Viscous Rate Effects in Shear Strength of Clay

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ABSTRACT

Viscous rate effects are commonly observed in clay soils in applications that involve high strain rates such as free falling penetrometer tests. Viscous rate effects are not well studied thus affecting reliable interpretation of these applications. This paper presents an investigation of viscous rate effects in clay using laboratory rate studies covering a wide range of strain rates (6 orders of magnitudes). The laboratory investigation has included conventional undrained triaxial compression tests and parallel plate rotational viscometer tests. The viscometer tests allow very high strain rates and are widely used in rheology, however their application to soils has been limited. The data from this study shows that the viscometer tests provide unreasonably low strength values and suggest limitations in the interpretation of the test.

Keywords: viscous rate effects, clay, triaxial test, rotational viscometer test

1 INTRODUCTION

Rate effects in soil shear strength have long been recognised, with Casagrande and Wilson (1951) reporting an increase in clay strength with increasing loading rate in unconfined compression tests. Since then, rate effects have been studied in various applications, and depending on the strain rate range these can be dominated by the effects of consolidation, at relatively slow rates, and viscous effects, at relatively high rates (Lehane et al. 2009). Viscous rate effects have not been well studied, and this limits the reliability of the interpretation of those applications that involve high strain rates such as free falling penetrometers (FFP), which can impact the soil at high velocity, up to 30 m/s. FFP are a potential tool for obtaining soil strength parameters rapidly and cost effectively at sites with poor accessibility, such as on the seabed. However, for this potential to be realised there is a need for a greater understanding of the viscous effects.

Proper allowance for rate effects is important in FFP as the penetration event involves decreasing velocity/strain rate from impact until the device comes to rest. Rate effects in FFP have been investigated mainly through laboratory model FFP tests (Dayal 1974, Hurst and Murdoch 1991, Stoll et al. 2007). In these model tests, viscous rate effects cause a significant increase in the soil shear strength as compared to that obtained from static tests. To address these rate effects, power-law or semi-logarithmic rate formulae have been proposed (Dayal 1974, Hurst and Murdoch 1991). However, the reliability of these rate formulae is dependent on the strain rate range and soil material properties. As the rate parameters in the empirical rate formulae differ from soil to soil they need to be determined individually and, as this is not generally possible, viscous rate effects cannot be reliably estimated.

In numerical investigations of FFP (Aubeny and Shi 2006, Abelev et al. 2009, Carter et al. 2010) the viscous rate effects have been addressed by adopting a conventional semi-logarithmic rate model that assumes the strength increases at a rate of 5% to 20% per log cycle of strain rate. These rate factors are obtained from rate studies involving element tests covering relatively low strain rates from triaxial and vane shear tests (Sheahan et al. 1996, Biscontin and Pestana 2001). The numerical studies suggest that the interpretation of FFP may be significantly affected by the rate factor, but the applicability of these rate models is not well established at the high strain rates relevant to FFP.

To provide further understanding of the factors influencing the rate effects element tests covering higher and wider strain rate ranges are required. Element tests such as triaxial tests and vane shear tests have been extensively used to investigate rate effects for lower strain rates (Sheahan et al. 1996,
Biscontin and Pestana 2001), but owing to technical limitations and difficulties in data interpretation, these element tests have seen little use for high strain rates. To address this limitation an investigation of the potential of a standard rheological test, the rotational viscometer test, has been carried out and is discussed below. In combination with triaxial tests, the study has covered a wide range of strain rates (6 orders of magnitudes) with triaxial tests covering the lower range (0.08 to 8 %/min) and rotational viscometer tests covering the upper range (6 to 60000 %/min).

2 EXPERIMENTAL STUDIES

Kaolin Q145 clay with the properties summarised in Table 1 was used in this study. Specimens for the two element tests were extracted from large one-dimensionally compressed clay cylinders prepared in a companion laboratory model FFP rate study discussed in Chow and Airey (2012).

<table>
<thead>
<tr>
<th>Table 1: Properties of kaolin Q145</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity</td>
</tr>
<tr>
<td>Plastic limit, PL</td>
</tr>
<tr>
<td>Liquid limit, LL</td>
</tr>
<tr>
<td>Coefficient of consolidation, c_v</td>
</tr>
<tr>
<td>Compression Index, C_c</td>
</tr>
<tr>
<td>Recompression Index, C_r</td>
</tr>
<tr>
<td>Critical state parameters (M, Γ, λ)</td>
</tr>
</tbody>
</table>

2.1 Triaxial Rate Study

The triaxial rate study involved nine K_o consolidated undrained compression tests (CK_oUC) covering 2 orders of magnitudes of strain rate from 0.08 to 8 %/min. These CK_oUC tests were monotonic constant rate of strain tests conducted on normally consolidated specimens with initial dimensions, 38 mm diameter x 76 mm high. Details of the apparatus, instrumentation and procedure are described in Chow and Airey (2012). The rate study involved specimens being one-dimensionally compressed to three pre-shear vertical effective pressures, \( \sigma'_v \) (62, 121 and 174 kPa), and for each of these pressures three strain rates (0.08, 0.8 and 8 %/min) were used during undrained shearing.

2.1.1 Test Results & Analysis

Data from tests conducted at the slowest, reference, strain rate of 0.08%/min, which also included CIU tests, were used to establish the critical state line, the parameters of which are given in Table 1. The critical state parameters have been used to estimate the reference undrained strengths in the rotational viscometer tests from the measured moisture contents. Results from the triaxial rate study are illustrated in Figure 1. A rate effect is clearly observed with the deviator stresses, q increasing with increasing strain rate, from 0.08 %/min to 8 %/min, for all three test series (Figure 1a). Similar rate effects have been reported in previous triaxial rate studies (e.g. Sheahan et al. 1996, Diaz-Rodriguez et al. 2005). To further investigate the strain rate effect on the undrained shear strength, the undrained shear strengths have been normalised by the vertical effective stresses at the start of shearing \( \left( s_u/\sigma'_v \right) \)

![Figure 1. Rate effects in triaxial tests](image-url)
and these are plotted against their respective strain rate on a logarithmic scale in Figure 1b. As reported in many other studies the normalised strength, $s_0/\sigma'_{vc}$ increases with increasing strain rate for all three test series. However, the results also show that the effects of rate of loading on $s_0/\sigma'_{vc}$ appear to reduce with increasing pre-shear vertical effective stress, $\sigma'_{vc}$. Depending on $\sigma'_{vc}$, the rate of increase in $s_0/\sigma'_{vc}$ ranges from 1.6 to 20% per log cycle for the strain rate range of 0.08 to 8 %/min. Similar dependency has been reported in the triaxial rate study reported by Diaz-Rodriguez et al. (2005) covering $\sigma'_{vc}$ between 300 to 40 kPa with the rate of increase in $s_0/\sigma'_{vc}$ varying from 17% to 41% per log cycle for the strain rate range of 0.02 to 13.3 %/min.

### 2.2 Parallel Plate Rotational Viscometer Rate Study

Rotational viscometer tests have been adopted in this study due to their ability to cover a wide range of shear strain rates (4 orders of magnitudes). Rotational viscometer tests have been used to determine the flow behaviours and shear strength properties of very soft clays and muds with high moisture contents (Torrance 1986, Locat and Ðemers 1988, Fakher et al. 1999). In these studies, coaxial (bob and cup) rotational viscometer tests were used to measure the yield stresses which were then correlated with the remoulded shear strengths measured from the Swedish cone test. It was reported by Torrance (1986) that coaxial rotational viscometer tests could not be used with clays with low moisture content (<LL) as shearing does not then occur through the full thickness of the annulus between the cylinders. In this study an alternative type of rotational viscometer, a parallel plate rotational viscometer (PPRV), has been used with the intention of avoiding these limitations.

#### 2.2.1 Test Setup and Procedure

The tests were conducted using a commercial rotational rheometer, Anton Paar Physica MCR 301, controlled by the manufacturer’s software, RheoPlus. The test involves shearing a thin cylindrical specimen 25 mm in diameter and approximately 2.1 mm high between two horizontal parallel circular plates by rotating the top plate at a pre-specified constant rate under controlled temperature (25 °C). The specimens were extracted from a single large clay sample using a 38 mm sampling tube and hence had similar initial moisture contents of approximately 36% (Table 2). Specimens were extruded and carefully trimmed down to size using a lubricated cutting ring that was 32 mm in diameter and 2.2 mm in height. Subsequently, the specimen was transferred to the bottom plate of the PPRV apparatus and further trimmed down to the required 25 mm diameter, and then the top plate was brought into contact with the specimen. The specimen was then sheared by rotating the top plate at the specified constant shear rate and the soil shear stress was determined from the torque measured at predetermined time intervals. The specimen trimmings during preparation and the specimen at the end of the test were collected and their moisture contents measured to enable comparison between the different tests.

#### 2.2.2 Test Results & Analyses

Ten tests covering the five strain rates (6, 60, 600, 6000 and 60000 %/min) were conducted. Repeat tests were conducted at each strain rate to check on the repeatability of the test results. The raw data produced from a test is comprised of the torques, $T$ measured at pre-determined time intervals, $t$. The raw data were then analysed to obtain the shear stress, $\tau$ and shear strain, $\gamma$ profile. For a specimen as shown in Figure 2a, the shear strain, $\gamma$ can be determined using Equation (1).

$$\gamma = \dot{\gamma} t = \left(\frac{r \theta}{h}\right) t$$

(1)

where $\dot{\gamma}$ is the shear strain rate, $t$ is the time, $r$ is the specimen radius, $\theta$ is the angular velocity and $h$ is the specimen height. The determination of the shear strength from the measured torque, $T$, is not straightforward as the shear stress varies with radius and time. For an element in the specimen with an area, $A$ equivalent to $rdldr$ (Figure 2a), the produced torque is the cross product of the displacement (radius) vector and the shear force vector equivalent to the product of $\tau$ and $A$. Assuming the specimen is in full contact with the top and bottom plates from $r = 0$ to $R$, the total torque acting on the specimen for the entire area can be determined from Equation (2).

$$T = \int_{0}^{\pi} \int_{0}^{R} \tau r^2 dr d\theta = 2\pi \int_{0}^{R} \tau r^2 dr$$

(2)
To determine the shear stress from Equation (2), the shear stress distribution along the radius of the specimen needs to be known. A linear shear stress distribution proportional to the radius is often assumed for a Newtonian fluid. As kaolin cannot be considered as a Newtonian fluid, it has been considered to be more realistic to adopt a shear stress distribution based on typical soil stress-strain behaviour as illustrated in Figure 2b. For a typical soil, the shear stress initially increases with increasing strain until it reaches a peak stress, \( \tau_{\text{peak}} \) at a corresponding strain, \( \gamma_{\text{peak}} \). Beyond \( \tau_{\text{peak}} \), the shear stress reduces with increasing shear strain and eventually reaches a residual state, \( \tau_{\text{residual}} \) at a corresponding strain, \( \gamma_{\text{residual}} \) where there is no more change in the shear stress with increasing strain. For ease of computation, a 3-phase linearised model has been adopted as shown in Figure 2b. Thus, the shear stress can be determined based on the idealised model as shown in Equation (3):

\[
T = 2\pi \int_0^R \tau r^2 dr
\]

\[
\tau = \begin{cases} 
   m\tau_w, & \gamma < \gamma_{\text{peak}} \\
   \tau_{\text{peak}} + n(\gamma_{\text{peak}} - \gamma), & \gamma_{\text{peak}} \leq \gamma < \gamma_{\text{residual}} \\
   \tau_{\text{residual}}, & \gamma \geq \gamma_{\text{residual}} 
\end{cases}
\]

\[
m = \frac{\tau_{\text{peak}} - \tau_{\text{residual}}}{\gamma_{\text{peak}} - \gamma_{\text{residual}}} 
\]

Equation (2) is subsequently integrated numerically to obtain the four parameters (\( \tau_{\text{peak}}, \tau_{\text{residual}}, \gamma_{\text{peak}}, \gamma_{\text{residual}} \)) by fitting the measured torque into the equation using a non-linear least-square method. Reasonable fitting was obtained as shown in Figure 3a. Using the fitting procedure, the four parameters (\( \tau_{\text{peak}}, \tau_{\text{residual}}, \gamma_{\text{peak}}, \gamma_{\text{residual}} \)) were determined for all tests and have been summarised in Table 2. In addition to the four parameters, Table 2 also lists the specimen height, \( h \), strain rate, \( \dot{\gamma} \), the initial and final moisture contents, \( w_i \) and \( w_f \), the peak torque, \( T_{\text{peak}} \) and the equivalent undrained shear strength, \( s_{u(\text{triaxial})} \) estimated using Equation (4) from the initial moisture content, \( w_i \).

\[
s_{u(\text{triaxial})} = s_{u(\text{ref})} \left[ 1 + \rho \log \left( \frac{\dot{\gamma}}{\dot{\gamma}_{\text{ref}}} \right) \right], \quad s_{u(\text{ref})} = \frac{M}{2} \left( \frac{1}{(1+\lambda G_s)} \right) \]

where \( M, \Gamma, \lambda \) are the critical state parameters obtained from the triaxial tests, \( G_s \) is the soil specific gravity, \( \rho \) is the rate of increase in \( s_{u(\text{ref})} \) per log cycle increase in strain rate, taken as 10% (approximate median value from the triaxial rate study) and \( \dot{\gamma}_{\text{ref}} \) is the slowest reference strain rate in the triaxial rate study, 0.08 %/min.

![Figure 2](image)

**Figure 2.** (a) A specimen experiencing shear in a PPRV test; (b) typical soil stress-strain behaviour

<table>
<thead>
<tr>
<th>Test No.</th>
<th>( h ) (mm)</th>
<th>( \dot{\gamma} ) (%/min)</th>
<th>( w_i ) (%)</th>
<th>( w_f ) (%)</th>
<th>( T_{\text{peak}} ) (mNm)</th>
<th>( \tau_{\text{peak}} ) (kPa)</th>
<th>( \tau_{\text{residual}} ) (kPa)</th>
<th>( \gamma_{\text{peak}} ) (%)</th>
<th>( \gamma_{\text{residual}} ) (%)</th>
<th>( s_{u(\text{triaxial})} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>2.20</td>
<td>6</td>
<td>35.4</td>
<td>32.6</td>
<td>5.21</td>
<td>1.32</td>
<td>-</td>
<td>2.21</td>
<td>-</td>
<td>24.06</td>
</tr>
<tr>
<td>2*</td>
<td>2.20</td>
<td>6</td>
<td>36.9</td>
<td>34.5</td>
<td>4.65</td>
<td>1.20</td>
<td>-</td>
<td>4.73</td>
<td>-</td>
<td>15.09</td>
</tr>
<tr>
<td>3</td>
<td>2.20</td>
<td>60</td>
<td>35.8</td>
<td>34.5</td>
<td>5.86</td>
<td>1.41</td>
<td>1.32</td>
<td>1.79</td>
<td>34.37</td>
<td>23.02</td>
</tr>
<tr>
<td>4</td>
<td>2.20</td>
<td>60</td>
<td>37.1</td>
<td>35.7</td>
<td>5.50</td>
<td>1.30</td>
<td>1.28</td>
<td>2.64</td>
<td>99.94</td>
<td>15.28</td>
</tr>
<tr>
<td>5</td>
<td>2.10</td>
<td>600</td>
<td>35.2</td>
<td>34.0</td>
<td>8.80</td>
<td>2.15</td>
<td>1.74</td>
<td>2.18</td>
<td>58.23</td>
<td>30.30</td>
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<tr>
<td>6</td>
<td>2.25</td>
<td>600</td>
<td>36.8</td>
<td>36.8</td>
<td>7.97</td>
<td>1.94</td>
<td>1.52</td>
<td>2.75</td>
<td>64.65</td>
<td>18.35</td>
</tr>
<tr>
<td>7</td>
<td>2.10</td>
<td>6000</td>
<td>36.7</td>
<td>35.3</td>
<td>8.60</td>
<td>2.12</td>
<td>1.40</td>
<td>5.44</td>
<td>139.65</td>
<td>20.16</td>
</tr>
<tr>
<td>8</td>
<td>2.15</td>
<td>6000</td>
<td>37.2</td>
<td>35.2</td>
<td>7.91</td>
<td>1.91</td>
<td>1.57</td>
<td>6.01</td>
<td>156.22</td>
<td>17.17</td>
</tr>
<tr>
<td>9</td>
<td>2.25</td>
<td>60000</td>
<td>36.8</td>
<td>35.6</td>
<td>12.07</td>
<td>2.13</td>
<td>2.85</td>
<td>2.98</td>
<td>35.38</td>
<td>20.72</td>
</tr>
<tr>
<td>10</td>
<td>2.20</td>
<td>60000</td>
<td>37.2</td>
<td>31.7</td>
<td>12.62</td>
<td>3.12</td>
<td>2.45</td>
<td>13.09</td>
<td>218.64</td>
<td>18.35</td>
</tr>
</tbody>
</table>

* \( T_{\text{peak}} \) and \( \tau_{\text{peak}} \) are taken at turning point in the stress-strain curve as no peak nor residual torque/stress is observed.
There appear to have been no reported attempts to use PPRV devices for clays with strengths of a few kPa and thus there is considerable uncertainty about the best procedure and method of interpretation. Some of the difficulties encountered included determining the best plate surface to minimise slip, lack of control over the normal stress applied, moisture content changes, and uncertainty about the method of interpretation. It is assumed that no slip occurs between the soil and the rotating plate, and that contact between the soil and the top plate is maintained. Although some preliminary tests were performed to assess the best plate surface, the tests take place in a sealed chamber, the normal stress is fixed in the software, and no observations are possible during the tests. Some loss of soil moisture occurred during the tests, as evident in the difference between the initial and final moisture contents reported in Table 2, but these moisture losses showed no obvious dependency on strain rate or specimen thickness. The moisture losses lead to uncertainty about the reference strength, and to questions about whether the tests are undrained as assumed. Also the absence of a peak in the torque for the tests at the lowest strain rate of 6 %/min (Fig. 3b), could not be accommodated by the assumed stress, strain response and for these tests the peak shear stress has been assumed to occur at the turning point. Possibly as a consequence of these experimental issues the peak shear stresses, $\tau_{\text{peak}}$ estimated from the PPRV tests are significantly lower than the equivalent shear strengths, $s_u(\text{triaxial})$ estimated using Equation (4) as shown in Table 2 and Figure 4a. This is unexpected as the PPRV tests use strain rates much higher than the triaxial tests. One contributing factor to the low $\tau_{\text{peak}}$ might be the uncertainty in the shear stress distribution across the specimen radius. To investigate this possibility, the shear stress estimated using both the assumed soil model/Equation (3) and the commonly assumed linear distribution (shear stress increases linearly with radius) are compared as shown in Figure 4b. The conventional linear distribution is found to produce higher shear stress than that estimated using Equation (3). However, despite the higher shear stress, both approaches still produce $\tau_{\text{peak}}$ that are much lower than $s_u(\text{triaxial})$.

3 DISCUSSION

To further investigate the PPRV results additional tests covering a wider range of moisture contents have been conducted. Figure 5 shows the measured $\tau_{\text{peak}}$ for all these tests and $s_u(\text{triaxial})$ determined using Equation (4). It can be seen that the PPRV test results show a similar variation of peak strength with moisture content as the triaxial tests, but the peak strengths are consistently about an order of
magnitude lower. This suggests that the low strengths are not a consequence of slip at the boundary, and that the PPRV tests may provide some useful information. For example, the results in Figure 5 suggest that the strength is not significantly affected by the strain rate for the high rates used in the PPRV tests, and this is in agreement with the observations from laboratory model FFP tests (Dayal 1974, Chow and Airey 2012), in which a plateau in rate effect is reached at high velocities.

![Figure 5. Additional PPRV tests covering wider range of moisture contents](image)

### 4 CONCLUSIONS

Laboratory rate studies using two element tests (triaxial and PPRV tests) covering a wide strain rate range (6 order of magnitudes) have been conducted to investigate viscous rate effects in kaolin. The triaxial tests covering the strain rate range from 0.08 to 8 %/min have shown a rate dependency with an increase in undrained shear strength per log cycle increase in strain rate of 6 to 20% (depending on $\sigma_{uc}^*$. The PPRV tests suggest there is little rate effect between strain rates of 60 and 60000 %/min, however, the tests indicate strengths that are much too low and thus the results from these tests cannot be considered to be reliable. More work is required to understand the reason for the low strengths from the PPRV tests and to further explore their potential before any conclusion can be drawn.

### REFERENCES


