady state seepage analysis using program FESEEP for Windows. This method enables a consistent analysis of the stability of earth slopes in which steady state seepage is occurring.

- Interactive analysis can be performed. After the entry and exit points of the circles on the surface of the slope are defined circles are automatically analysed by moving the mouse if the Analysis-Interactive (Mouse move) option is selected. If the Analysis-Interactive (Mouse click) option is selected then a circle is analysed only when the left button on the mouse is pressed. During the interactive analysis a record is kept of the circle analysed with the minimum factor of safety. This circle is plotted in red and its factor of safety value is tabulated in red.

- Non-circular failure surfaces can be analysed interactively using the Morgenstern and Price analysis. The non-circular failure surface is defined and modified by using the mouse. As the surface is updated the analysis is performed and the results continually updated and tabulated on the screen. Four side force functions are considered and the lines of thrust from each of these assumptions can be plotted.

- Soil layers can be assigned non-uniform strength properties, viz. cohesion (c) and friction angle (\(\varphi\)) values can be specified in the horizontal and vertical directions. This then provides the mechanism for analysing soils that exhibit anisotropic strength. Additionally, for each soil layer the values of c, \(\varphi\) can vary linearly below a specified elevation and therefore soils with linearly increasing anisotropic strength can be modelled.
3.2 A TYPICAL PROGRAM SESSION

A typical program session with XSLOPE will now be described in order to illustrate the program’s capabilities.

The steps involved in assembling the data file required to define the problem are:

- The first step in assembling the data required to define the stability problem being considered is to establish the geometric model. Divide the slope into the number of soil layers you want to model and number the layers sequentially downwards starting with the top layer set to layer number one. The base of the model is set at \( y = 0 \) and the material below the base is assumed to have infinite strength. Thus, circles that intersect the base of the model, the left or right hand vertical sides of the model are not analysed because they are assumed to have intersected with material with infinite strength. The \((x)\) coordinates assigned to the left and right hand ends of the model should be chosen so that the aspect ratio of the model is appropriate. If the results of the analysis indicate that the critical circle enters at the right hand side or exits at the left hand side of the model this may indicate that a lower factor of safety will be found if the lateral extent of the model is increased.

- The surface of each soil layer is defined by straight line segments. The changes in slope along the segments must be defined by nodes. These nodes must be assigned numbers which increase sequentially with the first node being assigned node number one. There is no restriction on the order in which you assign these node numbers. The definition of the surface of each layer must commence at the left hand end of the model and finish at the right hand end of the model.

- The second step is to consider how the pore pressures in each layer are to be calculated.

- The third step is to assemble the details of the surface loads if any are being applied.

- The fourth step is to tabulate the soil properties that are going to be assigned to each soil layer.

- This step involves assembling the remaining items of data that are required, e.g. earthquake loading coefficients, unit weight of water, etc. An important decision is what option should be selected for initially analysing circles. The recommended approach in most situations is to automatically generate the circle coordinates and initially specify 1,000 circles.

When these steps are completed invoke XSLOPE and select the Data option. A series of data entry grids, dialog boxes and text boxes are made available for entering the required data. When you are assembling a new data file the sequence that the data entry is performed is controlled by the program because an extensive set of checks are made on the integrity of the
XSLOPE - A Typical Program Session

data before the next data item can be entered. As the data is assembled it is recommended that you frequently invoke the Save Data command.

When the data has been assembled the Return To Graphics command displays the graphics form. Check the data graphically with the graphics options that are available. If the data requires modification the Data option should be selected and the data entry form will be displayed. Press the appropriate radio button (e.g. ● Nodal coordinates) and the data entry facilities for editing/entering the nodal coordinates will appear.

When the data has been assembled correctly select the Analysis-Show form option and the analysis form should be displayed. Press the Start command button and the analysis should commence.

When the analysis is completed a grid will be displayed with the results of the analysis of each circle. Several methods are available for verifying the results, i.e. scrolling through the grid that displays the results, inspecting the solution file (View Solution File command) or returning to the graphics form (Back to Graphics command) and plotting the results. The circles are plotted using the Circles option and the Grid option if you have defined a grid of circles. If the results of this analysis are not sensible careful inspection of the data that has been specified is required. However, if the results of this preliminary analysis are acceptable the recommended approach is to select the Analysis-Show form option again and now specify that you want more circles to be automatically generated, e.g. 5,000 circles. This can be done by checking the check box on the analysis form that has the description: Automatically generate circles No. = [ ] and entering the required number of circles in the text box.

After this analysis has been completed the critical circle should be re-analysed and the output option set to full tabular details. The critical circle should be plotted and the position noted. Some time should now be allocated to performing an interactive analysis, Analysis-Interactive (Move mouse) or the Analysis-Interactive (Move move) option. If during the interactive analysis you locate a circle with a lower factor of safety than the value calculated when the circle coordinates were automatically generated this circle should be re-analysed and the full tabular output produced. This can be done by selecting the Analysis-Show form option and checking the Analyse critical circle from the interactive analysis check box.

The final step is to check the validity of the solution by inspecting the full tabular output for the critical circle. Checks should be made of the values used in the computations (e.g. pore pressures, strength properties etc.). The solution should only be reported when all these values have been checked and deemed to be acceptable.
XSLOPE - INITIATING

3.3 INITIATING PROGRAM XSLOPE

After initiating XSLOPE a title page is displayed followed by the screen shown in Figure 3.

The actions performed when the command buttons are clicked are described below:

- **DIR**
  - Displays the common Windows file dialog control. If the cursor is in the “Data File (.d00)” text box the file dialog control has the first file filter set to “*.d??”. However, if the cursor is in the plot or solution file text boxes the file dialog control has the first file filter set to “*.p??” and “*.s??” respectively.

- **...**
  - Copies the filename that has been entered in the text box above the currently selected box into the remaining text boxes with the appropriate changes being made to the filename extensions.

- **Check Files Found**
  - After the filenames have been entered into the text boxes click this button. A check is then performed to determine which of the three files exist and check marks are placed in the appropriate boxes in the “Files Found” frame when a file is located. After the filenames have been correctly specified click the “Continue” command button and the main graphics screen will be displayed.
Figure 3 File Specification Screen
4. PROGRAM OPTIONS

After the “Continue” button has been clicked the screen is cleared and a set of options is displayed across the top line of the screen. The arrangement and appearance of the menus is shown below. Options can be highlighted and selected from the menu using the mouse.

Some of the options are also selected by clicking an icon along the toolbar. Referring to the labels “a” to “u” above and the icon immediately above the label, the option selected by clicking the icon is listed below.

- a Draw geometry
- b Paint layers
- c Draw loads
- d Label nodes
- e Label date
- f Draw all circles
- g Draw circles
- h Draw circle centres
- i Show analysis form
- j Interactive analysis of circles
- k Label scale
- l Non-circular analysis
- m Window-Define
- n Window-Specify
- o Window-Zoom
- p Window-Reset
- q Dump-Printer
- r Dump-Clipboard
- s Dump-PostScript (Mono) to disk
- t Dump-PostScript (Colour) to disk
- u Quit

Each primary menu option and any secondary menu options (if appropriate) are listed below in the order that they appear across the menu line. Detailed notes are provided for options which are not self-explanatory.

Data

This option is used to enter the data required for the analysis. When this option is selected a screen is displayed that is similar in appearance to Figure 4. The appearance will depend on whether a new data file is being assembled and what data options are being employed. When a new data file is being assembled the only radio button that will be initially visible is Maximum values. When the Next Data Item command button is clicked the Job title radio button will become visible and a text box for entering the job description. No checks are made of what is entered for the job description. A further click of the Next Data Item command button and the Unit weight of water radio button will become visible and a text box for entering the unit weight of water. For this data item and all subsequent data items a check on the validity of the entered data is made when the Next Data Item command button is clicked. If the data that has
been entered is not valid an informative message will be displayed and the data must be corrected before the next radio button becomes visible.

Thus, the data screen shown below is only displayed after the Data option has been selected and a complete data file has been established previously. Further, the Pore pressure grid radio button is disabled because in this analysis the pore pressures are being calculated without the use of a pore pressure grid.

![Data Screen](image)

**Figure 4  Data Screen**

Many of the data entry facilities are self explanatory and therefore only those considered to require explanation are considered further.

- **Soil layer definitions**

  This radio button is selected in order to enter or edit the nodes defining the top surface of each soil layer. The screen display is shown in Figure 5.
The node numbers can be separated by commas or spaces and there is no restriction on the number of nodes defined on each line of the text box. The spin buttons are used to step through the definitions of each soil layer.

**Soil properties**

This radio button is selected in order to enter or edit the soil properties. The screen display is shown in Figure 6. The “Pore Pressure Calculations” column will not be visible when the pore pressures in all the layers are calculated using the piezometric surface only or using pore pressure coefficients only. The method used for calculating the pore pressures in a layer can be changed by pointing at the text with the mouse cursor, e.g. “Piezometric surface” and clicking the left button on the mouse. The text in the column will then change to “Pore pressure coefficient” and this will then be the method used to calculate the pore pressure in that layer.
XSLOPE - Soil properties

In the example shown in Figure 6(a) the third layer has been highlighted and the Ins key pressed. When the Ins (Insert key) is pressed the pop-up menu shown in this figure is displayed. When soil layers are added the layers below the inserted layer have their layer number incremented by one. When soil layers are deleted the layers below the deleted layer have their layer number decremented by one. The number of soil layers and the layer definitions are automatically changed to reflect the insertion or deletion but remember you will have to go back and select the Soil layer definitions radio button and add the definitions for the newly inserted layer(s).

![Figure 6(a) Grid Used to Enter Soil Properties](image)

When the Non-Uniform Soil Properties command button is pressed the screen is updated and the grid used to enter the non-uniform soil properties is displayed as shown in Figure 6(b).

![Figure 6(b) Grid Used to Enter Non-Uniform Soil Properties](image)

Initially the values of each of the non-uniform strength properties are set to zero and the entry in the "NonU?" column is set to "No".
The "NonU ?" column specifies whether the non-uniform soil strength values are used to calculate the shear strength in each layer. The default setting is "No" and therefore the uniform values of cohesion and friction angle specified for the soil layer are used to calculate the shear strength within the soil layer and the non-uniform strength values are ignored.

The values used to calculate the shear strength within a layer can be changed by pointing at the text with the mouse cursor, e.g. “No” and clicking the left button on the mouse. The text in the column will then change to “Yes” and then the shear strength within that layer will be calculated using the non-uniform strength values specified for that layer.

The non-uniform values are described below:

- **Coh-h**: Cohesion in the horizontal direction.
- **Coh-v**: Cohesion in the vertical direction.
- **Coh-m**: Rate of change of cohesion values with depth.
- **Phi-h**: Friction angle in the horizontal direction.
- **Phi-v**: Friction angle in the vertical direction.
- **Phi-m**: Rate of change of friction angle values with depth.
- **y(surf)**: Elevation of the surface from which the rates of change are calculated.

- **NonU?**
  - **Yes**: The Non-uniform strength values are used to calculate the shear resistance within this layer.
  - **No**: The cohesion and friction angle values that have been specified for this layer are used to calculate the shear resistance within this layer.

### Piezometric surface

The points that define the position of the phreatic surface can be added or deleted by pressing the Ins key.

### Surface loads

Surface loads can be added or deleted by pressing the Ins key.
XSLOPE - Seismic Function

Seismic

This radio button is selected in order to enter or edit the step functions that define the variation of the seismic coefficients with depth. XSLOPE can model situations in which both the vertical and horizontal seismic coefficients vary with depth. In the example shown in Figure 7 the number of points required (NSEIS) to define the variation of these coefficients with depth is set to 3. Two step functions are defined by specifying NSEIS (y) coordinate values and the corresponding values of both the horizontal and vertical coefficients.

Figure 7 Seismic Function Details

Notes:

- The maximum number of points that can be used to define the step function is twenty.
- The step functions must be specified starting with the values at the maximum elevation and ending with the values at the minimum elevation.
- In many situations only a constant value for the horizontal seismic coefficient is used to model the loadings imposed due to earthquake activity. For this case specify only one point on the step function and specify the value for the horizontal seismic coefficient and set the vertical coefficient to zero. The (y) coordinate value is not used in the calculation and can therefore be specified as any convenient value.
- The points that define the step functions can be added or deleted by pressing the Ins key.
When the “Show form” option is selected the main analysis form is displayed. The form shown in Figure 8 has been displayed after an analysis was performed. The functions performed by most of the command buttons are considered self-explanatory. However, the check boxes require some further explanation. The number of circles that are automatically generated can be changed without returning to the data screen. This is done by clicking the check box **Automatically generate circles** No. = [___] and entering the number of circles in the text box.
The final step in the analysis procedure should be to re-analyse the critical circle and produce the full tabular output. This should then be thoroughly checked before the results from the analysis are reported. The critical circle (or any circle tabulated in the “Results” grid) can be re-analysed by firstly highlighting the circle in the results grid. Do this by moving the mouse cursor over the appropriate line in the results grid and pressing the left button on the mouse. Next, check the Analyse highlighted circle check box, click the Comprehensive summary table radio button and finally click the Start Analysis command button.

The circle with the lowest factor of safety from an interactive analysis can be re-analysed by checking the Analyse critical circle from the interactive analysis check box.

**Analysis → Interactive (Mouse move) → Interactive (Mouse click)**

When the Mouse move option is selected circles are automatically analysed by moving the mouse whereas if the Mouse click option is selected then a circle is analysed only when the left button on the mouse is pressed.

However, before any circles are analysed the entry and exit points of the circles on the surface of the slope must be defined. The details of how to define the entry and exit point are given below.

- Move the mouse cursor to a point on the surface. This point can be either the entry or exit point of the circles because the order in which the entry and exit points are defined does not have any effect.
- Press the left button on the mouse and the first point is "latched" onto the surface.
- Move the mouse cursor to a second point on the surface. Press the left button on the mouse and this second point is "latched" onto the surface.

These two points which have been defined are the entry and exit points for all the circles which are subsequently analysed until new entry and exit points are defined. New entry and exit points can be defined by pressing the right button on the mouse and then repeating steps one through three.

If the Mouse move option has been selected moving the mouse defines a new circle which is drawn and automatically analysed. The circle's centre coordinates, radius and factor of safety are tabulated on the first line of the results grid displayed in the top left hand corner of the plot (Figure 9).

If the Mouse click option has been selected moving the mouse defines a new circle and it is drawn. However, the circle is only analysed when the left button on the mouse is pressed.
When a circle is analysed the centre coordinates, radius and factor of safety are tabulated on the first line of the results grid.

During the interactive analysis a record is kept of the circle analysed with the minimum factor of safety. This circle is plotted in red and its factor of safety value is tabulated in red on the second line of the results grid.

Figure 9  Interactive Analysis
Non-Circular

This option is enabled when the NC icon is selected. When the Non-Circular icon is selected the geometry is drawn and an initial non-circular surface is generated, drawn, the Morgenstern & Price analysis performed and the results tabulated. The generated surface will normally consist of four points whose coordinates are generated from the coordinates of the points forming the top surface of the slope. The (x,y) coordinates of a point forming the non-circular surface can be changed in the following way. Move the cursor over the point, press the left button on the mouse, keep the button depressed and drag the point into the new position. As the coordinates of the point change, and therefore the definition of the failure surface changes, the analysis is performed and the new results tabulated at the top left hand corner of the screen.

The Non-Circular options available are shown below.

![Non-Circular menu options](Figure 10)

**Figure 10 Non-Circular menu options**

**Key Presses**

Referring to Figure 10, note that most of the options have a letter in square brackets at the end of the description. The letter in square brackets provides a short cut.
For example, the first option is **Add point [a]**. The "[a]" indicates that you can add a point to the definition of the non-circular surface by pressing the "a" key on the keyboard, moving the mouse to the point where you want to add the point and pressing the left button on the mouse. Alternatively, you can select **Non-Circular -> Add point [a]** from the menu, move the mouse to the point where you want to add a point and press the left button on the mouse.

The options that are available are now described.

**Add point [a]**

Add a point to the non-circular surface. Move the mouse to the point where you want to add a point and press the left button on the mouse.

**Delete point [d]**

Delete a point from the non-circular surface. Move the mouse to the point you want to delete and press the left button on the mouse.

**Keep as minimum [k]**

Store the coordinates of the previously analysed surface. This surface can then be re-analysed when required.

**Plot minimum circle [p]**

If a circular analysis has been performed, when this option is selected the circle with the minimum factor of safety from that analysis is drawn.

**Re-analyse minimum [r]**

The surface that has been stored with the **Keep as minimum [k]** is drawn and re-analysed.

**Side force function**

When this option is selected a frame is displayed as shown below in Figure 11.
Referring to the figure above, the eleven text boxes contain the user-specified values of the side force function used in the Morgenstern & Price analysis that XSLOPE performs. This side force function is one of four used by XSLOPE, the other functions are a constant value of one (referred to on the screen tabulation of the results as "Unity", sine curve ("Sine") and a trapezoidal function ("Trapez").

You can change these user-specified values of the side force function by entering new values in the text boxes. These new values are written to file "xslope.ini" when the Write values to "xslope.ini" command button is pressed. When XSLOPE is initiated it reads the contents of file "xslope.ini" and sets the values of the user-specified side force function to the values stored in this file.

Specify coordinates [s]

Normally you will change the coordinates of a point by dragging the point into a new position. However, in some situations you will want to specify the (x,y) coordinates of a point explicitly. When this option is selected, move the cursor over the point and press the left button on the mouse. A text box will appear and the current (x,y) coordinates of the point will be displayed. Enter the required values, press the Enter key and the point will be re-positioned. The x and y coordinates must be separated by a comma.
XSLOPE - Non-Circular Analysis

**Tension crack -> Apply**

The default setting is that a tension crack is not included in the analysis. If you want a tension crack to be included select this option. The default value for the depth of the crack is:

\[ 1.33 \times \text{cohesion} / \text{unitw} \]

The "cohesion" is the cohesion of the first soil layer and "unitw" is the unit weight of the first soil layer. You can change the default value for the depth of tension crack by selecting the **Tension crack -> Specify depth** option.

When a tension crack is included in the analysis a check mark will appear next to "Apply". Remove the tension crack from the analysis by selecting **Tension crack -> Apply** again.

**Tension crack -> Specify depth**

When this option is selected a frame is displayed that contains a text box with the current value for the tension crack depth. Enter the new value for the tension crack depth and press the **Ok** command button.

**View solution file** [v]

When this option is selected the solution file is displayed. You can use this option to inspect the results of the analyses that you have written to the solution file using the **Write solution summary to file** or the **Write full solution details to file** option.

**Write solution summary to file** [w]

A brief summary of the solution's convergence is written to the solution file. An example of a solution summary is shown below. A capture of a screen display after the **View solution file** option has been selected is shown in Figure 12.

**Write full solution details to file**

When this option is selected a comprehensive summary of the solution is written to the solution file. This includes tables detailing the values of the variables for each slice.
THRUST LINES

In the Morgenstern and Price analysis an assumption is made about the distribution of the interslice normal (E) and shear forces (X) in order to render the problem statically determinate. The assumption is that at an interslice the relationship between the forces can be expressed in the following way:

\[ X = \lambda \cdot f(x) \cdot E \]

where

- \[ X \] = shear force acting on the interslice
- \[ \lambda \] = scaling factor
- \[ f(x) \] = side force function
- \[ E \] = normal force acting on the interslice

XSLOPE computes the factor of safety for four different side force functions, i.e. a constant or no variation, sinusoidal variation, trapezoidal and a user specified variation. The different side force
functions result in different values for the factor of safety being calculated and lines of action along which the normal force (E) acts. These lines of action are referred to as "thrust lines".

A consistent solution requires that the line of thrust plot above the failure surface and below the surface of the slope. In many situations one or more of the side force functions will result in lines of thrust lying outside the acceptable region while the remaining functions produce acceptable lines of thrust. It is worth noting that in many of these cases the factor of safety is relatively insensitive to whether an acceptable thrust line has been calculated.

Analyses that involve cohesive soils can produce lines of thrust that plot outside the acceptable region near the top of the failure surface. This indicates that tension has occurred and that the use of a tension crack in the analysis may rectify the problem.

The line of thrust computed for a side force function is plotted when you move the mouse over the table in the top left hand corner of the screen and click the row corresponding to the side force function of interest. In the figure below the thrust line calculated using the trapezoidal side force function (third row on table) is plotted.

![Figure 13 Thrust Line Calculated from Trapezoidal Side Force Function](image)

In this example the comment column that forms part of the tabulation of results has the text "Converged - Check LT". This comment indicates that a converged solution has been obtained.
within the allowable 60 iterations but that the line of thrust lies outside the acceptable region and that you should plot the line of thrust. The line of thrust shown in Figure 13 just plots outside the acceptable region towards the top of the failure surface. Whether this is ignored and the factor of safety (= 1.39) is adopted becomes an engineering judgement that you must make.

ANALYSIS COMMENTS

The tabulation of the results of the non-circular analysis includes a comment column. The comments that appear in this column are described below.

Converged
A converged solution has been calculated and the line of thrust plots above the failure surface and below the surface of the slope at each interslice.

Converged - Check LT
A converged solution has been calculated BUT the line of thrust does not plot above the failure surface and below the surface of the slope at each interslice.

Not feasible
The iterative process has resulted in a solution that is not feasible. The definition of a feasible solution is defined by Morgenstern and Price (1967).

Not converged
XSLOPE performs the iterative process that is described by Morgenstern and Price (1967). It allows for 60 iterations. This comment indicates that a converged solution could not be found after 60 iterations.

PHI(Lamda,F)
The basis for solving the equations described by Morgenstern and Price (1967) is the Newton-Raphson method together with certain controls that are placed on the values of λ (a scaling factor in the relationship between the shear and normal forces) and F (Factor of Safety). The authors define a function Φ that must behave in a certain way from iteration to iteration. This comment indicates that this function does not behave in the required way.

Piping
If piping occurs at the base of one or more slices along the failure surface the word "Piping" will precede the descriptions described above.
Geometry

This option produces a plot of the geometry.

Chk_Mat

Notes:

(1) When the **Label** option is selected the geometry is drawn and the properties of each soil layer are tabulated.

(2) When the **Paint** option is selected the first soil layer is colour filled and the properties of the soil layers tabulated. The **Next**, **Previous** and **Clean** command buttons can be used to either step incrementally through the definition of the layers or check a particular layer’s definition.

(3) The default setting is to tabulate the values of cohesion, friction angle, unit weight and if one is used the pore pressure coefficient. This default setting is indicated by the check mark against the **Properties** option.
When the Non-Uniform Strength option is selected the check mark is removed from the Properties option and placed next to the Non-Uniform Strength option. Now when the Label or Paint options are selected the values tabulated are the non-uniform strength details. The values are described below:

Coh-h   Cohesion in the horizontal direction.
Coh-v   Cohesion in the vertical direction.
Coh-m   Rate of change of cohesion values with depth.
Phi-h   Friction angle in the horizontal direction.
Phi-v   Friction angle in the vertical direction.
Phi-m   Rate of change of friction angle values with depth.
y(surf) Elevation of the surface from which the rates of change are calculated.

NonU?   Yes

The non-uniform strength values are used to calculate the shear resistance within this layer.

No

The cohesion and friction angle values that have been specified for this layer are used to calculate the shear resistance within this layer.

Loads

When the Normal option is selected the tractions acting normal to the surface are plotted. The shear tractions are plotted when the Tangential option is selected. The With table and Without table options act as toggles. The Superimpose option can be used to superimpose the tractions acting normal to the surface onto a plot.
A rectangular “window” can be placed over the plot in the form of a “rubber-band” box. The size of the box can be changed using the mouse. When the position and size of the box have been set all subsequent plots only show the lines that were within this box but the resulting picture is drawn using the full size of the plotting tablet. A “window” can be placed on a previously defined “window” in order to magnify an area of particular interest. The method used to define the “window” using the mouse is described in more detail below.

**METHOD**

1. Invoke the **Window-Define** option. Move the cursor around the screen using the mouse. The (x,y) coordinates of the cursor are displayed in the top right hand corner of the plot.

2. Press the left button on the mouse and fix the position of one corner of the rectangular box.

3. When you move the mouse a “rubber-band” box will appear. Move the mouse and the box will shrink or expand. Position the box over an area of interest. The coordinates which are now displayed in the top right hand corner of the plot are those corresponding to the diagonally opposite corner of the rectangular box fixed in step 2.

4. Press the right button and set this “rubber-band” box as the current window. All subsequent options are plotted using this window until a new window is defined or the **Window-Reset** option is invoked.
The coordinates displayed in the top right hand corner of the plot are useful because they define the limits you set for your current window. If you note the coordinates of the corners of the rubber-band box it will be possible to reproduce the same window after the window has been reset to the full size using the **Window-Reset** option. However, because of the limitations of screen resolution it may not be possible to reproduce precisely the same window. This can occur because the graphics system firstly reads screen coordinates using the display's resolution and then maps these into the coordinates used to define the mesh geometry. Thus, there will be a minimum finite interval in both the (x) and (y) coordinates that can be plotted, and this depends on the minimum resolution that the display is capable of outputting. If this finite interval in coordinates does not allow you to define the window you require, then use the **Window-Specify** option. When you invoke this option you will be requested to input the coordinates of the bottom left hand and top right hand corners of the rectangular box using the same coordinate system that defines the mesh geometry.

### Window-Zoom

A point in the form of a cross \( \mathbf{+} \) can be placed over the plot and fixed in position by pressing the left button on the mouse. When the position has been set, all subsequent plots only show the lines that are within a rectangular box which has the position of the cross as it’s centroid. The size of the rectangular box is automatically generated and reduces each time the left button on the mouse is pressed. The resulting pictures are drawn using the full size of the plotting tablet and therefore the effect of “zooming in” is created. Each time the right button on the mouse is pressed the size of the rectangular box is increased until the box is larger than the full sized plot giving the effect of “zooming out” from the point.

### Circles

When the **All** option is selected all the circles that have been analysed are drawn. If the **Show centres** option has been checked the geometry is re-scaled to allow all the circle centres to be identified. Each circle centre is identified with a small box and all the circles are drawn.
When the **Draw** option is selected the plot produced depends on whether the **Show centres** option has been checked. If this option has not been checked the critical circle is drawn and colour coded red. The results from the analysis are displayed in a grid at the top left hand corner of the plot. Any circle can be drawn by moving the mouse cursor over the appropriate row in the grid and pressing the left button on the mouse. The **Clean** command button can be used to clean the screen and redrew the last circle that has been selected.

If the **Show centres** option has been checked the tabulation of the results is not included on the plot. Further, lines are drawn from the entry and exit points of the circle to the circle centre and the circle centre is labelled with the computed factor of safety. Before plotting commences, the scale used to plot the slope is adjusted to accommodate drawing to the circle whose centre has the maximum (y) coordinate of all the circles analysed. Thus, if circles with large (y) coordinates (compared to the height of the slope) have been analysed, this adjustment to the scale may result in a “postage stamp” sized plot of the slope. The **y_centre (Maximum)** option provides a mechanism to rescale the geometry. When this option is invoked a prompt is displayed requesting a (y) coordinate value. The (y) coordinate that you enter is then used to rescale the subsequent plots and the circles with centres that plot above this maximum value are ignored. *Care should be taken when using this option.* If the critical circle has a (y) coordinate greater than the (y) coordinate value specified the critical circle is not drawn.

**Grid**

With this set of options you can thoroughly investigate the results obtained from the grid of circles that you have specified. The contour plots are only useful if the FoS computed at each grid point is a very good estimate of the minimum FoS that can be obtained at the grid point. In order to minimise the discrepancy between the computed value and the minimum that is obtainable, fifty or more circles should be analysed at each grid point.
XSLOPE - Size, Dump

Size

This option is useful for re-positioning and reducing the size of the plots. The default plotting tablet is set at the maximum size available. On some plots tables of values may be displayed so that they overlap the line plots. With the Size option you can shift the plot away from these tables.

Dump

Notes:

(1) When the Printer option is selected the Windows print dialog control is displayed. Problems may occur when attempting to print at resolutions greater than 300 dpi on printers with insufficient memory. Thus, when this occurs error free printing is only ensured on high resolution printers when the 300 dpi printer driver is selected.

(2) The Clipboard option copies a portion of the screen to the clipboard. The menu, toolbar and the display of the (x) and (y) coordinates are stripped off the top of the screen bitmap before it is copied to the clipboard. This option provides a mechanism for transferring the XSLOPE plots into word processors or other Windows programs that are capable of importing bitmaps from the clipboard. The whole screen is copied to the clipboard whenever the “Print Screen” key on the keyboard is pressed.

(3) The PS(Mono) to Disk and PS(Color) to Disk options prompt you for the name of a disk file before writing Encapsulated PostScript instructions to the specified file. These PostScript files can then be copied to the system port to which the PostScript printer is connected or imported into a large variety of programs which can import and process PostScript files.
(4) The **Size** option prompts you to input the width of the hardcopy plot in millimetres. The corresponding height for a “square plot” is then calculated by XSLOPE. This option only has an effect on the **PS(Mono) to Disk** and **PS(Color) to Disk** options.

**Options**

The appearance of the plots produced by XSLOPE can be altered using this option.

**Notes:**

(1) When a check mark appears next to an option on the sub-menu list it indicates that this is the currently active choice.

(2) The **Background colour** option toggles the background colour from white to grey.

(3) The **Border on** option enables you to control whether the plot border is drawn. The plot border is the box that is drawn around each plot including the description of the option, e.g., “GEOMETRY”, and the problem title that is printed along the bottom of the plot. The default setting is that the border is drawn.

(4) It is assumed by default that the currently selected printer is monochrome and therefore a check mark appears next to **Monochrome printing** on the sub-menu list. If a colour printer is available select the **Colour printing** option. This will remove the check mark next to **Monochrome printing** and place a check mark next to **Colour printing**. Plots that are subsequently printed will be produced in colour.

(5) The **Font for labelling** option enables you to select the font used when nodes and soil layers are labelled. When this option is selected a standard Windows font dialog control is displayed that you use to select the font characteristics. By default nodes and soil layers are labelled with 8 point “Times New Roman” font.
XSLOPE - Options -> Set Colours

Options -> Set Colours

This option can be used to select the colour sets used to plot the circles and check the material properties. Different sets of colours can be used on the screen and printer. When this option is selected a screen similar to the one shown below is displayed with four frames. The frames have titles; "Circles - Screen", "Circles - Printer", "Soils - Screen" and "Soils - Printer". Each frame contains option buttons, command buttons and text boxes that display the Red-Green-Blue (RGB) values for the currently selected sets of colours. You can use the option buttons to select a particular colour and the text boxes to manually enter new RGB values to modify that colour. Additionally, the command buttons can be used to: [1] display the Window's colour dialog and then select a new colour from this dialog, [2] reset the colours in the frame to the default values or [3] update the colours to the newly selected RGB values.

On the example screen display shown above the colour dialog is included. In this example the colour that the fourth circle on the printer is drawn in is being adjusted. The fourth radio button down in the frame captioned “Circles - Printer” is selected and the colour dialog is displayed after the Show Colour Dialog command button is pressed. When a colour is selected from the dialog the RGB values that are displayed for the fourth circle are updated and the sloping coloured line to the right of the radio button redrawn in the newly selected colour. A newly selected colour is only used if the Update Colours command button is pressed.
XSLOPE - Options -> Set Colours

Note that the circle and soil colours are displayed in the order that they are plotted. The circle colour on the top row is the colour used to plot the circle with the minimum factor of safety (circle no.1), the circle colour displayed on the second row is the colour used to plot circle no.2, etc. Thus, the soil colour on the top row is the colour used to fill the first soil layer, the soil colour on the second row is the colour used to fill the second soil layer.

A command button is available on the form that can be used to write the RGB values to the file "Xslope.ini". If this file is available in the application directory when XSLOPE starts the RGB values are read from this file and are then subsequently used for plotting the circles and checking the material properties on either the screen or printer.

Help

This option displays the help topics that are available.
5. DETAILED NOTES

In this section detailed notes are provided on the data items and some of the computational features of XSLOPE.

5.1 SLOPE GEOMETRY

The following points should be noted when defining the problem:

1. The slope geometry may consist of a number of soil layers which are numbered sequentially from the surface down. The surfaces of separation between the soil layers are assumed to consist of a number of straight lines. These surfaces of separation must begin at \( x = x_{\text{min}} \) and end at \( x = x_{\text{max}} \) where \( x_{\text{min}} \) is the minimum (x) coordinate used to model the slope and \( x_{\text{max}} \) is the maximum (x) coordinate used to model the slope. Starting at the first point (\( x = x_{\text{min}} \)) and working across a surface of separation to \( x = x_{\text{max}} \), the points must have increasing x coordinates.

2. The points along the surfaces of separation are numbered arbitrarily. The base of the stability model is line \( y = 0 \). This line is not defined by point numbers.

5.2 MATERIAL PROPERTIES

Values for the cohesion, angle of friction and bulk unit weight must be provided for each layer. The units must be consistent. For example, a consistent set of units is:

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Equivalent Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion</td>
<td>kPa</td>
<td>tonnes force/m²</td>
</tr>
<tr>
<td>Bulk unit weight of soil</td>
<td>kN/m³</td>
<td>tonnes force/m³</td>
</tr>
<tr>
<td>Unit weight of water</td>
<td>kN/m³</td>
<td>tonnes force/m³</td>
</tr>
<tr>
<td>Traction</td>
<td>kPa/metre run</td>
<td>tonnes force/m²/metre run</td>
</tr>
</tbody>
</table>
5.3 NON-UNIFORM STRENGTH

The shear strength within a soil layer can be calculated using non-uniform strength values. The value of the cohesion and friction angle can vary with depth below a specified elevation and have different values in the horizontal and vertical directions.

In the analysis performed by XSLOPE the adopted variation of cohesion with direction is that proposed by Lo (1965) and is shown below:

\[ c_\theta = c_h + (c_v - c_h) \sin^2 \theta \]  

(5.1)

where

- \( c_h \) = cohesion on the horizontal plane
- \( c_v \) = cohesion on the vertical plane
- \( \theta \) = angle between the direction of a plane where \( c_\theta \) is measured and the horizontal direction

XSLOPE assumes that a similar variation occurs with the values of friction angle, i.e.

\[ \phi_\theta = \phi_h + (\phi_v - \phi_h) \sin^2 \theta \]  

(5.2)

where

- \( \phi_h \) = friction angle on the horizontal plane
- \( \phi_v \) = friction angle on the vertical plane
- \( \theta \) = angle between the direction of a plane where \( \phi_\theta \) is measured and the horizontal direction

The value of the cohesion on the horizontal plane \( c_h \) at a given depth is calculated in the following way:

\[ c_h = C(h) + (y - y) C(m) \]  

(5.3)

where

- \( C(h) \) = the specified value of cohesion in the horizontal direction. The heading in the data entry grid for this value is “Coh-h”. 

**XSLOPE - NON-UNIFORM STRENGTH**

\[ y_s = \text{the specified elevation below which the cohesion varies. The heading in the data entry grid for this value is “y(surf)”} \]

\[ y = \text{(y) coordinate at the mid-point of the slice base where the shear strength is being calculated.} \]

\[ C(m) = \text{the specified rate of variation of cohesion with depth. The heading in the data entry grid for this value is “Coh-m”} \]

If \((y_s - y) < 0\) then \((y_s - y)\) is set equal to zero, i.e. for all slice bases above the specified elevation \((y_s)\) the cohesion on the horizontal plane is set equal to \(C(h)\).

Similar expressions are used to calculate the cohesion on the vertical plane and the friction angles on the horizontal and vertical planes, i.e.

\[ c_v = C(v) + (y_s - y)C(m) \quad (5.4) \]

\[ \phi_h = \phi(h) + (y_s - y)\phi(m) \quad (5.5) \]

\[ \phi_v = \phi(v) + (y_s - y)\phi(m) \quad (5.6) \]

where

\[ C(v) = \text{the specified value of cohesion in the vertical direction. The heading in the data entry grid for this value is “Coh-v”.} \]

\[ \phi_h = \text{specified value of friction angle in the horizontal direction. The heading in the data entry grid for this value is “Phi-h”.} \]

\[ \phi_v = \text{specified value of friction angle in the vertical direction. The heading in the data entry grid for this value is “Phi-v”.} \]

Thus, the following procedure is followed by XSLOPE when calculating the shear resistance along the base of a slice.

1. The \((x,y)\) coordinates at the mid-point of the slice base are used to determine which soil layer the base of the slice lies in.

2. If non-uniform strength values are not being used in this layer the shear strength is calculated using the values of cohesion and friction angle specified for the layer.
(3) If non-uniform strength values are being used in this layer then equations 5.3, 5.4, 5.5 and 5.6 are used to calculate the values of cohesion and friction angles in the horizontal and vertical directions at the mid-point of the slice base.

(4) The angle that the base of the slice makes with the horizontal is then calculated and this is used in equations 5.1 and 5.2 to evaluate the value of cohesion $c_o$ and friction angle $\phi_0$ on this plane at this depth. These two values are then used to calculate the shear resistance provided by this slice.

Two situations where the non-uniform strength properties may be used are now considered and the method for specifying the data is detailed.

Example [1]

In most situations it is envisaged that the non-uniform strength values will be used to model soils/soft rocks that exhibit anisotropy for the cohesion but not the friction angle.

Consider a soil whose cohesion in the vertical direction is 50Kpa, in the horizontal direction 100Kpa, with a friction angle equal to zero and there is no variation in the cohesion values with depth. This situation would be modelled by setting:

<table>
<thead>
<tr>
<th>Coh-v</th>
<th>Coh-h</th>
<th>Coh-m</th>
<th>Phi-v</th>
<th>Phi-h</th>
<th>Phi-m</th>
<th>y(surf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that the value specified for "y(surf)" is not relevant because both "Coh-m" and "Phi-m" are set equal to zero and therefore the terms in equations 5.3, 5.4, 5.5, and 5.6 that include "y(surf)" are multiplied by zero.

Example [2]

Consider the situation where the undrained cohesion varies with depth at the rate of 10Kpa/metre and that at the surface of an 8m high cutting the cohesion is zero. The clay exhibits no anisotropy. This situation would be modelled by setting:

<table>
<thead>
<tr>
<th>Coh-v</th>
<th>Coh-h</th>
<th>Coh-m</th>
<th>Phi-v</th>
<th>Phi-h</th>
<th>Phi-m</th>
<th>y(surf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>
5.4 PORE PRESSURES

XSLOPE provides a method for conveniently treating pore pressures in nearly all situations encountered in practice. The pore pressure at the base of each slice can be calculated using either:

1. the piezometric surface specified for the slope
2. the pore pressure coefficient for the soil layer which contains the base of the slice,
3. a pore pressure grid. The values on the grid can be specified as data or generated using program FESEEP.

In most situations the use of a piezometric surface and/or pore pressure coefficients is appropriate. A particular case which occurs frequently in analysis is the case where there is zero pore pressure in the soil. The zero pore pressure case can be analysed by using a pore pressure coefficient ($r_p$) and setting it equal to zero.

The piezometric surface is defined by the coordinates of the points at the changes in slope of the straight lines which constitute this surface. These points are not numbered in the same way as the surface of separation defining the layers. For each slice the pore pressure is calculated from the depth below the surface to the base of the slice. Negative pore pressures above this piezometric surface are not considered.

The use of an equally spaced pore pressure grid is convenient when analysing a slope with an artesian layer. The pore pressure parameter IPORE (Appendix A - Data Item [9]) for this layer should be set to 3 and an equally spaced grid of pore pressures specified with values corresponding to the artesian pressure. In other layers IPORE should be set to 1 or 2. See Figure 14 for definitions of the parameters used in specifying an equally spaced grid of pore pressures.

In some situations it may be expedient to specify the pore pressures on an unequally spaced grid system. See Figure 15 for details of its specification. For example, when piezometer readings are available their positions are normally "scattered" in the $x$-$y$ plane. The provision of an unequally spaced grid pattern allows readings to be fitted onto the grid system.

When steady state seepage conditions exist special attention is required when considering the pore pressures. The pore pressure at the base of each slice should be calculated from the appropriate flow net. This results in each circle having its own unique piezometric surface. Therefore, specifying a single piezometric surface for more than one circle may lead to significant errors in some cases. A more accurate solution can be obtained by firstly performing a finite element seepage analysis using program FESEEP and then using the pore pressures from this analysis in the stability analysis. In this case program FESEEP can be used to generate the pore pressure grid.
The same problems occur if the pore pressures are transient, for example when considering the stability of an embankment constructed on a consolidating soil. In this case the pore pressures from a consolidation analysis should be used in the stability analysis.

Special consideration should be given to undrained analyses of partially saturated soils. XSLOPE uses the effective normal stress acting on the base of the slice when calculating the frictional resistance along the base. (Refer to (B6) in Appendix B). However, in undrained analyses this calculation must be done using the total normal stress. The apparent error in the calculation of the undrained strength does not occur because for fully saturated soils the undrained angle of friction $\phi_u = 0$. Therefore, the apparently erroneous use of the effective normal stress is cancelled by multiplication by $\tan \phi_u (= 0)$. However, in partially saturated soils the undrained angle of friction $\phi_u \neq 0$ and therefore the effective normal stress used in the calculation of the frictional resistance must be set equal to the total normal stress. This is accomplished by specifying the pore pressure in the partially saturated undrained soil layers as zero. This then ensures that the effective normal stress is equal to the total normal stress. This is accomplished most conveniently by employing a pore...
pressure coefficient $r_u$ in these layers and specifying its value as zero. It should be noted however that in most practical situations the undrained frictional resistance in partially saturated soils is ignored and the more conservative approach of setting $q_u = 0$ is adopted.

Subscripts at grid points are for $PORE(I,J)$

Figure 15 Parameters Used to Specify an Unequally-Spaced Pore Pressure Grid
5.5 EXTERNAL TRACTIONS

The analysis can take into account loadings of segments along the surface of the slope by normal and shear traction. The convention is that positive normal force acts in the direction into the soil. Positive shear causes clockwise rotation about an imaginary point just inside the soil and under the loaded segment. Node point numbers define the start and end of a segment. Shear and normal stresses are specified at these points and a linear distribution is assumed along the segment.

The submerged slope shown in Figure 16 (a) can be analysed by loading two segments with the normal traction (zero shear) shown in Figure 16 (b) and (c) and specifying a piezometric surface at the water level. The piezometric surface can be neglected if the soil below the water level is frictionless (i.e. $\theta = \varphi_u = 0$).

There are two restrictions when specifying the data for the surface traction.

The first restriction is that a traction cannot straddle one of the nodes used to define the surface of the slope. For example, consider a traction applied to a segment defined by (x) coordinates x-left and x-right respectively. If this traction straddles a node, (with (x) coordinate x-node, then this traction must be specified as two tractions. The first traction starts at x-left and ends at the x-node. The second traction starts at x-node and ends at x-right.

The second restriction is that the traction must be specified from left to right across the geometry, i.e. start with the traction with the minimum value of XL1 (Appendix A - Item [20]) and specify each traction in order of ascending XL1.
Figure 16  Submerged Slope (A) Modelled as (B) + (C)
5.6 SEISMIC LOADINGS

XSLOPE can model situations in which both the vertical and horizontal seismic coefficients vary with depth. Data item [26] (Appendix A) defines the number of points required (NSEIS) to define the variation of these coefficients with depth. Two step functions are defined in data item [27] by specifying NSEIS (y) coordinate values and the corresponding values of both the horizontal and vertical coefficients. The effect of an earthquake is then modelled by applying a set of horizontal and vertical forces at the centroid of each slice, where

\[ F(h)_i = a(h)_i W_i \]

\[ F(v)_i = a(v)_i W_i \]

where

\begin{align*}
  a(h)_i &= \text{horizontal seismic coefficient} \\
  a(v)_i &= \text{vertical seismic coefficient} \\
  F(h)_i &= \text{horizontal force applied at the centroid of the slice} \\
  F(v)_i &= \text{vertical force applied at the centroid of the slice} \\
  W_i &= \text{weight of the slice}
\end{align*}

The values \( a(h)_i \) and \( a(v)_i \) are calculated using the (y) coordinate of the slice centroid and linear interpolation of the step functions defined in data item [27].

Notes:

- The maximum number of points that can be used to define the step function is twenty.
- The step functions must be specified starting with the values at the maximum elevation and ending with the values at the minimum elevation.
- In many situations only a constant value for the horizontal seismic coefficient is used to model the loadings imposed due to earthquake activity. For this case specify only one point on the step function and specify the value for the horizontal seismic coefficient and set the vertical coefficient to zero. The (y) coordinate value is not used in the calculation and can therefore be specified as any convenient value.
5.7 AUTOMATIC GENERATION OF CIRCLES

Two algorithms are employed by XSLOPE to generate a user-specified number of slip circles automatically. The original algorithm is now only employed when 3,000 or fewer circles are automatically generated. If more than 3,000 circles are specified an algorithm which is a slight modification of the original is employed. The algorithms are considered simple but very effective. This basic procedure used in both algorithms is described in this section.

The "ideal" algorithm would return the centre coordinates and radius \((x_c, y_c, r)\) of the critical circle for the slope. In an attempt to implement this "ideal" algorithm many papers have been written describing a wide range of numerical techniques employed to locate the critical circle. Many of these techniques start by establishing that the factor of safety FoS can be expressed as a multi-variable function thus:

\[
FoS = G (v_1, v_2, v_3, \ldots, v_n)
\]  

where the variables \(v_1, v_2, \ldots, v_n\) define the geometry of the assumed failure surface.

Mathematical techniques are then employed in an attempt to minimise this function. A detailed investigation of the performance of five techniques which have been used previously is reported by Giam and Donald (1989). These methods prove efficient for some slopes but excessive computation may be required if local minima are encountered. The algorithm described in this section uses a different technique and has the important advantage that its effectiveness can be enhanced dramatically by intelligent specification of the slope data. This characteristic will be described in more detail later. The algorithm is based on the recognition that a circle can be defined by the circle's entry \((x_i, y_i)\) and exit \((x_o, y_o)\) points and an angle \(\delta\). These terms are defined in Figure 17. The automatic generation involves selection of an entry and exit point and then variation of the angle \(\delta\) from 5 to 90 degrees in the following steps: 5, 10, 20, 30, 40, 50, 60, 70, 80, 90. Thus, for a given pair of entry and exit points the maximum number of circles analysed is ten with centres varying from \((x_m, y_m)\) to \((x_{cmin}, y_{cmax})\). Note that when \(\delta = 0\) the circle degenerates to the straight line with end coordinates \((x_i, y_i)\) and \((x_o, y_o)\). The circle generated when \(\delta = 90^\circ\) has a centre \((x_m, y_m)\) and radius \(r = (x_o - x_m)\).

The entry and exit points used to generate the circles are determined in the following way. Consider the simple slope in Figure 18 that has four nodes (Node numbers 1, 2, 3 and 4) defining the surface of the slope. These nodes are used to establish which entry and exit points are to be used in the automatic generation. There are three stages in this generation process.

In the first stage the only nodes considered are those used to define the surface of the slope and those used to define any surface tractions. The automation commences with node 1 taken as the entry point \((x_r, y_r)\) and nodes 2, 3 and 4 consecutively treated as the exit point \((x_o, y_o)\).
Figure 17  Automatic Circle Generation - Definition of Terms

Figure 18  A Simple Slope with Four Surface Nodes
Figure 19  Circle Centres Ignored When "Connecting" Nodes 1 and 2

Figure 20  Stage 2: Mid-Point Nodes 5,6 and 7 Introduced
For each of these pairs of entry and exit points ten circles \((x_c, y_c, r)\) are generated, each circle corresponding to one of the ten values of \(\delta\). This procedure will be referred to as "connecting" two nodes. It should be evident that no useful circle centres are generated when nodes 1 and 2 are considered as entry and exit points. These circle centres are ignored because all circles passing through these points have an infinite value for the factor of safety, i.e. there is no mechanism for the circular arc of soil to rotate. This situation is illustrated in Figure 19.

After connecting node 1 to nodes 2, 3 and 4, node 2 is connected to nodes 3 and 4 and then node 3 is connected to node 4. Connecting nodes 3 and 4 produce no further circles for the same reason noted above when considering the connection of nodes 1 and 2. This then terminates the first stage of the generation process. After each useful circle \((x_c, y_c, r)\) is generated the user-specified number of circles is compared with the total number of circles generated. If the user-specified number of circles has been generated the analysis commences and no further generation takes place.

If required, the second stage of the automation involves the introduction of mid-points between the node points as illustrated in Figure 20. These new mid-point nodes are connected to the existing nodes and then the inter-connectivity of these new nodes is considered in a systematic way.

If more circles are required, the third and final stage of the automation process is undertaken. The difference between the two algorithms is the procedure used in this third and final stage. The original algorithm, that is now employed when 3,000 or less circles are specified, divides the width of the geometry into twenty equal segments (Figure 21) and the points at the ends of these segments are projected onto the surface of the slope.

Points projected onto surface of slope

![Points projected onto surface of slope](image)

Figure 21  Lateral Extent of Model Divided into Twenty Equal Segments
The projected points are indicated by the hollow circles on Figure 21. Projected points that coincide with existing node points, including the introduced mid-points, are ignored. The second algorithm introduces the new points onto the surface by firstly calculating the length along the top surface and then placing the points equidistant from each other. With these new points introduced into the scheme further connectivity of these points results in more circle coordinates being generated.

Having described the algorithms there are three important points to make:

(1) The very good performance of these algorithms is largely because of the entry and exit points chosen. These points are highly relevant to the stability of the slope being analysed. For example, the node at the toe of the slope is connected to many points along the slope's surface. Failure through the toe is commonly encountered. Further, the nodes which are used to specify the surface tractions are used as the entry points for many circles. There is also a strong possibility that one of these circles will be the critical circle.

(2) For each centre (x_c, y_c) only one radius is analysed and therefore maximum coverage in the Cartesian x-y plane is ensured. This maximum coverage also means that no minimisation of the factor of safety at a point is undertaken. However, the radius generated for each point is considered to be a very good one point selection because its value is based on entry and exit points that are relevant to the slope's stability.

(3) An important characteristic of these algorithms is that a simple alteration in the specification of the data results in a modification of the list of circles which are generated. For example, consider the simple slope shown in Figure 18. The four nodes which are used to model the changes in slope of the surface are the starting points used in the circle generation. The number of starting points can be increased by the introduction of extra "dummy" nodes into this surface definition. The placement of these extra nodes provides a mechanism for the user to specify a particular "class" of circles for generation. For example, if you wanted to thoroughly investigate the stability of circles which intersect the slope to the right of node 3 (Figure 18) then several "dummy" nodes can be included in the definition of the geometry. This is illustrated in Figure 22.
"Dummy" nodes 4, 5, 6 and 7

Figure 22   Extra Nodes Used to Define Surface of Slope
5.8 GENERATION OF CIRCLES FROM A USER-DEFINED GRID

In addition to the automatic generation of circles that has been described above, XSLOPE provides the facility to automatically generate a set of circles whose centres lie on a user-specified rectangular grid. The terms used to specify this grid are shown in Figure 23 and defined in Appendix A - Data Item [23].

The grid is positioned by specifying the coordinates of the bottom left hand corner of the rectangle \((X_{\text{bot}}, Y_{\text{bot}})\) and the top right hand corner \((X_{\text{top}}, Y_{\text{top}})\). The grid has NOX vertical grid lines and NOY horizontal grid lines, i.e. NOX * NOY * NOC circles. NOC is the user-specified number of circles analysed at each grid centre. The radii of the circles analysed are generated using the following algorithm. At each grid point the minimum and maximum possible circle radii, \(R_{\text{min}}\) and \(R_{\text{max}}\), are determined. The minimum radius \(R_{\text{min}}\) corresponds to the circle which just intersects with the surface of the slope. The maximum radius \(R_{\text{max}}\) corresponds to the circle with the maximum radius which does not intersect with either the vertical left hand side of the model \((x = x_{\text{min}})\), the vertical right hand side of the model \((x = x_{\text{max}})\) or the horizontal base of the model \((y = 0)\). The increment in radius \(\Delta R\) added to each circle is calculated as

\[
\Delta R = \frac{R_{\text{max}} - R_{\text{min}}}{NOC}
\]
The circles at each grid point have radii:

\[ R_{\text{min}} + \Delta R \]
\[ R_{\text{min}} + 2 \times \Delta R \]
\[ R_{\text{min}} + 3 \times \Delta R \]
\[ \cdots \]
\[ R_{\text{min}} + NOC \times \Delta R \]

The recommended technique to determine the position and radius of the critical circle is to analyse several thousand circles using the automatic generation facility described in the section above. From this preliminary analysis the size and position of a user-defined rectangular grid of circles can be determined. The grid should be centred over the critical circle and have dimensions that are appropriate to the dimensions used to model the slope. General rules are difficult to formulate, but what must be avoided is a grid in which the separation between the grid points is too large. If the grid points are too far apart an inadequate indication of the variation of factor of safety with position (x,y) will result.

If the analysis of this first rectangular grid results in the position of the critical circle lying on the perimeter of the grid, a further grid of circles must be analysed. This grid should be centred over the new critical centre. This procedure should be repeated until the critical centre coincides with an interior grid point.

This approach is a rational and methodical method for repeatedly overlaying a rectangular "window" on the contours of factor of safety and sampling estimates of the minimum factor of safety at an (x,y) position.

5.9 OUTPUT SPECIFICATIONS

Four types of output are available from XSLOPE. The results from the analysis of all circles can be output; the results from the circle with the minimum factor of safety at each centre considered; the results from the ten most critical circles; or finally a comprehensive summary table detailing the computations for an individual circle. This final option is only available when one circle has been specified for analysis.
6. PROGRAM FESEEP

6.1 DESCRIPTION OF PROGRAM

FESEEP for Windows (Balaam and Booker, 1998) provides an interface between the finite element
seepage analysis it performs and the slope stability analysis performed by XSLOPE. The FESEEP
program option, Control-Slope stability pore pressure grid generates a pore pressure grid that
can be used with XSLOPE. A user-specified rectangle or "window" is positioned over the finite
element mesh. This rectangle is divided into NDVGX equal divisions in the (x) direction and
NDVGY equal divisions in the (y) direction. This then results in (NDVGX+1) vertical grid lines,
(NDVGY+1) horizontal grid lines and (NDVGX+1) * (NDVGY+1) grid points.

The pore pressure at a grid point is calculated in the following way. Firstly, the finite element that
contains the grid point is determined and then linear interpolation of the finite element nodal values
of pore pressure is employed to calculate the pore pressure at the grid point. The generated pore
pressures are written to a file with the default extension ".POR". This file can then be read by
XSLOPE.

The geometric model used for the seepage and stability analysis normally differ in size and
therefore the coordinates of the bottom left corner of the pore pressure grid would normally have
different coordinates in the two analyses. The input requested by FESEEP places the rectangular
grid over a subset of the finite element mesh used for the seepage analysis. Once the grid is
positioned FESEEP generates the pore pressures at the grid points. The rectangular grid of pore
pressures can then be re-positioned within the stability analysis model (Data Item [14a] - Appendix
A) and used in the stability analysis.

6.2 GENERATING A PORE PRESSURE GRID

The example problem considered in Section 7.2 is also used in this section to describe the steps
involved in using a pore pressure grid generated by program FESEEP from the results of the finite
element seepage analysis.

The finite element model adopted for the seepage analysis of a 15m high embankment constructed
at a slope of 2:1 is shown in Figure 24. In this example the steady state seepage pore pressures are
calculated for heavy rainfall onto the top surface. The flow net obtained from the finite element
analysis is shown in Figure 25.
Figure 24 Finite Element Mesh Used for Steady State Seepage Analysis

Figure 25 Flow Net from Finite Element Analysis
In this example it is considered appropriate that the pore pressure grid used in the stability analysis should have 1m divisions in both the (x) and (y) directions. The stability model (see Section 7.2) that has been adopted for the analysis of this slope is only 45m wide compared to the seepage analysis model which is 60m wide. Therefore, the number of divisions in the (x) direction is set equal to 45. The crest height of both the seepage and stability models is 15m.

The pore pressure grid is generated by program FESEEP when the Control-Slope stability pore pressure grid-Specify option is selected, the appropriate values entered and the Write File command button pressed (see Figure 26). Options are available for displaying the pore pressure grid and the generated pore pressure values. Figure 27 shows a plot generated by FESEEP when the pore pressure grid is superimposed on a contour plot of the pore pressures. The positioning of the pore pressure grid in the stability model is shown in Figure 28.

Although the dimensions of the grid are set the positioning of the grid in the stability model does not have to coincide with the seepage model. In this example the origin of the grid in the seepage model is (0.,0.). However, in the stability model the origin of the grid does not have to be specified as (0.,0.).
Figure 27  F.E. Seepage Analysis - Placement of the Pore Pressure Grid

Figure 28  Pore Pressure Grid Superimposed over Critical Circle Plot
7. EXAMPLES

7.1 ANALYSIS OF A SUBMERGED SLOPE

In this example the submerged slope in Figure 29 is considered.

![Figure 29 Example 1 - A Submerged Slope](image)

The coordinates of the node points and soil properties are tabulated below.

<table>
<thead>
<tr>
<th>NODE COORDINATES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td><strong>Point</strong></td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>
EXAMPLE 1 - SUBMERGED SLOPE

SOIL PROPERTIES

<table>
<thead>
<tr>
<th>Layer</th>
<th>Cohesion (kPa)</th>
<th>Friction Angle (Degrees)</th>
<th>Bulk Unit Weight (kN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>40</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>35</td>
<td>19.2</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>0</td>
<td>19.6</td>
</tr>
</tbody>
</table>

Pore Pressures and Water Loadings

The water that submerges the slope is modelled in the following way:

1. Specify a piezometric surface at the water level. Note that a piezometric surface must start at the left hand end of the geometry and finish at the right hand end of the geometry.

2. Apply equivalent normal tractions to the surface of the slope that are exerted by the water. The tractions are:

   a. Traction on the horizontal surface between (x) coordinates XL1 = 0.0 and XL2 = 20.0

      XL1: Normal traction = (25-10) * 9.81
            = 147.15 kN/m² / metre run

      XL2: Normal traction = (25-10) * 9.81
            = 147.15 kN/m² / metre run

   b. Traction on the inclined surface between (x) coordinates XL1 = 20.0 and XL2 = 38.0

      XL1: Normal traction = (25-10) * 9.81
            = 147.15 kN/m² / metre run

      XL2: Normal traction = 0

   c. There is no SHEAR applied by the water loading.

XSLOPE provides a facility for automatically generating the loads exerted onto the segments along the surface of a slope submerged by water. When the loads details are entered specify “0” for the number of loads and press the Automatically generate surface loads on submerged slopes radio button. This is shown below.

-60-
The circle with the minimum factor of safety after one thousand circles have been analysed is shown below in Figure 30.

**Figure 30  Example 1 - Minimum FoS After 1000 Circles Analysed**
EXAMPLE 1 - SUBMERGED SLOPE

The circle with the minimum factor of safety has then been analysed again and the output code IOUT (Data Item [25] - Appendix A) has been set equal to four which generates an output file that contains tables with the full details of the computation. The data file is listed below.

DATA FILE

[MAXIMUM VALUES]
[Slices Layers Ndes/l Ndes/t Loads Piezo PP_XG PP_YG Circs]
50 15 40 75 15 15 21 21 10000

[TITLE]
Stability analysis of earth slope

[WATER] Unit weight of water

9.810

[GEOMETRY] No. of layers

4

[GEOMETRY] No. of nodes defining each layer surface

4 5 2 2

[GEOMETRY] Node points defining each layer surface (New line - each layer)

2 3 5 6

2 3 4 7 8

2 9

1 10

[GEOMETRY] Node point, (x)-coordinate, (y)-coordinate

1 0.000 7.000

2 0.000 10.000

3 20.000 10.000

4 50.000 21.500

5 50.000 35.000

6 70.000 35.000

7 56.000 21.500

8 70.000 30.000

9 70.000 10.000
EXAMPLE 1 - SUBMERGED SLOPE

10 70.000 7.000

[PORE PRESSURE FLAG] IPIEZ = (1, 2, 3 or 4)
1

[MATERIAL PROPERTIES] Layer, Cohesive, Friction angle, Bulk unit weight, (R(u))
1 5.000 40.000 18.000
2 0.000 35.000 19.200
3 20.000 0.000 19.000
4 40.000 0.000 19.600

[NON-UNIFORM STRENGTH PROPERTIES] Layer, Coh: (x)(y)(m), ... Phi: (x)(y)(m), ...
1 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0
2 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0
3 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0
4 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0

[PIEZOMETRIC SURFACE] (IPIEZ=1, 2 or 4?) NPP - No. of points along surface
2

[PIEZOMETRIC SURFACE] (IPIEZ=1, 2 or 4?) (x, y) coords. of NPP points
0.000 25.000
70.000 25.000

[LOADS] LOADS - No. of external tractions
1

[LOADS] (LOADS > 0) (x)-Left, (x)-Right, Normal-L, Shear-L, Normal-R, Shear-R
water

[CIRCLE ROTATION] IROTAT = 0 Clockwise = 1 Anti-clockwise
0

[CIRCLE CODE] NCODE >0 Specify =0 Grid -n Automate n "g" Graphics
-1000
EXAMPLE 1 - SUBMERGED SLOPE

[CIRCLE GRID] {NCODE=0}  Xbot, Ybot, Xtop, Ytop, NOX, NOY, NOC

[CIRCLES] {NCODE > 0}  (x)-Centre, (y)-Centre, Radius

[OUTPUT CODE]  IOUT = (1,2,3 or 4)

1

[SEISMIC COEFFICIENTS-FUNCTIONS] Number of points defining functions

0

[SEISMIC COEFFICIENTS FUNCTION]  Point No., y_coord, Horiz_k, Vertical_k

OUTPUT FILE

The output file is considered self-explanatory although the following should be noted.

The total weight of the sliding mass of soil is provided towards the end of the output along with the coordinates at the centroid of this mass. The total weight of soil may be useful when assessing whether this circle represents a “failure” of the slope or a localized slip, i.e. the circle merely “grazes” the slope. The coordinates at the centroid could be used to perform hand calculations on the stability of this sliding mass when additional restraints (e.g. anchor loads) are included in the analysis.
7.2 ANALYSIS OF A SLOPE WITH STEADY STATE SEEPAGE

In this example the stability of an embankment constructed using compacted clayey sands, and subjected to prolonged heavy rainfall is considered. The embankment is 15 metres high and has been constructed at a slope of 2:1, i.e. an angle of 26.5° to the horizontal. The clayey sand has the following soil properties; $c' = 5$, $\phi' = 35°$, $\gamma_{\text{bulk}} = 18$ kN/m³.

The steady state seepage due to the prolonged rainfall has been analysed using the finite element program FESEEP for Windows (Balaam and Booker, 1998). The finite element mesh used in this analysis is shown in Figure 24, the flow net generated is shown in Figure 25 and the pore pressure contours are shown in Figure 27 (See section 6.2). Program FESEEP includes an option that has been employed to generate a pore pressure grid for use in the stability analysis.

Four thousand circles have been analysed. Piping has occurred in some the circles that intersect with the toe of the embankment. When piping occurs in the critical circle “PIPING!” appears in the text box to the right of the text box in which the factor of safety is written. The piping that has occurred in some of the circles has been identified by inspecting the output file. In the tabulation of the factors of safety, an example is shown below for circle number 154, the “(1)” at the end of this line indicates that piping has occurred in this circle.

<table>
<thead>
<tr>
<th>Circle NO.</th>
<th>x(centre)</th>
<th>y(centre)</th>
<th>Radius</th>
<th>F.o.S (Bishop)</th>
</tr>
</thead>
<tbody>
<tr>
<td>154</td>
<td>8.882</td>
<td>26.77</td>
<td>23.06</td>
<td>0.9905 (1)</td>
</tr>
</tbody>
</table>

Further investigation has been undertaken by re-analysing several circles individually and selecting the solution file output option **Comprehensive summary table**. An inspection of the detailed tabulation for each slice shows that the piping has occurred in the first slice, i.e. the slice near the toe of the embankment. The piping has occurred because the computed normal stress acting on the base of the first slice is slightly less that the pore pressure interpolated from the seepage pore.
pressure grid. The pore pressure is calculated using linear interpolation and because of the rapid increase in the pore pressure value near the toe of the embankment with increasing (x) coordinate the interval size adopted for the pore pressure grid (1m) may be a little too coarse in this region. Additionally, the volume of the slice is calculated by multiplying the depth of the soil layer down the mid-line of the slice by the width of the slice. (The analysis is formulated for 1m thickness in the third dimension). The difference between this value and the actual volume of the first slice may also contribute to the normal stress on the base of the slice being less than the interpolated pore pressure which then results in the “piping” condition. However, because the piping “occurs” in only one slice and it is most likely due to a numerical issue and not the embankment design, the factors of safety computed have been adopted. Note that in slices where piping occurs the shear resistance along the base of the slice are set to zero. It is imperative that when piping is reported further investigation of the solution is undertaken to determine whether the piping is localized on one or two slices and due to this type of numerical problem or whether the analysis is predicting that piping will occur in the soil mass. A design in which piping occurs is flawed and must not be adopted.

The circle with the minimum factor of safety after 4000 circles has been analysed is shown below in Figure 31. This style of plot has been produced by selecting the Circles-Show centres option and then selecting the Circles-y_centre (Maximum) option and entering a value of 33.0.

Figure 31 Minimum FoS after 4000 Circles Analysed
The data file used for the analysis of the steady state seepage case is shown below.

**DATA FILE**

[MAXIMUM VALUES]

[Slices Layers Ndes/l Ndes/t Loads Piezo PP_XG PP_YG Circs]

50 15 40 75 15 15 51 51 10000

[TITLE]

Slope Stability with Steady State Seepage

[WATER] Unit weight of water

9.810

[GEOMETRY] No. of layers

1

[GEOMETRY] No. of nodes defining each layer surface

3

[GEOMETRY] Node points defining each layer surface (New line - each layer)

1 2 3

[GEOMETRY] Node point, (x)-coordinate, (y)-coordinate

1 0.000 0.000
2 30.000 15.000
3 45.000 15.000

[PORE PRESSURE FLAG] IPIEZ = (1,2,3 or 4)

4

[PORE PRESSURE CODE] {IPIEZ=4} IPORE=(1,2, or 3) for a layer

3

[PORE PRESSURE GRID] {IPIEZ=4} IGRID = (1,2 or 3)

3

[PORE PRESSURE GRID] {IPIEZ=4} {IGRID=3} X0GRD, Y0GRD

0.000 0.000

[PORE PRESSURE GRID] {IPIEZ=4} {IGRID=3} Filename - program XGRID

E:\FWinXslope\Ex2Seepage.por
EXAMPLE 2 - STEADY STATE SEEPAGE

[MATERIAL PROPERTIES] Layer, Cohesion, Friction angle, Bulk unit weight, \( R(u) \)

1 5.000 35.000 18.000

[NON-UNIFORM STRENGTH PROPERTIES] Layer, Coh: \( (x)(y)(m) \), ... Phi: \( (x)(y)(m) \), ...

1 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0

[PIEZOMETRIC SURFACE] \( \{ I_{PIEZ}=1,2 \text{ or } 4? \} \) NPP - No. of points along surface

0

[PIEZOMETRIC SURFACE] \( \{ I_{PIEZ}=1,2 \text{ or } 4? \} \) (x,y) coords. of NPP points

0

[LOADS] LOADS - No. of external tractions

0

[LOADS] \( \{ LOADS > 0 \} \) (x)-Left, (x)-Right, Normal-L, Shear-L, Normal -R, Shear-R

[CIRCLE ROTATION] IROTAT = 0 Clockwise = 1 Anti-clockwise

0

[CIRCLE CODE] NCODE >0 Specify =0 Grid -n Automate n "g" Graphics

-4000

[CIRCLE GRID] \( \{ NCODE=0 \} \) Xbot, Ybot, Xtop, Ytop, NOX, NOY, NOC

[CIRCLES] \( \{ NCODE > 0 \} \) (x)-Centre, (y)-Centre, Radius

[OUTPUT CODE] IOUT = (1,2,3 or 4)

1

[SEISMIC COEFFICIENTS-FUNCTIONS] Number of points defining functions

0

[SEISMIC COEFFICIENTS FUNCTION] Point No., y_coord, Horiz_k, Vertical_k
APPENDIX A - SPECIFICATION OF THE INPUT DATA

XFIT

In two situations the (x) coordinate of a point can be specified using the word xfit (or XFIT). These two situations are when you are specifying the coordinates of a node on the surface of the slope or specifying the coordinates of the piezometric surface. Thus, the nodal coordinate of a point on the surface of the slope could be specified as:

xfit,15.6

XSLOPE then calculates the (x) coordinate to “fit” the point onto the surface of the slope at the (y) coordinate or elevation (in this example 15.6). This is useful when adding “dummy” nodes to the surface definition in order to modify the set of circles that are automatically generated. It is also useful when specifying the coordinates of the piezometric surface. The water level (i.e. (y) coordinate) is known, using “xfit” the corresponding (x) coordinate does not have to be manually calculated. You can not “xfit” a coordinate onto a horizontal line.

DATA LAYOUT

The specification of the data set for XSLOPE is given below. The data items must adhere to the format listed below and must be assembled in the specified order. Each data group or data item must be structured as follows. The first line of the group is a text description of the type of data contained in the group. (These descriptions can be changed to the user's own choices, i.e. the contents of these text lines are not processed in any way by XSLOPE). This first line is then followed immediately by all items of data in the group. The end of the data group is signalled by a blank line. This is mandatory because XSLOPE checks for the presence of this blank line when all the data for a particular data group or data item has been read.

The contents of each group are set out below. The text descriptions of each data item are those generated by XSLOPE. If these descriptions include statements in "curly braces", e.g. {IGRID=2}, data need only be specified for the data group if the statement(s) in the set(s) of braces are true.

[1] MAXIMUM VALUES

[Slices Layes Ndes/l Ndes/t Loads Piezo PP_XG PP_YG Circs]

Slices Number of slices each circle is divided into.
Layers Maximum number of soil layers.
Ndes/l Maximum number of nodes used to define a layer.
Ndes/t Maximum number of nodes used to define the geometry.
### DATA SPECIFICATION

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loads</td>
<td>Maximum number of external loads.</td>
</tr>
<tr>
<td>PP_XG</td>
<td>Maximum number of vertical grid lines used to define a pore pressure grid.</td>
</tr>
<tr>
<td>PP_YG</td>
<td>Maximum number of horizontal grid lines used to define a pore pressure grid.</td>
</tr>
<tr>
<td>Circs</td>
<td>Maximum number of circles specified for an analysis.</td>
</tr>
</tbody>
</table>

#### [2] [TITLE]

A descriptive title for the analysis. This will appear at the bottom of the plots produced by the graphics module within XSLOPE.

#### [3] [WATER] Unit weight of water

WATER The unit weight of water. The value must be consistent with the units used for length, cohesion and the applied tractions.

#### [4] [GEOMETRY] No. of layers

LAYERS The number of soil layers that the slope is divided into.

#### [5] [GEOMETRY] No. of nodes defining each layer surface

NP(1) The number of points required to define the changes in slope along the surface of layer L.

Note: The definition of the surface of the top layer (layer no. 1) must include the nodes that are required to specify the applied surface tractions. There must be a node at the left and right hand ends of each traction.

NP(LAYERS)
[6] [GEOMETRY] Node points defining each layer surface (New line - each layer)

NCON(L,1) The number of the points that are used to define the changes in slope along the surface of layer L. The maximum value of I for each layer is NP(L). If there are more than 10 points defining a layer specify 10 points per line.

NCON(L,1)

[7] [GEOMETRY] Node point, (x)-coordinate, (y)-coordinate

NODE The node point number and the (x,y) coordinates of the point. Start a new line for each point. The node numbers must be in consecutive and ascending order.

X(NODE) Y(NODE)

[8] [PORE PRESSURE FLAG] IPIEZ = (1,2,3 or 4)

IPIEZ IPIEZ = 1; The piezometric surface is used for calculating pore pressures in all layers.

IPIEZ = 2; The piezometric surface is used for calculating pore pressures in some layers - pore pressure coefficients \( r_u \) are used in the remaining layers.

IPIEZ = 3; Pore pressure coefficients are used for calculating pore pressures in all layers.

IPIEZ = 4; A grid of pore pressures is used for calculating pore pressures in some layers.

IF IPIEZ ≠ 4 GOTO data Item 15.
DATA SPECIFICATION

[9] [PORE PRESSURE CODE] {IPIEZ=4} IPORE=(1,2 or 3) for a layer

IPORE(1) IPORE = 1; The piezometric surface is used for calculating pore pressures in this layer.

IPORE(2) IPORE = 2; A pore pressure coefficient $r_p$ is used in this layer.

IPORE(3) IPORE = 3; A grid of pore pressures is used in this layer. The pore pressures are specified as data or generated using program FESEEP (Finite Element Seepage Analysis)

IPORE(LAYERS)

[10] [PORE PRESSURE GRID] {IPIEZ=4} IGRID=(1,2 or 3)

IGRID IGRID = 1; An equally spaced grid of pore pressures is used. The pore pressures are input as part of the data set.

IGRID = 2; An unequally spaced grid of pore pressures is used. The pore pressures are input as part of the data set.

IGRID = 3; The grid of pore pressures has been generated by the finite element seepage analysis program FESEEP.

Skip data item 11 if IGRID ≠ 1.

[11] [PORE PRESSURE GRID] {IPIEZ=4}{IGRID=1}X0GRD,Y0GRD,NOXG,DXGRD,NOYG,DYGRD

X0GRD (x) coordinate of bottom left hand corner of grid.

Y0GRD (y) coordinate of bottom left hand corner of grid.

NOXG Number of vertical grid lines.

DXGRD Spacing between vertical grid lines.

NOYG Number of horizontal grid lines.

DYGRD Spacing between horizontal grid lines.
Skip data items 12a, 12b and 12c if IGRID ≠ 2.

[12a] [PORE PRESSURE GRID]{IPiez=4} {IGRID=2} NOXG, NOYG
    NOXG   Number of vertical grid lines.
    NOYG   Number of horizontal grid lines.

[12b] [PORE PRESSURE GRID]{IPiez=4} {IGRID=2} (x) coords.
    XGRD(1) The (x) coordinates of the vertical grid lines.
    XGRD(2) Specify 10 coordinates per line.
    ...
    ...
    XGRD(NOXG)

[12c] [PORE PRESSURE GRID]{IPiez=4} {IGRID=2} (y) coords.
    YGRD(1) The (y) coordinates of the horizontal grid lines.
    YGRD(2) Specify 10 coordinates per line.
    ...
    ...
    YGRD(NOYG)

Skip data item 13 if IGRID = 3.

[13] [PORE PRESSURE GRID]{IPiez=4} {IGRID=1,2} Pore pressures
    PORE(1,1) Pore pressure at the intersection of the \(i^{th}\) horizontal grid line
    PORE(1,2) and the \(j^{th}\) vertical grid line. Start at the origin of the grid
    PORE(1,3) (bottom left hand corner) and work across the first horizontal
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grid line (I=1, J=1 to NOXG) and then the second (I=2, J = 1 to NOXG) etc. until all pore pressures are specified.

Specify 10 pore pressures per line and start a new line for each horizontal grid line.

PORE(NOYG,NOXG)

**Skip data items 14a and 14b if IGRID ≠ 3.**

[14a]  [PORE PRESSURE GRID] {IPiez=4} {IGRID=3} X0GRD, Y0GRD

X0GRD  (X0GRD,Y0GRD) positions the grid generated by program FESEEP within the stability model. These are the coordinates of the point where the bottom left hand corner of the grid will be placed. The size of the grid is part of the input for program FESEEP.

Y0GRD

[14b]  [PORE PRESSURE GRID] {IPiez=4} {IGRID=3} (*.POR) filename from FESEEP

GFILE Name of file specified when running program FESEEP. It contains the size of the grid and the pore pressure at each of the grid points.

If IPiez = 2 or 3 skip data item 15a.

If IPiez = 4 and IPORE(I) = 2 in any layer skip data item 15a.

[15a]  [MATERIAL PROPERTIES] Layer,Cohesion,Friction angle,Bulk unit weight, {R_u}

L The layer number.

COH(L) Cohesion; in units consistent with the unit weight of water.

PHI(L) Friction angle in degrees.

UNITW(L) Bulk unit weight; in units consistent with the unit weight of water.

Start a new line for each layer.

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GO TO data item 16.

[15b] [MATERIAL PROPERTIES] Layer,Cohesion,Friction angle,Bulk unit weight,\{Ru\}

L The layer number.

COH(L) Cohesion; in units consistent with the unit weight of water.

PHI(L) Friction angle in degrees.

UNITW(L) Bulk unit weight; in units consistent with the unit weight of water.

RU(L) The pore pressure coefficient ru (no units). Set RU(L) < 0 for layers in which the pore pressure is to be calculated using the piezometric surface or pore pressure grid.

Start a new line for each layer.

[16] [NON-UNIFORM STRENGTH PROPERTIES] Layer, Coh:(x)(y)(m), Phi:(x)(y)(m), y(top),Uniform?

L The layer number.

COH_H(L) Cohesion in the horizontal direction.

COH_V(L) Cohesion in the vertical direction.

COH_M(L) Rate of change of cohesion with depth.

PHI_H(L) Friction angle in the horizontal direction.

PHI_V(L) Friction angle in the vertical direction.

PHI_M(L) Rate of change of the friction angle with depth.

Y_SURF(L) Elevation below which the change in cohesion and friction angle are calculated.

NONU(L) =0 Shear strength in this layer is not calculated using the non-uniform strength values. The shear strength is calculated using the cohesion and friction angle values specified in data item [15].

=1 The non-uniform strength values are used to calculate the shear strength in this layer.
DATA SPECIFICATION

[17] [PIEZOMETRIC SURFACE] {IPIEZ=1, 2 or 4} NPP - No. of points along surface

NPP
The number of points required to define the changes in slope along the piezometric surface.

Note:
If IPIEZ = 3 or IPIEZ = 4 and no layer has IPORE(i) set to 1; then
(a) Set NPP = 0
(b) Include two lines for data item [18]. The first line should be a text description of the data required to define a piezometric surface and the second line should be blank.

[18] [PIEZOMETRIC SURFACE] {IPIEZ=1, 2 or 4} (x,y) coords. of NPP points

XP(I)
The (x,y) coordinates of the NPP points that define the changes in slope along the piezometric surface.
YP(I)
Start a new line for each point.

[19] [LOADS] LOADS - No. of external tractions

LOADS
The number of segments along the surface of the slope that are subjected to applied tractions.

If LOADS = 0 include two lines for data item 20. The first line should be a text description of the data required to specify load details and the second line should be blank.

[20] [LOADS] {LOADS > 0} (x)-Left,(x)-Right,Normal-L,Shear-L,Normal-R,Shear-R

XL1(I)
The (x) coordinate at the left hand end of the traction.
XL2(I)
The (x) coordinate at the right hand end of the traction.
PLN(I)
The normal stress applied at the left hand end of the traction.
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PLS(I) The shear stress applied at the left hand end of the traction.
PRN(I) The normal stress applied at the right hand end of the traction.
PRS(I) The shear stress applied at the right hand end of the traction.

Start a new line for each traction.

Note:

If the slope is partially (or fully) submerged the surface tractions due to the water can be automatically generated. In order to generate these tractions:

(a) Count the loads due to the water as ONE load. For example, if there were 3 external tractions due to building loads then the number of LOADS would be specified as 4, i.e. the 3 external loads + the water loads.

(b) Specify the water loads first. Do this by using the word "WATER" or "water". After this line specify the details of the other tractions, in the example above there would be another three lines of data, i.e. four lines in total.

[21] [CIRCLE ROTATION] IROTAT  =0 Clockwise  =1 Anti-Clockwise

IROTAT  IROTAT = 0; The circular slip occurs by clockwise rotation of the soil mass.

IROTAT = 1; The circular slip occurs by anti-clockwise rotation of the soil mass.

[22] [CIRCLE CODE] NCODE ( > 0 Specify) (=0 Grid) (-n Automate n circles)

NCODE  NCODE > 0; Input the number of circles that are to be analysed. The centre coordinates and radii will then be specified individually.

NCODE = 0; A set of circles are automatically generated from a rectangular grid. The details of this grid are specified by the user.

NCODE = -n; n circles will be automatically generated. For example, a value of -200 results in 200 circles being generated.
If NCODE ≠ 0 include two lines for data item 23. The first line should be a text description of the data required to specify a grid of circles and the second line should be a blank.

[23] [CIRCLE GRID] {NCODE=0} Xbot, Ybot, Xtop, Ytop, NOX, NOY, NOC

XBOT  XBOT is the (x) coordinate of the left hand bottom corner of the rectangular grid.

YBOT  YBOT is the (y) coordinate of the left hand bottom corner of the rectangular grid.

XTOP  XTOP is the (x) coordinate of the right hand top corner of the rectangular grid.

YTOP  YTOP is the (y) coordinate of the right hand top corner of the rectangular grid.

NOX  NOX is the number of vertical grid lines.

NOY  NOY is the number of horizontal grid lines.

NOC  NOC is the number of circles analysed from each grid point.

If NCODE ≤ 0 include two lines for data item 24. The first line should be a text description of the data required to specify individual circles and the second line should be a blank.

[24] [CIRCLES] {NCODE>0} (x)-Centre, (y)-Centre, Radius

XCEN(I)  The (x) coordinate of the centre of the i\textsuperscript{th} circle.

YCEN(I)  The (y) coordinate of the centre of the i\textsuperscript{th} circle.

RAD(I)  The radius of the i\textsuperscript{th} circle.

A total of NCODE circles must be detailed.

Start a new line for each circle.
DATA SPECIFICATION

[25] [OUTPUT CODE] IOUT = (1,2,3 or 4)

IOUT

IOUT = 1; The solution file will contain the results for all the circles analysed.

IOUT = 2; The solution file will contain the results for only one circle at each unique centre that was considered. The output is for the circle with the minimum factor of safety at the centre.

IOUT = 3; The solution file will contain the results from the 10 circles with the smallest factors of safety.

IOUT = 4; A comprehensive summary table of the calculation is generated. This output code is only permitted if one circle is being analysed, i.e. NCODE = 1.

[26] [SEISMIC COEFFICIENTS-FUNCTIONS] Number of points defining functions

NSEIS

The number of points used to define the variation of both the horizontal and vertical seismic coefficients.

Notes:

(a) The maximum number of points is twenty.

(b) Specify one point if these coefficients have constant values.

[27] [SEISMIC COEFFICIENTS-FUNCTIONS] Point No., y_coord, Horiz_k, Vertical_k

NO_POINT

The point number. These must be in ascending order from 1 to NSEIS.

Y_COORD

The (y) coordinate. These values must be specified in descending order.

HORIZ_K

The horizontal seismic coefficient.

VERTICAL_K

The vertical seismic coefficient.

Notes:

(1) A value of NSEIS equal to one signals that these coefficients have constant values and therefore the value of
the (y) coordinate is not used in the calculation. Thus, the (y) coordinate value can be specified as any convenient value. For example, if only horizontal forces are being modelled and the horizontal seismic coefficient is .2, then the input for this line would be:

1,0.,.2,0.

where the value of Y_COORD has been specified as 0.

(2) Start a new line for each point.

(3) A total of NSEIS points must be specified.
APPENDIX B - METHODS OF ANALYSIS

BISHOP’S SIMPLIFIED METHOD

Numerous two-dimensional limit equilibrium methods have been proposed for estimating the stability of earth slopes. Some of these methods are restricted to circular failure surfaces (e.g. Swedish, Bishop’s Simplified) whereas others have addressed the problem of non-circular surfaces (e.g. Janbu, Morgenstern-Price, Sarma, Spencer).

In all these methods of analysis it is assumed that the soil fails simultaneously along the failure surface, (i.e. incremental yielding along the failure surface is not considered) and obeys the Mohr-Coulomb failure criterion. In these methods of analysis the soil above the assumed failure surface is divided into n "slices" (Figure 1) with vertical boundaries and the failure surface is approximated by n piecewise linear segments that are defined by n+1 points. When the equations of statics (horizontal, vertical and moment equilibrium) are considered the problem is found to be statically indeterminate. In Bishop's simplified method the factor of safety equation is derived by taking moments about the centre of the circle and eliminating the static indeterminancy by setting the interslice shear forces to zero. Several useful papers have been written (e.g. Fredlund, Krahn and Pufahl, 1981; Bardet and Kapuskar, 1989) in which the analysis has been rationalised and the assumptions employed and differences between the various methods are clearly stated. Previously, the similarities and differences between the various methods were obscure because of the lack of uniformity in formulating the factor of safety.

In these analyses it is assumed that the factor of safety against shear failure is the same for each slice, i.e. incremental yielding along the failure surface is not considered. Therefore, the factor of safety for the slope can be estimated by comparing the shear stress along the base of each slice required to maintain limit equilibrium with the available shear strength ($\tau_s$) of the soil. Thus, adopting an effective stress approach, the factor of safety (FoS) is defined as:

$$\tau = \frac{\tau_s}{\text{FoS}} = \frac{c'}{\text{FoS}} + \frac{(\sigma_n - u) \tan \phi'}{\text{FoS}} \quad (B1)$$

- $\tau$ = shear stress acting on the base of the slice.
- $c'$ = cohesion of the soil along the base of the slice.
- $\sigma_n$ = normal stress acting on the base of the slice.
- $u$ = pore pressure at the base of the slice.
- $\phi'$ = friction angle of the soil along the base of the slice.
Bishop (1955) developed a general expression for the factor of safety from consideration of moment equilibrium of the volume of soil above the failure arc. The original derivation was in terms of effective stress. The general expressions for the moment equilibrium of the failure arc, vertical equilibrium of a slice and the Mohr-Coulomb failure criterion for the soil along the circular failure surface can be expressed as:

Moment Equilibrium

\[ M_O = M_R \]  \hspace{1cm} (B2)

Overturning Moment \((M_O)\)

\[ \sum W R (1 + a_v) \sin \alpha + M_{EXT} + \sum a_h W (y_c - \bar{y}) \]  \hspace{1cm} (B3)

Restraining Moment \((M_R)\)

\[ \sum R T \]  \hspace{1cm} (B4)

Vertical Equilibrium

\[ (N' + u_l) \cos \alpha + T \sin \alpha = W(1 + a_v) + V + (X_{n+1} - X_n) \]  \hspace{1cm} (B5)

Mohr Coulomb

\[ T = \frac{c'}{FoS} + \frac{N' \tan \phi'}{FoS} \]  \hspace{1cm} (B6)

where the summations are for \(n\) slices and;

\[ a_h = \text{seismic coefficient to take account of dynamic horizontal forces.} \]

\[ a_v = \text{seismic coefficient to take account of dynamic vertical forces} \]

\[ l = \text{the length BC (Figure 2).} \]
METHODS OF ANALYSIS

\( M_{\text{EXT}} \) = overturning moment due to the external surface tractions.
\( N' \) = effective normal force on the base of the slice.
\( R \) = radius of the circle.
\( T \) = shear force acting along the base of the slice.
\( V \) = vertical force applied along the top of the slice due to the external surface tractions
\( W \) = total weight of the slice of soil.
\( X_{n,n+1} \) = the vertical shear forces.
\( y_c \) = \((y)\) coordinate of the circle centre.
\( y \) = \((y)\) coordinate of the slice centroid.
\( \alpha \) = angle between the horizontal and the tangent at the centre of the base of the slice.

The effective normal force on the base of a slice, \( N' \), is

\[ N' = N - ul \] (B7)

Rearranging equation (B6) gives the following expression for \( N' \)

\[ N' = (T.FoS - c'/l) \cot \varphi' \] (B8)

Substituting equation (B8) into the vertical equilibrium equation (B5) gives a general expression for the shear force on the base of a slice, thus

\[ (T.FoS - c'/l) \cot \varphi' \cos \alpha + T \sin \alpha = W(1 + a_v) + V - ub + (X_{n+1} - X_n) \] (B9)

Rearranging (B9) leads to

\[ T = \frac{W(1 + a_v) + V - ub + (X_{n+1} - X_n) + c'/l \cot \varphi'}{FoS \cos \alpha \cot \varphi' + \sin \alpha} \] (B10)
where
\[ b = \text{Width of the slice} = l \cos \alpha \]  

(B11)

Further rearranging of equation (B10) gives the following expression for the shear force \( T \)

\[
T = \frac{\tan \phi' \sec \alpha}{FoS} \left[ \frac{W(1 + a_v) + V - r_u V_T + (X_{n+1} - X_n) + c'/b \cot \phi'}{1 + \frac{\tan \alpha \tan \phi'}{FoS}} \right]
\]

(B12)

where \( r_u \) is the pore pressure coefficient, i.e.

\[ r_u = \frac{u b}{W} \]  

(B13a)

and

\[ V_T = W(1 + a_v) + V \]  

(B13b)

This general expression for the shear force can be substituted into the expression for moment equilibrium. This leads to:

\[
\sum R \frac{\tan \phi' \sec \alpha}{FoS} \left[ \frac{W(1 + a_v) + V - r_u V_T + (X_{n+1} - X_n) + c'/b \cot \phi'}{1 + \frac{\tan \alpha \tan \phi'}{FoS}} \right]
\]

\[ = \sum W(1 + a_v) R \sin \alpha + M_{EXT} + \sum a_h W(y_c - \bar{y}) \]  

(B14)
or rearranging

\[ \text{FoS} = \sum \left\{ c b + \tan \phi' \left( W(1 + a_v) + V - r_u V_T - (X_n - X_{n+1}) \right) \right\} \frac{1}{k_a} \ \cdot \ B \]  

(B15)

where

\[ k_a = \frac{\sec \alpha}{1 + \tan \alpha \tan \phi'} \]  

(B16)

and

\[ B = \sum W(1 + a_v) \sin \alpha + \frac{M_{\text{EXT}}}{R} + \sum a_h W (y_c - \bar{y}) \frac{1}{R} \]  

(B17)

Bishop found that the factor of safety is relatively insensitive to the distribution of interslice forces, i.e. quite different distributions of interslice forces could be adopted and the factor of safety would be only slightly altered. The distributions adopted by Bishop satisfied equilibrium, gave a satisfactory factor of safety against shear failure along the interslices and finally provided a reasonable line of thrust between the slices. Thus, the assumption adopted in Bishop's simplified method in order to render the problem statically determinate is that:

\[ \sum k_a (X_n - X_{n+1}) \tan \phi' = 0 \quad (\text{Assumption}) \]  

(B18)

and therefore the expression for the factor of safety that is computed by program XSLOPE is:

\[ \text{FoS} = \sum \left\{ c b + \tan \phi' \left( W(1 + a_v) + V - r_u V_T \right) \right\} \frac{1}{B} \]  

B(19)

The factor of safety (FoS) appears on both sides of the equation and therefore an iterative method of solution is required. The iterative solution for the FoS of the slope usually converges satisfactorily in less than four iterations.
METHODS OF ANALYSIS

MORGENSTERN AND PRICE’S METHOD

Non-circular failure surfaces are analysed by XSLOPE using a method of analysis first published by Morgenstern and Price in 1965. The numerical solution to the equations published in that paper was fully described in a subsequent publication by these two authors in 1967. This numerical solution has been coded and implemented in XSLOPE. A further publication by Chen & Morgenstern (1983) details several extensions to this analysis that eliminate some of the inconsistencies in the original analysis and according to the authors appeared to overcome the problems of convergence that can occur when solving the equations. These extensions have not been implemented in XSLOPE but it must be noted that the authors concluded that the extensions resulted in variations in the factor of safety that were of little practical significance.

Assumption

In order to render the problem statically determinate the authors make an assumption about the resultant of the shear (X) and normal force (E) that act on the vertical plane within the mass of sliding soil. The assumption is that the angle of the resultant to the horizontal direction can be described by an expression of this form:

\[ X = \lambda f(x) E \] (B20)

where

\( \lambda \) is a constant scaling factor that is calculated as part of the analysis in order to satisfy the equations of equilibrium.

\( f(x) \) assumed interslice force function.

Interslice Force Function

The function, \( f(x) \), is an assumed function that can take any prescribed form in principle but the behaviour of the soil will place certain limitations on what would be deemed to be a reasonable variation in the angle that the resultant force makes with the horizontal. For example, Morgenstern and Price suggested that the function can be an estimate from available elasticity solutions or alternatively from intuitive assumptions. For example, they note that for most slopes the higher the rate of curvature of the slip surface the greater the ratio between the shear (X) and horizontal force (E).

Alternatively, a function of the form published by Fan, Fredlund and Wilson (1986) could be adopted. These authors have presented plots of side force functions obtained from linear finite element analyses of homogeneous slopes with various slope angles. The results of these analyses have been used to derive a generalised empirical interslice force function. However, in most situations it is envisaged that a non-circular analysis is performed because of the presence of a weaker stratum that can not be adequately analysed using circular failure surfaces. The effect of the presence of a weaker stratum on the interslice side force functions obtained from finite element
analyses is unknown and therefore the relevance of the side force function proposed by these authors is questionable.

XSLOPE performs four analyses of each non-circular failure surface that is defined. The analyses are for the interslice force functions shown in Figure B1.

![Interslice Force Functions Analysed by XSLOPE](image)

**Figure B1**  Interslice Force Functions Analysed by XSLOPE

**Summary of Equations**

Morgenstern and Price (1965, 1967) adopted the axes directions as shown in Figure B2. The line of thrust is the computed line along which the resultant of the interslice shear and normal forces act. In the first publication (1965) the authors present a detailed derivation of the governing equations. Moment equilibrium about the centre of the base of a slice provides one of two governing differential equations. The second equation is established from the force equilibrium normal and tangential to the base of a slice and the Mohr-Coulomb failure criterion that specifies the relationship between the normal and tangential forces that act on the base of a slice. The statical indeterminancy of this problem is then eliminated by introducing an assumption about the relationship between the interslice normal and shear forces (Equation B20).
In the subsequent publication (1967) the authors summarised the governing equations and provided a detailed method for numerically solving the equations. The equations are summarised below.

**Moment equilibrium**

\[ X = \frac{d}{dx} (E_y) - y \frac{dE}{dx} \]  

**Force equilibrium and Mohr-Coulomb criterion**

\[ \frac{d}{dx} \left[ (Kx + L/E) \right] = N_x + P \]  

**Assumption**

\[ X = \lambda f(x) E \]  

where

\[ K = \lambda k \left( \frac{\tan \phi'}{FoS} + A \right) \]
METHODS OF ANALYSIS

\[
L = \lambda m \left( \frac{\tan \phi'}{FoS} + A \right) + 1 - \frac{A \tan \phi'}{FoS} \quad (B24)
\]

\[
N = p \left( \frac{\tan \phi'}{FoS} + A - r_u \left(1 + A^2\right) \frac{\tan \phi'}{FoS} \right) \quad (B25)
\]

and

\[
P = \frac{c'}{FoS} \left(1 + A^2\right) + q \left( \frac{\tan \phi'}{FoS} + A - r_u \left(1 + A^2\right) \frac{\tan \phi'}{FoS} \right) \quad (B26)
\]

and within each slice

\[
\frac{dW}{dx} = px + q \quad (B27)
\]

\[
f(x) = kx + m \quad (B28)
\]

\[
A = \frac{dy}{dx} \quad (B29)
\]

and \(y_c\) is the equation of the line of thrust.

The technique used to solve these equations is the Newton-Raphson method combined with certain controls that are placed on the values of FoS and \(\lambda\).

**Lines of thrust**

If convergence of the solution is obtained then in order for the solution to be accepted a check must be made of the calculated position of the line of thrust. If the line of thrust falls outside the sliding mass of soil then tension must exist within the sliding mass. In many situations one or more of the interslice force functions that are analysed may result in lines of thrust that fall outside the sliding mass of soil whilst the remainder result in lines of thrust that fall completely within. Further, in these situations the factors of safety computed for each of the interslice force functions may not vary significantly and therefore the results from the analysis may be considered acceptable.

However, in some situations the lines of thrust for all the interslice force functions will lie outside the sliding mass of soil. In these situations it must be concluded that the failure surface is unlikely or further analyses performed that include a tension crack. The inclusion of a tension crack into the analysis may eliminate the occurrence of tension and produce lines of thrust that lie fully within the sliding mass of soil.
REFERENCES


Culmann, K. (1866) *Die Graphische Statik*, Zurich.


REFERENCES


REFERENCES


