Experimental test on steel storage rack components

Research Report No R899

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Kim J.R. Rasmussen, MScEng, PhD

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Abstract:
Steel storage racks are commonly used worldwide to store goods on pallets and represent complex and challenging freestanding structures to design. In the current competitive market, storage racks have to be able to carry heavy loads while being designed as lightly and economically as possible. Often unbraced, their stability may depend solely on the pallet beam to column connector and on the base plate connection to the floor characteristics (Baldassino and Bernuzzi (2000)). Moreover, to allow bracings and pallet beam connections, the web and flanges of the upright section are perforated at regular intervals along the length affecting the axial and bending resistance of the section (Hancock (1998)). The main international racking specifications recognise the importance of accurately determining the properties of individual components of storage racks and require testing for this purpose. This report presents experimental results from tests performed on individual components, namely coupon tests, stub-column tests, pallet beam to column connection tests, base plate floor connection rotational and uplift tests, upright frame shear tests and four point bending tests of uprights. Clarification of the guidance provided by the European Standard EN 15512 (2009) for the base plate to floor connection test is presented as is an alternative test set-up to the upright frame shear test described in EN 15512 (2009), allowing accurate determination of the in-plane global stiffness of an upright frame.

Keywords:
Steel storage racks, individual component tests, Base plate tests, Upright frame shear tests.
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Experimental test on steel storage rack components
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# Introduction

The experimental test on steel storage rack components was conducted in October 2009. The study aimed to evaluate the performance of different components under various loading conditions. The test outcomes provided valuable insights into the structural behavior of the racks.

## Experimental Test Setup

The experimental setup was designed to simulate real-world conditions. Key components included:

- **Base Plate to Floor Connection** – Rotational Stiffness Tests
  - **Base Plate Behavior**
  - **Method 1**
  - **Method 2**
  - **Results and Comparison**

- **Base Plate to Upright Connection** – Uplift Stiffness Calibration Tests
  - **General**
  - **Experimental Test Setup**
  - **Experimental Test Results and Multi-Linear Approximation**

- **Shear Stiffness of Upright Frame Tests**
  - **General**
  - **Literature Review**
  - **Alternative Test Setup No1**
  - **Alternative Test Setup No2**

- **Four Points Bending Test on Uprights**
  - **General**
  - **Test Set-Up**
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Appendix 2: Portal frame test, calculation of the second moment of area $I_b$ of the portal beam
Appendix 3: Portal frame test, test comments
Appendix 4: Upright frame shear test, Timoshenko shear formula for upright frame in the longitudinal and transverse direction
Appendix 5: Upright frame shear test, boundary conditions for FE analysis
Appendix 6: Upright frame shear test, upright and bracing section characteristics, Tested upright frame dimensions
1 INTRODUCTION

Rarely seen by the general public, steel storage racks are extensively used in industry for storing goods on pallets. They are freestanding structures mainly made from cold-formed steel profiles and are able to carry heavy loads while being designed as lightly as possible. Different racks are available on the market and are described in Pekoz and Winter (1973). “Selective rack” is the most common type of rack in which each racking structure is separated from another by aisles allowing each pallet to be always accessible. Figure 1 shows a typical storage rack.

Figure 1: Typical storage rack with base plate and pallet to column connector shown

This report describes experimental tests on individual storage rack components performed in the structures laboratory of the School of Civil Engineering at the University of Sydney. All components tested are part of a drive-in racking system but observations reported in this report apply to steel storage racks in general. Contrary to selective racks, in drive-in racks, pallets are stored on beam rails, one after each other on the first-in last-out principle. When storing the same goods or located in expensive locations such as industrial freezers, by minimising floor allocation, drive-in racks are often an attractive alternative to selective racks.

The main international racking design specifications such as the RMI (2008) (Rack Manufacturers Institute), Australian Standard AS 4084 (1993) and European Standard EN 15512 (2009) recognise the importance of investigating individual component characteristics but can have different approaches to determine their behaviours. Coupon and stub-column tests on uprights, portal beam to upright connection rotational stiffness tests, base plate to floor connection rotational and uplift stiffness tests, shear tests on upright frames and four point bending tests on uprights have been experimentally investigated and are reported in the present report.

The EN 15512 (2009) Standard has been recently published and supersedes the FEM (1998) specification (Fédération Européenne de la Manutention). The test procedures are similar in the two specifications but there are some differences as mentioned in the present report.
Prior to testing, the RMI (2008), AS 4084 (1993) and EN 15512 (2009) require the material properties to be obtained by means of tensile coupon tests. This report presents the results of tests on coupons cut from the upright used for the component experimental tests.

Storage racks upright are typically perforated in the flange to connect frame bracing members and in the web to connect pallet beams. These perforations can reduce the bending and axial capacities of the upright (Hancock (1998)). All three previously mentioned main specifications impose compression testing on a short length of upright (stub-column tests) to determine the effect of perforations on the upright load carrying capacity. Three stub column tests are presented in the present report on the upright type used in the component experimental tests.

Often unbraced, the stability of steel storage racks is sensitive to the behaviour of the base plate to floor connection and to the pallet beam to column connector behaviour (Baldassino and Bernuzzi (2000)). The RMI (2008), AS 4084 (1993) and EN 15512 (2009) agree on the method for determining the stiffness and strength of the pallet beam to column connectors and impose testing by proposing one or two alternative test set-ups referred to as “cantilever test” and “portal frame test”. Yet, different approaches are taken to determine the base plate to floor connection characteristics; while the RMI (2008) and AS 4084 (1993) use a formula for the base plate rotational stiffness based on the base plate dimensions and the modulus of elasticity of the floor, the EN 15512 (2009) recommends experimental testing for determining both the stiffness and strength of the connection.

A literature review on the pallet beam to column connector shows that the portal frame test appears to be a more accurate way to determine the stiffness of the portal beam to upright connector. Experimental results from the portal frame test are presented in the present report. Due to the unusual bolted connection between the portal beam and the upright, results show sudden changes in the connector stiffness while performing the test.

The base plate rotational stiffness has been tested according to the EN 15512 (2009), however uncertainty in the EN 15512 (2009) procedure leads to considering two alternative test set-ups. Both alternatives have been investigated resulting in one alternative being found superior to the other. Experimental results and clarification of the test procedures are presented in this report.

While performing experimental tests on a full-scale braced drive-in rack structure, it was observed that the base plate lifts up at the base when the upright is subjected to a tension force. The uplift stiffness is found to influence the overall behaviour of the storage rack (Gilbert and Rasmussen (2009a)) but is not accounted for in the three above mentioned specifications. A test method to determine the base plate uplift stiffness is presented in this report.

The transverse shear stiffness of upright frames is required to accurately evaluate the cross-aisle deflection and elastic buckling load of upright frames (Sajja et al. (2006)). The RMI (2008) and AS 4084 (1993) evaluate this shear stiffness using shear stiffness formulae given by Timoshenko and Gere (1961). The EN 15512 (2009) imposes the testing of a length of upright frame in the longitudinal direction and the evaluation of the transverse shear stiffness using Timoshenko and Gere (1961)’s shear stiffness theory. However, it is unclear if the effects contributing to the shear deformation of upright frames would manifest themselves to a different extend in the transverse and longitudinal directions. An alternative to the EN 15512 (2009) test set-up is proposed and investigated in this report by testing an upright frame in both bending and shear in the transverse direction. The results allow the upright frame cross-aisle displacement to be accurately modelled and the proposed approach is believed to be superior to the EN 15512 (2009) approach.
Finally, four points bending tests are performed to accurately determine the flexural rigidity of the upright about the major and minor axes, taking into account the presence of perforations. Results are analysed and presented.

2 COUPON TESTS

2.1 General

The RMI (2008), AS 4084 (1993) and EN 15512 (2009) recommends material properties to be accurately determined by means of tensile coupon tests. The resulting material yield stress $f_y$ and Young’s modulus $E$ are needed for most component tests.

Tensile coupon tests have been performed on the steel uprights in accordance with Australian Standard AS 1391 (2007). The upright type used in the present report is referred to as RF12519, has a nominal thickness of 1.9 mm, a nominal width of 125 mm, is a rear flange upright type and is roll-formed from nominally G450 steel to Australian Standard AS 1397 (2001). The measured cross-section dimensions of a RF12519 specimen is given in Appendix 1. Two steel sheet coils were used to roll-form the uprights used for the component tests presented in this report. Three coupons are cut from flat lengths of each steel coil. For each coil, two of these coupons are tested to failure using a 40 mm gauge extensometer and the remaining coupon is fitted with strain gauges on each side of the narrow strip to accurately measure the Young’s modulus by loading and unloading the coupon twice in the elastic range. Table 1 gives the measured thickness of each coil after etching off the galvanising layer.

<table>
<thead>
<tr>
<th>Thickness coil n°1 (mm)</th>
<th>Thickness coil n°2 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.91</td>
<td>1.87</td>
</tr>
</tbody>
</table>

*Table 1: Measured thickness of steel coil*

The coupons were tested in a 300 kN capacity MTS Sintech testing machine with a strain rate of $2.78 \times 10^{-4}$/s.

<table>
<thead>
<tr>
<th>Coil n°</th>
<th>Coupon n°</th>
<th>Dynamic yield stress (MPa)</th>
<th>Static yield stress (MPa)</th>
<th>Dynamic ultimate strength (MPa)</th>
<th>Static ultimate strength (MPa)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>501.4</td>
<td>444.0</td>
<td>537.0</td>
<td>501.8</td>
<td>13.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>498.8</td>
<td>451.9</td>
<td>536.6</td>
<td>505.1</td>
<td>11.5</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>500.1</td>
<td>447.9</td>
<td>536.8</td>
<td>503.4</td>
<td>12.4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>522.4</td>
<td>472.4</td>
<td>556.1</td>
<td>523.7</td>
<td>12.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>525.3</td>
<td>468.4</td>
<td>557.5</td>
<td>522.3</td>
<td>11.9</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>523.9</td>
<td>470.4</td>
<td>556.8</td>
<td>523.0</td>
<td>12.2</td>
</tr>
</tbody>
</table>

*Table 2: Coupon test results*

2.2 Coupon test results

The complete set of stress-strain curves is presented in Figure 2 and Figure 3. The yield stress $f_y$ is obtained from these curves as the 0.2% proof stress and is summarised in Table 2, as are the ultimate tensile strength $f_u$ and uniform elongation $\varepsilon_u$ after fracture of each coupon. The dynamic values shown in Table 2 are obtained when pausing the test at the yield stress and at the tensile strength, while the static values are obtained after pausing the tests for at least 2 minutes at the
yield stress and at the tensile strength. Table 3 gives the Young’s modulus obtained from the coupons fitted with strain gauges, obtained from the stress-strain curves shown in Figure 3.

<table>
<thead>
<tr>
<th>Coil n°</th>
<th>Test n°</th>
<th>Young’s modulus E (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>219397</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>212769</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>216083</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>223721</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>217412</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>220567</td>
</tr>
<tr>
<td>1 and 2</td>
<td>Average coil n°1 and 2</td>
<td>218325</td>
</tr>
</tbody>
</table>

Table 3: Coupon test with strain gages, Young’s modulus

![Figure 2: Coupon test results (a) steel coil no1 and (b) steel coil no2](image)

![Figure 3: Coupon test with strain gages, Young’s modulus (a) steel coil n°1 and (b) steel coil n°2](image)

3 STUB COLUMN TESTS

3.1 General

Storage rack uprights are typically perforated in the flange to connect frame bracing members and in the web to connect pallet beams. These perforations reduce the bending and axial capacities of the upright (Hancock (1998)). The RMI (2008), AS 4084 (1993) and EN 15512 (2009) impose compression testing on a short length of upright, short enough to avoid column buckling, as specified in Clause 8.1.2 of the Australian Standard AS/NZS 4600 (2005) or in Part VIII of the AISC (1996) manual.
The main purpose of the stub column tests is to determine the Q-factor defined as,

$$Q = \frac{P_u}{f_y \cdot A_{\text{net}_{\text{min}}}} \leq 1$$  \hspace{1cm} (1)$$

where $P_u$ is the experimental stub column strength, $f_y$ is the measured yield stress given in Section 2.2 and $A_{\text{net}_{\text{min}}}$ is the minimum net area defined as the minimum area of a plane passing through the cross-section of the upright, including perforations. The Q-factor is used in cold-formed steel design formulas as a reduction factor accounting for the effect of local buckling and postbuckling of members with perforations.

Three RF12519 upright specimens were tested in a 2000 kN capacity Dartec testing machine in which the lower platen is fixed while the cross head of the machine itself is mounted on a half sphere bearing which could rotate so as to provide full contact between the platen and the specimen, as shown in Figure 4. The lengths of upright are milled flat at each end. The RF12519 specimens are all roll-formed from the steel coil n°1 introduced in Section 2.

![Figure 4: Stub column test on RF12519 upright – Test set-up and failure mode](image)

### 3.2 Stub column test results

Table 4 gives the specimen characteristics where $A_{\text{net}_{\text{min}}}$ is accurately determined by subtracting the maximum hole area $A_{\text{holes}}$ from the gross area $A_{\text{gross}}$. $A_{\text{holes}}$ is determined by measuring the diameter of the holes on the stub column specimens and multiplying it by the measured base metal coil thickness, and $A_{\text{gross}}$ is determined by measuring the flat coil width and multiplying it by the measured base metal coil thickness.

<table>
<thead>
<tr>
<th>Upright</th>
<th>Measured thickness (mm)</th>
<th>Measured coil width (mm)</th>
<th>$A_{\text{gross}}$ (mm$^2$)</th>
<th>$A_{\text{net}_{\text{min}}}$ (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF12519</td>
<td>1.91</td>
<td>385.2</td>
<td>735.7</td>
<td>686.1</td>
</tr>
</tbody>
</table>

*Table 4: Stub column test specimen characteristics*
The speed of the ram and the ultimate load supported by each specimen are shown in Table 5. The dynamic values shown in Table 5 are obtained when pausing the test at the ultimate load and the static values are obtained after pausing the test for at least 2 minutes at the ultimate load.

The Q-factor is calculated from Equation 1 using the average static yield stress in Table 2 and the average static ultimate load in Table 5 and is found to be equal to 0.994.

<table>
<thead>
<tr>
<th>Upright</th>
<th>Test n°</th>
<th>Measured specimen length (mm)</th>
<th>Cross head speed (mm/min)</th>
<th>Dynamic peak load (kN)</th>
<th>Static peak load (kN)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF12519</td>
<td>1</td>
<td>500</td>
<td>0.2</td>
<td>309.8</td>
<td>302.5</td>
<td>0.984</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>497</td>
<td>0.2</td>
<td>314.1</td>
<td>306.8</td>
<td>0.998</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>497.2</td>
<td>0.2</td>
<td>314.0</td>
<td>307.3</td>
<td>0.999</td>
</tr>
<tr>
<td>Average</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>312.6</td>
<td>305.5</td>
<td>0.994</td>
</tr>
</tbody>
</table>

Table 5: Stub column test results

4 PORTAL BEAM TO UPRIGHT CONNECTION TESTS

4.1 General

A literature review of the behaviour of pallet beam to upright connectors shows that the portal frame test is an accurate way to determine the stiffness of the pallet beam/portal beam to upright connector when analysing the stability of the rack in the down-aisle direction (sway motion). This section reviews the method for determining the pallet beam/portal beam to upright connection characteristic from the main racking design specifications.

Portal beam to upright bolted moment connections are not frequently used in drive-in and drive-through storage racks, where horizontal beams are typically connected to the upright using “tab connectors” as detailed in Section 4.2.1. However, tab connectors are costly to manufacture, they require the cold-forming of the connector, punching or pressing the tabs in the connector, welding the connector to the beam and attaching a locking pin system to the connector (see Figure 5). Studies presented in Section 4.2.2 show that bolted moment connection between cold-formed steel members are economical and feasible, and thus, may represent an attractive economical alternative to tab connectors.

This section presents cyclic experimental tests on bolted moment portal beam to upright connections of drive-in and drive-through storage racks. The non-linear cyclic behaviour of the bolted connection is presented and explained. Due to the nature of the connection, experimental results show sudden changes in the connection stiffness value, which includes a large amount of looseness. However, contrary to tab connectors, where looseness is initially present in the connector, looseness in the tested connection is only encountered after applying a non-negligible moment to the connection. A literature review of bolted moment connections shows that looseness in the connection is acknowledged but often disregarded in determining the connection characteristics, as previous studies focused on structures not sensitive to P-Δ effects. However, storage racks are slender structures, sensitive to second-order effects, and the RMI (2008), AS 4084 (1993) and EN 15512 (2009) therefore require P-Δ effects to be considered in the design, implying the tab connector looseness to be taken into account, especially for unbraced racks.

Finite Element results show that first yield may develop in the structure before the moment applied to the bolted connection induces slippage in the connection, and this section concludes
with proposing two methods, with different degree of complexity, for designing drive-in and drive-through racks with bolted moment connections.

4.2 Literature review

4.2.1 Pallet beam to upright connector in storage racks

In storage racks, pallet beams and portal beams are typically connected to uprights by means of beam end connectors. Often connectors are fitted with tabs which are inserted into the perforations of the upright web and secured in place by locking pins. Different types of connectors are described and tested in Markazi et al. (1997). Figure 5 shows an example of a beam end connector.

![Figure 5: Example of beam end connector](image)

Often unbraced, the stability of storage racks in the down-aisle direction is provided by the beam end connectors and by the base plate connection (Baldassino and Bernuzzi (2000)). Moreover, the axial capacity or “pull-out” capacity of the pallet beam to upright connector influences the mode of collapse of steel storage racks when subjected to forklift impact at the bottom of an upright (McConnel and Kelly (1983)). If the “pull-out” capacity of the connector is low then the collapse is likely to be confined whereas if the “pull-out” capacity of the connector is high, collapse is likely to be progressive.

Partly due to the diversity of beam end connectors, determining the stiffness and strength of the connector analytically is not currently practical (Bernuzzi and Castiglioni (2001) and Markazi et al. (1997)) and the main international racking design codes impose testing to determine these properties. Two beam end connector test set-up alternatives are proposed by the RMI (2008), AS 4084 (1993) and FEM (1998), the choice of the test set-up being the responsibility of the designer. However, only one test set-up is now proposed by the recent EN 15512 (2009).

In the first test set-up, referred to as “cantilever test” and shown in Figure 6, a short length of pallet beam is connected to a short length of upright. The upright is rigidly connected at both ends and the pallet beam is connected at the centre of the upright. A force is applied near the extremity of the pallet beam and measurements are taken of the displacements at the force
location and/or the rotation of the pallet beam near the connector. Even if the test set-ups in the previously mentioned design codes are very similar, slight differences in the test set-up dimensions, procedure and position of transducers are encountered, as described in Harris (2006). The test set-up allows forces to be applied cyclically to measure the opening stiffness, closing stiffness and looseness of the connector. Examples of earthquake related cyclic tests on beam end connectors are reported in Bernuzzi and Castiglioni (2001).

In the second test set-up, referred to as “portal frame test”, two lengths of upright are connected by a pallet beam to form a portal frame, pinned at its base. This alternative set-up is not included in the EN 15512 (2009). The pallet beam is loaded with its service load, typically pallets are used to load the beam requiring the use of two parallel portal frames. A horizontal load is then applied at the level of the top of the pallet beam and horizontal displacements are recorded at the same elevation. The FEM (1998) requires the test to be performed to failure while the RMI (2008) and AS 4084 (1993) only require the maximum applied load to be twice the maximum horizontal design load. Other minor differences in the test set-up dimensions and procedures are reported in Harris (2006). Figure 7 shows the portal frame test set-up from the AS 4084 (1993).

As the portal frame test allows the average stiffness of one connector closing up and of one connector opening to be determined under the sway motion of the frame, the results from the portal frame tests are generally used for sway analysis while results from the cantilever tests are usually used for the design of beams and connectors (Sarawit and Pekoz (2002)).
Harris (2006) compared the two experimental set-ups and found that the connector stiffness values obtained from the cantilever test are typically half the connector stiffness values obtained from the portal frame test. Harris explained the difference by the closing-opening effect of the connector in the portal frame test. Indeed, when the portal beam is loaded with its service pallet load, the left and right connectors are closing up, as shown in Figure 8 (a), and when the portal frame is subsequently subjected to a horizontal load the left connector in Figure 8 (b) keeps closing up while the right connector starts opening. Typically, cyclic cantilever tests show that when reversing the load, the stiffness does not follow a linear path and the unloading stiffness is significantly higher than the loading stiffness (Abdel-Jaber et al. (2005, 2006)), resulting in two different stiffness values for the right and left connectors when performing a portal frame test and explaining the difference between the stiffness values obtained from the cantilever and portal frame tests.

![Figure 8: Portal frame test deformation (a) application of pallet load and (b) application of lateral load](image)

4.2.2 Bolted moment connections between cold-formed steel members

Investigations to determine the moment-rotational behaviour of bolted moment connections between cold-formed steel members have been reported using gusset plates between members by Chung and Lau (1999), Wong and Chung (2002) or Lim and Nethercot (2003, 2004b), and with members bolted directly together by Zaharia and Dubina (2006) or Uang et al. (2009). These investigations were motivated by the growing trend of using cold-formed steel members in buildings in the 1990’s and the lack of design rules for this type of connection (Chung and Lau (1999)).

Studies by Chung and Lau (1999), Wong and Chung (2002) and Lim and Nethercot (2003) showed that bolted moment connections between cold-formed steel members are feasible and economical with an ultimate capacity of the connection up to 85% of the capacity of the connected members. The stiffness of bolted moment connections can also be used to reduce the effective length in member design (Zaharia and Dubina (2006)).

A typical experimental moment-rotation curve of a bolted moment connection is given by Dubina and Zaharia (1997) and is reproduced in Figure 9. High initial connection stiffness followed by a slippage of the connection is encountered, and in the last phase, the connection stiffens when bolts are in bearing until failure. While being a significant part of the connection behaviour, the “initial slip rotation” in Figure 9 is often acknowledged but generally ignored in the design when calculating the connection stiffness $K_i$ shown in Figure 9 (Dubina and Zaharia (1997), Yu et al. (2005) and Lim and Nethercot (2003, 2004a, 2004b)). Disregarding the initial slip rotation was justified by Kitipornchai et al. (1994) who studied, using Finite Element analysis, a cantilever truss in bending and a braced transmission tower under vertical and
horizontal loads, and showed that the connection slippage influences the deflection of the structure but not the ultimate load. On the other hand, by experimentally also investigating a cantilever truss in bending, Zaharia and Dubina (2006) showed that the axial load in the diagonal bracings prevents the initial rotational slippage of the connection, yet the deflection of the tested truss was influenced by the bolt slippage. However, the previously reported studies were carried out on structures mainly in bending in which second-order P-Δ effects were insignificant, and for such structures, ignoring the initial connection looseness is acceptable. For slender structures like storage racks, sensitive to P-Δ effects, the looseness of the connection cannot be ignored and needs to be addressed in the design as stipulated by the RMI (2008), AS 4084 (1993) and FEM (1998).

Figure 9: Typical moment-rotation characteristic of bolted connection (from Dubina and Zaharia (1997))

Uang et al. (2009) and Sato and Uang (2009) experimentally investigated the cyclic behaviour of cold-formed steel bolted moment connections for the earthquake design of one story framing systems (e.g. freestanding mezzanine). A test set-up similar to the cantilever test set-up shown in Figure 6 was used to characterise the cyclic behaviour of the connection. The results showed a similar behaviour to the experimental results presented in Section 4.3.4.1. Uang et al. (2009) proposed a simple model to evaluate the overall connection stiffness, and the stiffness associated with the bolts in bearing. They expressed the bearing resistance $R_B$ as (Fisher (1965)),

$$ R_B = R_{ult} \left[1 - e^{-\mu(\delta_{br}/25.4)\lambda} \right] $$

where $R_{ult}$ is the ultimate bearing strength of the member and $\delta_{br}$ is the bearing deformation of the bolt hole. Values for coefficients $\mu$ and $\lambda$ were given by Crawford and Kulak (1971) but are valid for thicker members than cold-formed steel members, for which the bearing deformation is not associated with bolt deformation. For cold-formed steel members, Uang et al. (2009) proposed the values $\mu=5$ and $\lambda=0.55$ with,

$$ R_{ult} = 2.1dtF_u $$

In Equation 3, $t$ is the ply thickness, $d$ is the bolt diameter and $F_u$ is the ultimate tensile strength. A procedure for calculating the seismic design moments and forces in cold-formed members is also developed in Sato and Uang (2009), taking into account bolt slippage and yielding of the bolt holes due to bolts in bearing. While considering bolt slippage in the design, the previous study is intended for seismic design, not for the ultimate or serviceability limit state.
4.3 Experimental tests

4.3.1 Choice of test set-up

In drive-in and drive-through rack structures, pallets are stored on beam rails, and portal beams are only located at the top of the rack and are not intended for supporting pallets. However, their stiffness contributes to the side-sway stability of the rack. The portal beam end connection stiffness can be determined using the test set-ups described in Section 4.2.1.

The bolted moment connection tested consists of 13 mm diameter circular holes punched in both the portal beam web and the upright web, and two M12 bolts as shown in Figure 10. All members are roll-formed from flat steel sheets and bolt holes are punched during the rolling process.

Due to the symmetry of the connection, the closing up and opening stiffness values are expected to be similar. Moreover, as intended for drive-in and drive-through rack applications, no vertical load needs to be applied to the portal beam while performing a portal frame test, and hence no initial rotation of the connection occurs before applying the lateral load. Consequently, similar stiffness values should be expected to be obtained from a cantilever test and portal frame test. However, by providing a statistically more accurate stiffness by testing two connectors at once instead of one, as in the cantilever test, the portal frame test approach is used here to determine the characteristic of the connection.

4.3.2 Experimental set-up

The portal frame test set-up is shown in Figure 11. Two 700 mm high uprights are welded to 6 mm thick end plates bolted to hinges which are clamped to the strong floor rails as shown in Figure 11. The vertical distance between the centre of the hinges to the bottom of the uprights is equal to 65 mm. The distance between the upright centrelines is chosen to be 1475 mm which corresponds to an actual drive-in rack configuration. Only one portal frame is used per test.

A modification is proposed to the testing procedure in the RMI (2008), AS 4084 (1993) and FEM (1998), in which the lateral load and horizontal displacements are applied and recorded, respectively, at the top of the portal beam, and in which the connection stiffness is then determined for that location. The procedure used in this test program aims to define the
connection stiffness at the centreline axis of the portal beam, and hence the lateral load is applied and horizontal displacements are recorded at this axis.

The lateral load is applied through a pinned joint at the shear centre plane of the upright using a stroke controlled 100 kN hydraulic jack. Due to misalignments, the jack applies the lateral load 17 mm below the centreline of the portal beam. The portal beam is bolted to the upright applying a 20 N.m torque per bolt following the manufacturer standard procedures.

![Figure 11: Portal frame experimental test set-up (a) set-up and (b) photo of actual set-up](image)

For each upright, two LVDTs (Linear Variable Differential Transformer) record the horizontal displacements at the centreline of the portal beam. Two additional LVDTs record the horizontal displacements of the hinge above the strong floor beam rail to indicate if sliding occurs. The LVDT numbering is given in Figure 11 (a).

### 4.3.3 Connection stiffness

#### 4.3.3.1 International racking design codes

The RMI (2008), AS 4084 (1993) and FEM (1998) give similar approaches to determining the average beam end connector stiffness. The RMI (2008) and AS 4084 (1993) calculate the lateral displacement $\delta$ at the top of the portal beam for one portal frame loaded with a lateral load of $2H$ in terms of the average beam end connector stiffness $k_{RMI,AS}$ as,

$$\delta = \frac{Hh^3}{3EI_c} + \frac{Hh^2L}{6EI_b} + \frac{Hh^2}{k_{RMI,AS}}$$

(4)

where $E$ is the Young’s modulus, $h$ is the vertical distance from the floor to the top of the portal beam, $I_c$ and $I_b$ are the second moments of area of the portal beam and the upright respectively and $L$ is the horizontal distance between the upright centrelines. By solving Equation 4, the average beam end connector stiffness $k_{RMI,AS}$ is given for a regular test with two portal frames with a lateral load of $2H$ as,

$$k_{RMI,AS} = \frac{1}{\frac{2\delta}{Hh^2} - \frac{h}{3EI_c} - \frac{L}{6EI_b}}$$

(5)
When no vertical load is applied to the portal beam, the FEM (1998) gives the moment $M_{FEM}$ in the connector when a lateral force $F$ is applied to a test set-up with two portal frames as,

$$M_{FEM} = \frac{Fh}{4} \left(1 - \frac{d}{L}\right)$$

(6)

where $h$ is the vertical distance from the bottom hinge to the top of the portal beam, $d$ is the width of the face of the upright and $L$ is the distance between the central lines of uprights. The FEM (1998) expresses the rotation $\theta_{FEM}$ of the beam end connector as,

$$\theta_{FEM} = \frac{\delta}{h} - F \left(\frac{h^2}{12EI_c} + \frac{hL}{24EI_b}\right)$$

(7)

where $\delta$ is the average lateral displacement at the top of the portal beam. The $M_{FEM}-\theta_{FEM}$ experimental curve can be used to either derive a bi-linear curve in which the connector stiffness $k_{FEM}$ is calculated as “a line through the origin which isolates equal area between it and the experimental curve, below the design moment ... $M_{Rdc}$” or a multi-linear curve. The design moment $M_{Rdc}$ is defined in Sections 13.3.3 and A.2.4.5.1 of the EN 15512 (2009) Specification.

### 4.3.3.2 Modified stiffness equations

The above equations are modified to consider the height of the hinges at the bottom of the uprights in the test set-up shown in Figure 11, and the beam connection stiffness $k$ is calculated with respect to the intersection of the upright and portal beam central lines.

The total sway deformation of the portal frame can be broken into two parts with identical internal forces: a portal sway deformation with rigid connections as shown in Figure 12 (a) and a portal frame deformation due only to the connection rotations as shown in Figure 12 (b). In Figure 12, points A and F represent the centre of the hinges, points B and E represent the bottom of the uprights and points C and D are the intersections of the central lines of the uprights and portal beam.

The vertical eccentricity $\Delta F$ of the point of application of the load in Figure 12 represents less than 2.5% of the upright height and its effect is ignored in the calculation of the portal beam to upright connection stiffness $k$.

![Figure 12: Portal frame deformation broken into two parts (a) portal sway and (b) connection stiffness](image-url)
points B and C respectively and in terms of the horizontal displacements $\delta_{B,1}$ and $\delta_{C,1}$ at point B and C respectively as,

$$M_C = 4k_c\theta_C + 2k_c\theta_B - 6\frac{k_c}{h_1}(\delta_{C,1} - \delta_{B,1}) = -\frac{F}{2}h$$  (8)

$$M_B = 2k_c\theta_C + 4k_c\theta_B - 6\frac{k_c}{h_1}(\delta_{C,1} - \delta_{B,1}) = \frac{F}{2}(h - h_1) = -M_C \left(\frac{h - h_1}{h}\right)$$  (9)

where $h$ and $h_1$ are the vertical distance from the centre of the hinge to the portal beam centreline and the vertical distance from the top of the hinge to the portal beam centreline respectively. $k_c$ is the upright stiffness given as,

$$k_c = \frac{EI_c}{h_1}$$  (10)

where $E$ is the Young’s modulus and $I_c$ is the second moment of area of the upright. The end moment $M_C$ at point C can be expressed in terms the portal beam in double curvature as,

$$M_C = -6k_b\theta_C$$  (11)

where $k_b$ is the portal beam stiffness given in terms of its second moment of area $I_b$ and of the horizontal distance $L$ between the centreline of the uprights as,

$$k_b = \frac{EI_b}{L}$$  (12)

Combining Equations 8 and 9, the displacements $\delta_{B,1}$ and $\delta_{C,1}$ are given as,

$$\delta_{C,1} - \delta_{B,1} = -M_C \left(\frac{3h - h_1}{h}\right) \frac{h_1}{6k_c} + h_1\theta_C$$  (13)

Incorporating the expression of $\theta_C$ given in Equation 11 into Equation 13, the displacements $\delta_{B,1}$ and $\delta_{C,1}$ are now expressed as,

$$\delta_{C,1} - \delta_{B,1} = -M_C \left[\left(\frac{3h - h_1}{h}\right) \frac{h_1}{6k_c} + \frac{h_1}{6k_b}\right]$$  (14)

For the portal frame deformation due to the connection rotation in Figure 12 (b), the end moment $M_C$ at point C is expressed in terms of the connection rotation $\theta_H$ and in terms of the connection stiffness $k$ as,

$$M_C = -k\theta_H$$  (15)

and the horizontal displacements $\delta_{B,2}$ and $\delta_{C,2}$ at points B and C respectively are given in terms of the connection rotation $\theta_H$ as,

$$\delta_{C,2} - \delta_{B,2} = \theta_H h_1$$  (16)
Adding Equations 14 and 16 and substituting $\theta_H$ in Equation 16 by the expression of $\theta_H$ given in Equation 15, the combined total displacements $\delta_B$ and $\delta_C$ at point B and C respectively is given by,

$$\delta_C - \delta_B = -M_C \left[ \frac{3h-h_1}{h} \right] \frac{h_1}{6k_c} + \frac{h_1}{6k_b} + \frac{h_1}{k} \right]$$ \hspace{1cm} (17)

where,

$$\delta_b = \delta_{b,1} + \delta_{b,2} \hspace{1cm} (18)$$

$$\delta_c = \delta_{c,1} + \delta_{c,2} \hspace{1cm} (19)$$

Using the expression for $M_C$ in terms of the applied load $F$ in Equation 8, the connection stiffness $k$ in Equation 17 is given as,

$$\frac{1}{k} = \frac{2(\delta_C - \delta_B)}{Fh h_1} - \frac{1}{6k_c} \left( \frac{3h-h_1}{h} \right) - \frac{1}{6k_b} \hspace{1cm} (20)$$

Equation 20 can be simplified by eliminating $\delta_B$ through the following manipulations: first from Figure 12 (a),

$$\delta_{b,1} = \theta_B (h - h_1) \hspace{1cm} (21)$$

Second, combining Equations 8 and 9 the displacements $\delta_{b,1}$ and $\delta_{c,1}$ can be expressed as,

$$\delta_{c,1} - \delta_{b,1} = M_C \left( \frac{3h-2h_1}{h} \right) \frac{h_1}{6k_c} + \frac{h_1}{h} \theta_B \hspace{1cm} (22)$$

Using the expression for $M_C$ in terms of the applied load $F$ given in Equation 8 and substituting $\theta_B$ in Equation 22 by the expression of $\theta_B$ given in Equation 21, Equation 22 gives,

$$\delta_{c,1} - \delta_{b,1} = -F \left( \frac{3h-2h_1}{h} \right) \frac{(h-h_1)h_1}{12k_c h} + \frac{h_1}{h} \delta_{c,1} \hspace{1cm} (23)$$

Third from Figure 12 (b),

$$\frac{h-h_1}{h} = \frac{\delta_{b,2}}{\delta_{c,2}} \hspace{1cm} (24)$$

which gives,

$$\delta_{c,2} - \delta_{b,2} = \frac{h_1}{h} \delta_{c,2} \hspace{1cm} (25)$$

Adding Equations 23 and 25, $\delta_C - \delta_B$ is expressed in terms of $\delta_C$ as,

$$\delta_C - \delta_B = \frac{h_1}{h} \left[ -F \left( \frac{3h-2h_1}{h} \right) \frac{(h-h_1)}{12k_c} + \delta_C \right] \hspace{1cm} (26)$$
Finally Equation 26 can be substituted into Equation 20 to eliminate $\delta_B$ as,

$$ k = \frac{2}{F h^2} \left[ -\frac{F}{12 k_c} \left( \frac{3h - 2h_1}{h} \right) + \delta_C \right] \left[ \frac{1}{6k_c} \left( \frac{3h - h_1}{h} \right) - \frac{1}{6k_h} \right] $$

(27)

the portal beam to upright connection stiffness $k$ is now given in terms of the applied load $F$ and of the lateral displacement $\delta_C$ at the portal beam centreline. For $h$ equal $h_1$, Equation 27 is equivalent to the RMI (2008) and AS 4084 (1993) connector stiffness $k_{RMLAS}$ expression in Equation 5.

4.3.4 Experimental tests

Experimental tests have been performed on three different portal frames. The first two portal frames were tested in the elastic range and the third portal frame was tested beyond the elastic range. However, due to excessive rotation of the loading device, see Figure 13, the maximum moment capacity was not reached for the third portal frame.

4.3.4.1 Experimental results

For each portal frame, at least two complete loading cycles were performed. For the second portal frame, three series of test were performed to capture the complete moment-rotation behaviour of the connection. All tests were performed at a stroke rate of 6 mm/min.

The experimental moment-rotation curves are given in Figure 14, Figure 15 and Figure 16 in which the moment in the connection is calculated by substituting the measured value of force $F$ into Equation 8, the connection rotation $\theta_H$ is calculated using Equation 15 and the experimental stiffness $k$ is obtained from Equation 27. In Equation 27, $\delta_C$ is taken as the average recorded displacement obtained from LVDT 1 and LVDT 2 shown in Figure 11, the upright flexural rigidity $E I_c$ is taken as the experimental mean value found in Section 8.3.1 and the portal beam flexural rigidity $E I_b$ is calculated by theoretically approximating the second moment of area $I_b$ of the portal beam with perforations; detailed calculations of $I_b$ are given in Appendix 2. In the absence of experimental data, the average Young’s modulus of the two steel coils experimentally obtained for the upright and given in Table 3 is used for the portal beam.

Table 6 summarises the values used for calculating the experimental connection stiffness $k$ in Equation 27.
### Table 6: Values used in Equation 27

<table>
<thead>
<tr>
<th>h (mm)</th>
<th>h₁ (mm)</th>
<th>L (mm)</th>
<th>EI₁ (kN.mm²)</th>
<th>k₁c (kN.mm)</th>
<th>EI₂ (kN.mm²)</th>
<th>k₂c (kN.mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>690</td>
<td>625</td>
<td>1475</td>
<td>2.899×10⁸</td>
<td>4.639×10⁵</td>
<td>3.608×10⁸</td>
<td>2.471×10⁵</td>
</tr>
</tbody>
</table>

**Figure 14:** Connection moment-rotation curve for portal frame 1

**Figure 15:** Connection moment-rotation curve for portal frame 2

**Figure 16:** Connection moment-rotation curve for portal frame 3
Three distinct phases are observed in the portal beam to upright connection behaviour shown in Figure 14 to Figure 16. The first phase occurs during initial loading of the connection and when unloading or reloading the connection. During phase 1, the connection stiffness is high and the portal frame mainly deforms in a portal sway mode as shown in Figure 12 (a). The high connection stiffness during phase 1 is a result of the friction forces developed between the portal beam and the upright due to the torque applied to the bolts. The applied moment does not exceed the resisting frictional moment during phase 1 and the portal beam is effectively restrained from sliding relatively to the upright as shown in Figure 17 (a).

The second phase occurs when the moment applied to the connection overcomes the friction forces and the portal beam rotates with essentially no further increase in the applied moment as illustrated in Figure 17 (b). During phase 2, the connection stiffness is low and the portal frame mainly deforms due to the connections opening and closing up as shown in Figure 12 (b).

During phase 2 the portal beam rotates up to an angle where the bolts come into contact with both the upright and the portal beam, thus reaching the rotation of the portal beam shown in Figure 17 (c) and initiating phase 3. During phase 3 the connection stiffness is again high and the portal frame mainly deforms in a portal sway mode as shown in Figure 12 (a). A picture of the connection deformation during phase 3 is given in Figure 18.

When unloading, the moment applied to the connection has again to overcome the frictional moment to initiate the rotation of the portal beam in the opposite direction as illustrated in Figure 17 (d) and (e). Consequently the stiffness is the same as in phase 1.
4.3.4.2 Experimental observations

It is observed from Figure 14 to Figure 16 that the moment-rotation curves are not symmetrical about the origin which is a result of small initial out of plumb of the frame.

The extent of phase 2 can be calculated as a function of the nominal tolerance between the upright and portal beam bolt holes and the bolt diameters. Per beam this tolerance is ± 0.5 mm, which corresponds to a total tolerance of ± 1 mm per bolt. The vertical distance \( d \) between the two bolts is equal to 100 mm which gives the maximum connection angle \( \theta_{H} \) at the end of phase 2 of,

\[
\theta_{H} (\text{end phase 2}) = \frac{2 \text{ mm}}{100 \text{ mm}} = 0.02 \text{ rad}
\]  

From Figure 14 to Figure 16 it is observed that the total connection rotation during phase 2 is consistently 0.08 rad, which is twice the connection rotation of ± 0.02 rad obtained from Equation 28, implying that the connection looseness cannot simply be calculated from nominal hole tolerances and spacing between holes. For the tested connection, bearing occurs on the threaded part of the bolt, i.e. on a reduced bolt diameter than the nominal diameter of 12 mm considered in Equation 28. The measured external diameter of the threaded part of the bolt is 11.84 mm and the internal diameter is 10.60 mm which corresponds to a total tolerance per bolt between ± 1.16 mm and ± 2.40 mm depending on the part of the thread in bearing, and a connection looseness between ± 0.023 rad and ± 0.048 rad in the range of the measured looseness of ± 0.04 rad.

The phase 3 behaviour is different during the first loading cycle compared to subsequent cycles as shown in Figure 19 (a). During the first loading cycle, phase 3 starts at a lower rotation \( \theta_{H} \) and with a lower connection stiffness than subsequent cycles. This phenomenon is attributed to the grooves cut in the upright by the nuts when the bolts start sliding in phase 2 as shown in Figure 19 (b). As the grooves are cut during the first loading cycle, a higher moment develops in the connection during this cycle compared to subsequent cycles where the nuts simply slide in the grooves, as illustrated in Figure 19 (a).

Other observations are reported in Appendix 3.
4.3.4.3 Portal frame n°3 experimental results

Figure 20 (a) shows the experimental moment-rotation curves for the failure test performed on portal frame n°3. The test was terminated before reaching the maximum moment because of excessive rotation of the loading device, as shown in Figure 13. As shown in Figure 20 (a), phase 3 follows an essentially linear path until the bearing pressure produces local yielding of the bolt holes, as illustrated in Figure 20 (b). Using the upright and portal beam characteristics given in Table 7 (with the ultimate tensile strength $F_u$ of the portal beam material taken as the ultimate tensile strength of the upright in the absence of experimental data), the moment-rotation model for the third phase proposed by Uang et al. (2009) and summarised in Section 4.2.2 is also plotted in Figure 20 (a). Good agreement is found between the experimental results and the Uang et al. (2009)’s model.

<table>
<thead>
<tr>
<th>Member</th>
<th>Name</th>
<th>Bolt diameter (mm)</th>
<th>Thick (mm)</th>
<th>Yield stress (MPa)</th>
<th>Ultimate strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upright</td>
<td>RF12519</td>
<td>12</td>
<td>1.9</td>
<td>$459^{(1)}$</td>
<td>$513^{(1)}$</td>
</tr>
<tr>
<td>Portal beam</td>
<td>SB15019</td>
<td>12</td>
<td>1.9</td>
<td>$450^{(2)}$</td>
<td>--</td>
</tr>
</tbody>
</table>

$^{(1)}$: From average coupon test values, $^{(2)}$: Nominal value

Table 7: Beam characteristics
4.4 Cyclic multi-linear moment rotation curve

The moment-rotation curves shown in Figure 14 to Figure 16 can be idealised by multi-linear moment-rotation curves for the purpose of Finite Element modelling. Average stiffness values \(k_1\), \(k_2\) and \(k_3\) for phases 1, 2 and 3 respectively as well as average constant moments \(M_1\), \(M_2\) and \(M_3\) for multi-linear moment-rotation equations for each phase are given in Table 8. The stiffness values given in Table 8 are averages calculated from the moment-rotation equations of all tests obtained by performing a first-order polynomial interpolation for each phase. The first loading cycle for phase 3 is ignored in the calculations.

Figure 21 compares the experimental test results and the multi-linear curves given in Table 8. For the purpose of the comparison, the experimental results are translated horizontally to be symmetrically centred about the origin.

<table>
<thead>
<tr>
<th></th>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_1) (kN.mm/rad)</td>
<td>96326 (*)</td>
<td>3289 ± 286</td>
<td>59345 ± 2091</td>
</tr>
<tr>
<td>(M_{i,1}) (kN.mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k_2) (kN.mm/rad)</td>
<td></td>
<td>± 286</td>
<td></td>
</tr>
<tr>
<td>(M_2) (kN.mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k_3) (kN.mm/rad)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M_3) (kN.mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*): \(M_{i,1}\) depends on the point of change between loading and unloading

Table 8: Connection stiffness

Figure 21: Experimental and multi-linear moment-rotation curves

4.5 Design of drive-in and drive-through racks with bolted moment portal beam to upright connections

The experimental results in Section 4.3.4.1 are used to study the influence of the portal beam to upright bolted moment connection characteristic on the design of drive-in and drive-through racks.

Storage racks are slender structures, sensitive to second-order effect, and consequently the main international racking specifications (RMI (2008), AS 4084 (1993) and EN 15512 (2009)) require P-\(\Delta\) effects to be incorporated in the design. These specifications consider the initial looseness in tab connectors as frame imperfections (out-of-plumb), which is generally accounted for in the design by means of notational horizontal forces \(F_{\text{out-of-plumb}}\),
\[ F_{\text{out-of-plumb}} = \theta \cdot W \]  

(29)

where \( \theta \) is the out-of-plumb angle and \( W \) is the vertical load. For braced and unbraced racks, the AS 4084 (1993) requires the connection looseness to be taken into account while the EN 15512 (2009) only requires the connection looseness to be considered for unbraced racks. When experimentally testing the cyclic shear stiffness of upright frames in the cross-aisle direction, Godley and Beale (2008) encountered similar cyclic behaviour to that reported in Section 4.3.4.1. They recommended incorporating the measured looseness in the rack out-of-plumb and to design the rack based on the stiffness \( k_3 \) associated with the bolts in bearing as shown in Figure 22. While rational and easy to implement, this recommendation may lead to conservative results in some cases, especially for the tested portal beam to upright connections which experience a significant amount of looseness. By ensuring a sufficient torque in the bolts during installation, the looseness in a bolted moment connection only develops after a significant moment is applied to the connection, and in normal operating conditions, no bolt slippage may occur. If first yield occurs at a lower moment in the bolted connection than the one initiating phase 2, then considering the looseness in the connection would considerably increase the bending in the upright due to the combined effect of the increased horizontal forces and the P-Δ effect.

As a worked example illustrating this remark, a 7.5 meter high drive-in rack, with two rail beam elevations, four pallets deep, six bays wide is designed to carry 1200 kgs pallet load following current industry practice. The design is carried out using the proprietary software RAD (Dematic (2006)), developed in-house by Dematic Pty. Ltd. The spine and plan bracings run over two bays and the rack features two upright frames in the cross-aisle direction. The maximum action-to-capacity ratios given by RAD are equal to 0.96 for member check and to 0.88 for serviceability check. The section properties of the upright, portal beams, rail beams and frame bracings are shown in Table 9.

<table>
<thead>
<tr>
<th>Member</th>
<th>Gross area (mm²)</th>
<th>( I_{\text{major axis}} ) (mm⁴)</th>
<th>( I_{\text{minor axis}} ) (mm⁴)</th>
<th>( J ) (mm⁴)</th>
<th>Warping (mm⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upright</td>
<td>441.5</td>
<td>6.603×10⁵</td>
<td>2.778×10⁵</td>
<td>331.1</td>
<td>1.264×10⁹</td>
</tr>
<tr>
<td>Portal beam</td>
<td>559.1</td>
<td>1.746×10⁶</td>
<td>1.683×10⁵</td>
<td>672.8</td>
<td>1.011×10⁹</td>
</tr>
<tr>
<td>Rail beam</td>
<td>640.8</td>
<td>1.450×10⁶</td>
<td>6.067×10⁵</td>
<td>771.1</td>
<td>4.638×10⁹</td>
</tr>
<tr>
<td>Frame bracing</td>
<td>283.7</td>
<td>2.662×10⁵</td>
<td>9.928×10⁴</td>
<td>212.8</td>
<td>1.337×10⁶</td>
</tr>
</tbody>
</table>

Table 9: Section properties for the FE rack
A Finite Element model of the rack is built in Abaqus (2005), similarly to the Finite Element model calibrated against experimental test results for a full-scale drive-in rack structure (Gilbert and Rasmussen (2009a)). The base plate rotational stiffness is modelled as a linear rotational spring with stiffness determined for the axial force in the uprights. Pallets are not modelled in the Finite Element analysis. The out-of-plumb is introduced as horizontal point forces acting at the intersections between rail beams and uprights, the magnitude of which are proportional to the vertical loads acting at the particular beam level. Pallet loads are introduced as vertical uniformly distributed forces on the rail beams. The rack is fully loaded.

The portal beam to upright rotational stiffness is introduced in the FE model using four different approaches: (i) using the multi-linear curve shown in Figure 23 and introduced in Section 4.4, with an arbitrary ultimate moment of 2500 kN.mm (accurate solution), (ii) using the stiffness $k_3$ as shown in Figure 22, with an arbitrary ultimate moment of 2500 kN.mm and with an additional out-of-plumb $\Phi_1$ due to the looseness in the connection, (iii) using a linear rotational stiffness $k_1$ without an ultimate moment and an additional out-of-plumb $\Phi_1$, and (iv) using a pinned connection without an additional out-of-plumb $\Phi_1$.

The out-of-plumb angle $\theta$ is determined following the AS 4084 (1993) recommendation as,

$$\theta = \frac{1}{2} \Psi_0 \left(1 + \frac{1}{n}\right) + \Phi_1$$  

(30)

where $\Psi_0$ is the initial out-of-plumb equal to 0.007 rad for braced manually operated racks and $n$ is the number of bays interconnected. $\Phi_1$ is the connection looseness, equal to 0.04 rad herein as in Figure 14 to Figure 16. To summarise, in the analyses without an additional out-of-plumb (approaches (i), (iii) and (iv)), $\Phi_1$ is taken equal to zero and an out-of-plumb angle $\theta$ equal to 0.00408 rad is considered. In the analysis with an additional out-of-plumb due to the connection looseness (approach (ii)), an out-of-plumb angle $\theta$ equal to 0.04408 rad is considered.

Figure 24 (a) and Figure 24 (b) plot the down-aisle displacement at the top front of the rack and the bending moment for the critical upright respectively, for the four portal beam to upright types of connection. It can be seen that considering the additional out-of-plumb in the connection (approach (ii)) considerably increases the down-aisle motion of the rack and the bending moment in the upright, leading to excessive internal forces in the upright. However, considering a linear stiffness $k_1$ (approach (iii)) or a released moment (approach (iv)) for the portal beam to upright connection leads to results close to the accurate solution at the factored design load. This implies
that the base plates and the spine and plan bracings provide the major contributions in restraining
the down-aisle displacement of the rack as acknowledged in the EN 15512 (2009) in which $\Phi_1 = 0$ for braced racks.

The same rack is therefore now studied in a drive-through configuration with the spine bracing
removed. In the drive-through rack configuration, RAD gives a design pallet load of 690 kg with
action-to-capacity ratios equal to 0.64 for member check and 1.00 for serviceability check.
Figure 25 plots the corresponding curves to those shown in Figure 24 but for the drive-through
rack configuration. It can be observed, that incorporating the portal beam to upright connection
looseness in the out-of-plumb (approach (ii)) still leads to excessive displacements of the rack
and bending moment in the upright compared to the accurate approach (i). It can also be
observed that considering a linear stiffness $k_1$ (approach (iii)) in the design leads to similar
results to the accurate solution at the design load of the rack, emphasising that slippage does not
occur in the portal beam to upright connections at the design load. However, contrary to the
drive-in rack configuration, the stability of the rack now depends solely on the rotational
stiffness of the base plate connection and the portal beam to upright connection, and releasing
the moment between portal beams and uprights (approach (iv)) leads to excessive displacements
and moments compared to the accurate solution.

The behaviour of the bolted moment connection between portal beams and uprights can then be
incorporated in the design of drive-in (braced) and drive-through (unbraced) racks using two
alternative approaches. First, a linear stiffness $k_1$, associated with the initial stiffness of the connection before slippage occurs, can be used to design the rack without an additional out-of-plumb due to looseness in the connection. In this approach, it needs to be checked that at the design load the moment in the portal beam to upright connection is less than the moment initiating slippage. Second, the complete moment rotational curve can be modelled using a multi-linear curve as shown in Figure 23 (approach (i)), thus ensuring representative displacements and internal forces in the rack members. Especially for drive-through racks, these design recommendations would lead to lighter structures than if designed with an additional out-of-plumb $\Phi_1$ and/or a pinned connection between the upright and portal beam.

4.6 Summary

This section reviews the current practices for determining the pallet beam/portal beam stiffness for storage racks and the characteristics of bolted moment connections between cold-formed steel members. By combining high moment capacity and stiffness, bolted moment connections represent an economical and feasible alternative to tab connectors for portal beam to upright connections for drive-in and drive-through storage racks. Experimental test results show significant looseness in the bolted connection after a high initial rotational stiffness. Current research on bolted moment connections focus on structures not sensitive to second-order P-$\Delta$ effects and ignore looseness in the connector. Yet, the P-$\Delta$ effect is important for storage racks and international racking specifications require the connector looseness to be incorporated in the design, especially for unbraced racks. Contrary to tab connectors in which the looseness is initially present in the connection, Finite Element results on drive-in and drive-through racks show that it is likely that at the design load, the moment in the bolted portal beam to upright connection is lower than the one inducing slippage in the connection, and consequently the looseness in the connection can be disregarded. If this is not the case, connection looseness must be considered in the design. Finally, this section presents methods for the design of drive-in and drive-through racks with bolted moment portal beam to upright connections.

5 BASE PLATE TO FLOOR CONNECTION – ROTATIONAL STIFFNESS TESTS

5.1 General

The European Standard EN 15512 (2009) recommends testing to determine the base plate rotational stiffness, the stiffness being derived from the maximum observed moment. A test set-up is suggested in the EN 15512 (2009) Specification, however, several aspects of the test need clarification, including the test set-up and the transducer arrangement required to measure the rotation of the upright relative to the floor.

Based on the test set-up suggested in the EN 15512 (2009) Specification, tests were performed on the particular base plate used for the full-scale tested drive-in rack. Results showed that for low axial loads in the upright section, no maximum bending moment can be reached under large deformations, leading to difficulties in calculating the base plate stiffness using the EN 15512 (2009) guidelines.

This section analyses the different components affecting the base plate stiffness and gives recommendations for the location of the transducers when performing base plate tests to the EN 15512 (2009) Specification. Two alternative base plate test set-ups are investigated. The merits of both test set-ups are discussed, concluding that one alternative is superior to the other. This section also proposes an alternative to determining the maximum bending moment based on the
ultimate deformation when no maximum bending moment can be reached. Stiffness values obtained from tests are compared to stiffness values obtained using available guidelines.

5.2 Base plate behaviour

Storage racks are connected to the floor by the use of base plate assemblies, a typical base plate assembly is shown in Figure 26. The assemblies are designed to transmit axial forces and moments to the floor. Typically, their moment strength and stiffness depend on the axial force in the upright Godley et al. (1998).

5.2.1 Base plate deformation

The deformation of a base plate assembly subjected to an applied moment can be broken into four main components. Each of these deformations contributes to the total rotation of the upright and may not be prominent at the same time. The concrete block under the base plate will deform locally (Figure 27 (a)), the bracket of the base plate assembly will bend (Figure 27 (b)), the upright itself will bend and rotate relative to the base plate assembly (Figure 27 (c)), and a combination of flexure and yield lines will form in the base plate allowing the assembly to rotate (Figure 27 (d)).
5.2.2 Transducer location

In the base plate test set-up shown in the EN 15512 (2009) Specification, two LVDTs are positioned on either side of the upright to measure its rotation. However, the precise position of these transducers is not specified. Several base plate tests can be found in the literature (Baldassino and Zandonini (2003), Godley (2007)) but they do not specify the chosen location of the transducers, calling for clarification and standardisation of the transducer location.

In a structural analysis model, the base plate assembly will be either modelled as a connection element or node with linear or non-linear properties. The properties are based on the measured moment-rotation response, and hence, the measured rotation should include all four components of rotation. It is therefore important that the transducers be placed such that they capture all components of rotation over the length of the base plate assembly and not more.

![Figure 28: Appropriate location of the displacement transducers](image)

In view of the contributions to rotation shown in Figure 27, to capture the total rotation of the upright, the transducers should be located just above the boundary between the base plate assembly and the upright. Indeed, having the transducers below this line will only partially measure the rotation of the upright and having the transducers well above this line will include flexural deformations of the upright and lead to conservative results for the stiffness. Figure 28 shows an appropriate position for the transducers to accurately capture the rotation of the upright when performing base plate tests.

5.3 Base plate test set-up

The EN 15512 (2009) recommends testing to determine the base plate stiffness for a range of axial loads. A test set-up is suggested in Section A.2.7.2 of the Specification, reproduced in Figure 29. However, it is permitted to use an alternative test set-up if the alternative method “accurately models the real structural condition”.

In the test arrangement proposed in the EN 15512 (2009), two lengths of upright less than 600 mm long are symmetrically connected to a concrete block representing the floor, as shown in Figure 29. Standard base plate assemblies are used and connected to the concrete block using standard fixing devices. Jack no1 simulates the axial load in the upright while jack no2 applies a lateral force on the concrete block to create a moment in the base plate assembly. As stipulated in the EN 15512 (2009), at the beginning of the test, the load is increased in jack no1 to the desired axial load and kept constant during the test. The load in jack no2 is then gradually increased until the load reaches a maximum.

The EN 15512 (2009) Specification stipulates that the concrete block must be “free to move in the horizontal plane but restrained from rotating about its vertical axis”. However, restraining the block from rotating about its vertical axis and having a pin between the block and jack no2, as shown in Figure 29, is inconsistent.

In view of this inconsistency, two methods were tested. Method 1 had a pinned connection between the concrete block and jack no2, allowing the concrete block to rotate about its vertical axis. In Method 2, the concrete block was connected to jack no2 in a way which restrained its rotation about its vertical axis but allowed displacements in the horizontal plane. The two methods and the associated formulae to obtain the moment-rotation curves are explained hereafter. Results obtained from the two methods are discussed, concluding that Method 2 is superior to Method 1.

5.3.1 Method 1

The test arrangement for Method 1 is in accordance with Figure A.11 of the EN 15512 (2009) Specification, using a pinned connection between jack no2 and the concrete block. The concrete block is free to move in the horizontal plane and free to rotate about its vertical axis. The moment-rotation formulae are given in Equations 31 to 34, derived in accordance with Figure 30 with reference to Figure 31 (a) which illustrates the test set-up. In Figure 31 (a), the connection between the concrete block and the jack no2 is achieved by a frictionless half-round, allowing no shear to be transmitted to jack no2.
Assuming that the transducers measure displacements relative to a fixed reference, the rotation $\theta_{b,i}$ of each base plate relative to the concrete block is given by,

$$\theta_{b,12} = \theta_{12} - \theta_{56} = \frac{\delta_1 - \delta_2}{d_{12}} - \frac{\delta_5 - \delta_6}{d_{56}}$$

(31)

$$\theta_{b,34} = \theta_{34} + \theta_{56} = \frac{\delta_1 - \delta_3}{d_{34}} + \frac{\delta_5 - \delta_6}{d_{56}}$$

(32)

where $\delta_i$ represents the displacement of transducer i in Figure 31 (a). The second order moments applied to the two base plates are given by,

$$M_{12} = \frac{F_1 L}{4} + F_1 \left( \Delta - \frac{\theta_{56} L_e}{2} \right)$$

(33)

$$M_{34} = \frac{F_1 L}{4} + F_1 \left( \Delta + \frac{\theta_{56} L_e}{2} \right)$$

(34)

where $\Delta$ is the displacement of the concrete block, obtained as,

$$\Delta = \frac{\delta_5 + \delta_6}{2}$$

(35)
In this test, it is likely that one base plate will fail before the other. The rotation of this particular base plate will continue to increase and a plastic hinge will eventually form, whereas the rotation in the other base plate will essentially stop or revert to elastic unloading, causing the concrete block to rotate. Results discussed later will show this phenomenon. Due to the asymmetrical behaviour at failure, only one moment-rotation curve out of two can be used for determining the base plate stiffness (the one which fails) and no average moment-rotation curve can be calculated.

5.3.2 Method 2

The test arrangement for Method 2 is shown in Figure 31 (b). The concrete block is connected to jack n°2 using rigid plates and bolts. The holes in the plate connected to jack n°2 are slotted, allowing it to slide relative to the plate bolted to the concrete block, and the bolts are loosely tightened so as to minimise the shear force transmitted between the concrete block and jack n°2. Special care is taken to restrain the rotation of jack n°2 by applying a lateral restraint to the jack near the connection to the concrete block.

![Figure 32: Forces and deflections – Method 2](image)

Referring to Figure 31 (b) and Figure 32, the rotation \( \theta_{b,ij} \) of each upright relative to the concrete block is given by,

\[
\theta_{b,12} = \theta_{12} = \frac{\delta_1 - \delta_2}{d_{12}}
\]

(36)

\[
\theta_{b,34} = \theta_{34} = \frac{\delta_4 - \delta_3}{d_{34}}
\]

(37)

The second order moment applied to each base plate is given by,

\[
M_{12} = \frac{F_2 L}{4} + \frac{M_R L}{4(L + L_c)} + F_1 \Delta
\]

(38)

\[
M_{34} = \frac{F_2 L}{4} - \frac{M_R L}{4(L + L_c)} + F_1 \Delta
\]

(39)

where \( M_R \) is the moment exerted by the restraining system, as shown in Figure 32.

By restraining the concrete block from rotating about its vertical axis, it is possible for both base plates to fail. The average rotation \( \theta_b \) of the base plates and the average moment \( M_b \) applied to the base plates can thus be calculated,
\[
\theta_b = \frac{1}{2}(\theta_{12} + \theta_{34}) = \frac{1}{2}\left(\frac{\delta_1 - \delta_2}{d_{12}} + \frac{\delta_4 - \delta_3}{d_{34}}\right)
\]

(40)

\[
M_b = \frac{1}{2}(M_{12} + M_{34}) = \frac{F_i L}{4} + F_i \Delta
\]

(41)

Equations 40 to 41 are identical to the equations given in Section A.2.7.3 of the EN 15512 (2009) Specification.

Photographs of the test set-up for Method 2, including the restraining system are shown in Figure 33.

![Photograph of Method 2](image)

(a) (b)

Figure 33: Photographs of Method 2, (a) test set up and (b) restraining system

5.3.3 Results and comparison

Method 2 allows both base plates to fail and thus the moment-rotation curve becomes an average of the moment-rotation curves of the two base plates. Statistically, the mean curve is a better result than the single lower-bound curve Method 1 produces when a pin is used between the concrete block and jack no2.

Moreover, it is essential to perform the test with jack no2 driven in displacement control, while jack no1 is best driven in load control. Method 2 readily allows the test to reach and pass the maximum bending moment when jack no2 is driven in displacement control because the rigid connection between jack no2 and the concrete block allows the transfer of both compressive and tensile forces. On the contrary, in Method 1, unless the pinned connection between jack no2 and the concrete block is designed to transfer tension, a catastrophic failure will occur when, under large displacements, the lateral load changes from compression to tension. For these reasons, Method 2 is superior to Method 1.

Two tests at an axial load of 100 kN were performed with a pinned connection between jack no2 and the concrete block (Method 1). Fifteen tests at axial loads equal to 0 kN, 33 kN, 100 kN, 150 kN and 200 kN were performed (three tests per axial load) with the concrete block restrained from rotating (Method 2). The specimens used for the 100 kN axial load tests were retested up to a base plate rotation of 0.1 rad. Values of the upright characteristics (detailed in Sections 2.2 and 8.3.1) and base plate assembly dimensions are summarised in Table 10.
The results obtained using Method 1 are shown in Figure 34 for the two tests. Both $M_{12}$ and $M_{34}$ determined from Equations 33 to 34 are plotted. For test 1, only one base plate failed while the rotation in the other essentially stopped increasing. For test 2, both base plates failed but not at the same rate. It can be seen that the initial stiffness values are similar for all moment-rotation curves.

![Figure 34: Moment-rotation curves from Method 1 – 100 kN axial force](image)

The results obtained using Method 2 are shown in Figure 35 for all tests. The bending moment $M_b$ is in this case determined from Equation 41. It can be observed that the initial stiffness values are similar for axial loads greater than 100 kN. The tests were paused at the maximum bending moment to obtain the maximum static moment. These pauses can be seen on the 150 kN and 200 kN axial load test curves where the load drops at the maximum bending moment. In all tests, the rotational restraint of the concrete block forced both base plates to fail.

![Figure 35: Moment-rotation curves from Method 2](image)

Due to looseness in the system, initial rotations of the uprights were recorded when applying the axial load. The axial load gradually closed pre-existing gaps causing all components of the base plate assembly to be in full contact. The curves in Figure 34 and Figure 35 were shifted on the horizontal axis to have the initial part of the curve passing through the origin.

5.4 Maximum bending moment

In the EN 15512 (2009) Specification, the base plate stiffness is calculated based on the characteristic failure moment $M_{k}$, which is derived from the maximum moments $M_{ti}$ of several or
numerous tests. According to the EN 15512 (2009) Specification, “the test component shall be deemed to have failed when (a) the applied test loads reach their upper limit, (b) deformations have occurred of such a magnitude that the component can no longer perform its design function.” It is also stated that observations shall be made until jack n² load reaches a maximum.

However, tests results show that the load in jack n² may reach a maximum value before large deformations occur in the base plate and/or before a maximum bending moment is reached. Indeed, after jack n² has reached its maximum load, the decreasing rate of the jack force F₂ may not be sufficient to overcome the P-Δ effect in Equation 41 and the moment in the base plate may still increase.

This is demonstrated in Figure 36, in which the load in jack n² is plotted against the moment applied to the base plate for the 33 kN, 100 kN, 150 kN and 200 kN axial load tests. It can be noticed that the moment applied to the base plate is still increasing when the lateral load reaches a maximum, this being more noticeable for low axial loads. For the 150 kN and 200 kN axial load tests, the bending moment corresponding to the lateral load reaching a maximum is about 5 to 10% less than the maximum bending moment. The point at which the lateral load reaches its maximum is therefore not a good indicator for when the maximum moment capacity of the base plate assembly is reached.

Figure 36: Moment-lateral force curves for (a) 33 kN axial force, (b) 100 kN axial force, (c) 150 kN axial force and (d) 200 kN axial force. Each figure contains the experimental curves for three nominally identical tests.

For the 200 kN axial load tests, it can be also noticed that the P-Δ effect significantly affects the moment when applying the axial load (jack n¹) at the beginning of the test. The assembly moves sideway, thus inducing initial moments, as shown in Figure 36 (d).

Maximum bending moments were only reached for the 150 kN and 200 kN axial load tests. These maximum bending moments correspond to the formation of inelastic local buckles in the upright, as shown in Figure 37 (a).
In the 0 kN, 33 kN and 100 kN axial load tests, failure was initiated by the formation of plastic hinge lines in the base plate, as shown in Figure 37 (b). As deformations in the base plate increase, the base plate is still able to withstand load due to the development of tensile membrane stresses. Because of this membrane action, the 0 kN, 33 kN and 100 kN axial load tests were performed beyond 0.1 rad and no maximum bending moment was reached. The same phenomenon were reported by Godley (2007).

![Figure 37: Failure mode for (a) 150 kN and 200 kN axial load (local buckling) and (b) 0 kN, 33 kN and 100 kN (development of plastic hinges)](image)

When no maximum bending moment can be reached, an alternative to the failure moment $M_k$ has to be found for calculating the base plate stiffness. Large deformations where the connection is considered not to fail have been encountered by Kosteski and Packer (2003) and Yura et al. (1980). Kosteski and Packer (2003) set the ultimate deformation limit to 3% of the width of the rectangular hollow sections used or 3% of the diameter of the circular hollow sections used. This criterion corresponds approximately to 3 times the first yield deformation. Yura et al. (1980) set the ultimate deformation limit to 4 times the first yield deformation. These coefficients of 3 or 4 are commonly used as indicators for whether a plastic hinge is able to occur and sustain plastic deformations. It is recommended that a deformation limit of 4 times the yield deformation be used as a criterion for determining the ultimate moment of base plate assemblies.

![Figure 38: Example on how to calculate the ultimate moment for (a) 33 kN axial load and (b) 100 kN axial load](image)

Figure 38 show examples of how to calculate the ultimate moment using a rotation limit equal to 4 times the first yield deformation. As shown in Figure 38, the first yield deformation is
calculated at the intersection between a line representing the elastic stiffness deformation and a line representing the plastic stiffness deformation.

5.5 Discussion on base plate stiffness modelling

5.5.1 Serviceability base plate stiffness

The RMI (2008) Specification uses the following expression for the base plate stiffness:

\[ M = \frac{1}{12}bd^2E_c\theta \]  

(42)

where \( b \) and \( d \) are the depth and width of the upright section respectively, and \( E_c \) represents the Young’s modulus of the floor, concrete in our case typically equal to 30,000 MPa. Equation 42 is derived from Salmon et al. (1955) who considered the concrete deformation under the base plate, as shown in Figure 27 (a), but not the deformation of the base plate assembly itself.

Sarawit (2003) improved the RMI (2008) Equation 42 and proposed two formulae for determining the moment-rotation relation of the concrete floor, depending of the base plate geometry. For the base plate geometry of the tests reported herein, the following expression for the stiffness \( k_b \) applies:

\[ \theta \cong \frac{7}{25}bd^2E_c\theta \Rightarrow k_b = \frac{7}{25}bd^2E_c \]  

(43)

The rotation of the upright itself occurring over the length of the base plate assembly, as shown in Figure 27 (c), may be determined from Equation 44, which is based on a single cantilever beam with a lateral point load at the top as given in Figure 39; i.e. a system that is statically equivalent to the EN 15512 (2009) Specification base plate test set-up. The first order stiffness \( k_u \) of the base plate assembly is given by:

\[ M = F\frac{L}{2} \]

\[ \theta = \frac{Fa}{2EI_c(L-a)} \]

\[ \Rightarrow M = k_u\theta = \frac{EI_cL}{(L-a)a}\theta \Rightarrow k_u = \frac{EI_cL}{(L-a)a} \]  

(44)

where \( E \) and \( I_c \) are the Young’s modulus and the moment of inertia of the upright section, respectively, \( L \) is twice the length of the tested upright and \( a \) is the location of the transducers where the base plate assembly rotation is calculated. By determining the rotation of the upright itself at the same location as the transducers in Figure 28, \( k_u \) allows comparing the stiffness of a fixed upright at its base to the base plate test stiffness results. Base plate test set up dimensions are given in Table 10.
To consider both the deformation of the concrete floor and the rotation of the upright, $k_b$ is reciprocally added to $k_u$ as:

$$k_{bu} = \frac{1}{\frac{1}{k_b} + \frac{1}{k_u}}$$  \hspace{1cm} (45)$$

The stiffness lines for $k_u$ and $k_{bu}$ are plotted against experimental test results in Figure 40, obtained using Method 2 as experimental set-up. Little difference can be noticed between $k_{bu}$ and $k_u$ because the stiffness $k_b$ proposed by Sarawit (Equation 43) is high compared to $k_u$. Both $k_{bu}$ and $k_u$ give good approximations to the observed base plate initial stiffness, particularly at high levels of axial load. This implies that the initial deformation of the base plate assembly consists primarily of the contributions shown in Figure 27 (a) and Figure 27 (c). For serviceability limit state calculations, it is likely that base plate assemblies will deform in their elastic range and $k_{bu}$ may be used for determining global serviceability deformations.

![Figure 39: Static of a single upright base plate test](image)

![Figure 40: Comparison of proposed stiffness](image)

<table>
<thead>
<tr>
<th>Upright section characteristic</th>
<th>Base plate test dimensions</th>
<th>Base plate assembly dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (MPa)</td>
<td>I (mm$^3$)</td>
<td>Depth b (mm)</td>
</tr>
<tr>
<td>218325</td>
<td>$1.33e+6$</td>
<td>98</td>
</tr>
</tbody>
</table>

*Table 10: Upright section characteristic and base plate test dimensions*
5.5.2 Ultimate limit state design base plate stiffness

Godley (2007) has shown that increasing the base plate stiffness above a certain value does not increase the load carrying capacity of the rack, i.e. the rack may be considered to be fixed at the base. Godley proposes the following stiffness $k_h$ for this limiting value,

$$M = k_h \theta = \frac{EI_c}{h} \theta \Rightarrow k_h = \frac{EI_c}{h}$$

where $E$, $I_c$ and $h$ are the Young’s modulus, the moment of inertia of the upright section and the distance from the floor to the first beam respectively.

$k_h$ is also plotted against experimental test results in Figure 40 as are the design stiffness values $k_{33kN}$, $k_{100kN}$, $k_{150kN}$ and $k_{200kN}$ for the 33 kN, 100 kN, 150 kN and 200 kN axial loads, respectively, calculated by the bi-linear procedure specified in Section A.2.4.5.2 of the EN 15512 (2009). A typical distance from the floor to the first beam of 1200 mm is assumed for plotting $k_h$ in Figure 40.

For the particular base plate tested, it can be observed that, in the presence of axial load in the upright, the base plate stiffness $k_h$ is lower than the initial stiffness and the EN 15512 (2009) design stiffness values. As stated by Godley (2007), stiffness values greater than $k_h$ would yield similar frame load carrying capacities, and as $k_h$ is lower than the EN 15512 (2009) design stiffness, designing the rack with fully fixed base plate assembly would produce an accurate frame load carrying capacity.

Thus, in terms of design and in the presence of axial load in the upright, $k_h$ being close to the initial stiffness and greater than $k_h$, a simple fixed boundary condition for the particular tested base plate assembly would give an accurate approximation for ultimate and serviceability limit states. This suggests that for similar base plate geometries, if experimental test results were not available, a fixed ended condition at the base can be used to predesign the loaded steel storage rack.

5.6 Multi-linear moment rotation curve for Finite Element analysis

The base plate moment-rotation curves shown in Figure 35 are idealised by multi-linear moment-rotation curves for the purpose of Finite Element modelling. While performing the full-scale drive-in rack tests, the axial load in the uprights is not expected to exceed 100 kN and only the 0 kN, 33 kN and 100 kN axial load test results are approximated by multi-linear curves this section. For each test, first-order polynomial interpolations are used and the average stiffness value for each linear segment is reported in Table 11. Figure 41 compares the multi-linear approximations with test results.
Axial load (kN) | Multi-linear approximation | Point 1 (moment, rotation) (kN.mm, rad) | Point 2 (moment, rotation) (kN.mm, rad) | Stiffness (kN.mm/rad) |
---|---|---|---|---|
0 | 1<sup>st</sup> straight line | (0, 0) | (135, 0.00223) | 60541 |
   | 2<sup>nd</sup> straight line | (359, 0.01405) | (596, 0.03088) | 14086 |
   | 3<sup>rd</sup> straight line | (596, 0.03088) | (818, 0.05803) | 8173 |
   | 4<sup>th</sup> straight line | (818, 0.05803) | (M>818, 0>0.05803) | 5027 |
   | 5<sup>th</sup> straight line | (M>818, 0>0.05803) | | |
33 | 1<sup>st</sup> straight line | (0, 0) | (1663, 0.00271) | 614615 |
   | 2<sup>nd</sup> straight line | (1663, 0.00271) | (2090, 0.00682) | 103985 |
   | 3<sup>rd</sup> straight line | (2090, 0.00682) | (2677, 0.03468) | 21064 |
   | 4<sup>th</sup> straight line | (2677, 0.03468) | (2984, 0.06226) | 11102 |
   | 5<sup>th</sup> straight line | (2984, 0.06226) | (M>2984, 0>0.06226) | 5025 |
100 | 1<sup>st</sup> straight line | (0, 0) | (4377, 0.00373) | 1174263 |
   | 2<sup>nd</sup> straight line | (4377, 0.00373) | (5966, 0.01801) | 111273 |
   | 3<sup>rd</sup> straight line | (5966, 0.01801) | (6630, 0.04553) | 24143 |
   | 4<sup>th</sup> straight line | (6630, 0.04553) | (M>6630, 0>0.04553) | 8333 |

Table 11: Base plate multi-linear curves

![Graph](a)

![Graph](b)

![Graph](c)

![Graph](d)

Figure 41: Experimental and multi-linear moment-rotation curves, (a) 0 kN axial load, (b) 33 kN axial load, (c) 100 kN axial and (d) all multi-linear curves

5.7 Summary

This section presents base plate test results for a range of axial loads in the upright. This section also clarifies and explains the base plate test set-up proposed in the EN 15512 (2009) Specification for measuring the stiffness of steel storage rack base plates. An appropriate location of the transducers is proposed for best capturing the base plate behaviour. An alternative test set-up is also proposed, which allows both base plates to fail without triggering catastrophic failure.
When base plate tests do not reach a maximum moment, a deformation limit criteria can be used to calculate the maximum bending moment. It is proposed herein that a deformation limit of four times the yield deformation limit is appropriate for determining the ultimate moment from base plate tests.

It is shown that the initial stiffness of base plate connections is derived mainly from the bending of the upright and the elastic deformation of the floor. For the particular base plate assembly tested, considering a fully fixed boundary condition for the base plate would give satisfactory results for preliminary design for both the frame load carrying capacity and serviceability deformations in the presence of axial load in the upright.

6 BASE PLATE FLOOR CONNECTION – UPLIFT STIFFNESS CALIBRATION TESTS

6.1 General

While international rack design specifications do not mention base plate uplift, the full-scale drive-in rack tests (Gilbert and Rasmussen (2009a)) show that this effect can have a significant influence on the global rack behaviour. A braced storage rack mainly resists a down-aisle lateral force by axial tension and compression in the spine bracing uprights as shown in Figure 42 (a), and when the axial force in the upright in tension overcomes the axial compressive force due to the pallet loading, base plate uplift occurs. The stiffness against uplift is related to the base plate geometry. By rotating the spine bracing as a rigid body, the base plate uplift significantly reduces the down-aisle stiffness of the rack as shown in Figure 42 (b) and consequently increases the second order P-Δ effect due to the pallet loading. Uplift also affects the cross-aisle stiffness of the rack as it causes upright frames to rotate as rigid bodies as shown in Figure 43, consequently also increasing the second order P-Δ effect in the cross-aisle direction. Furthermore, by increasing the down- and cross-aisle displacements of the rack, considering base plate uplift may be important for the serviceability limit state.

Figure 42: Braced storage rack deformation when subjected to a down-aisle lateral load (a) without base plate uplift and (b) with base plate uplift
Figure 43: Cross-aisle deformation of a braced storage rack when subjected to a down-aisle lateral load

Figure 44 shows typical base plate uplift deformations caused by spine bracing during the full-scale drive-in rack tests presented in Gilbert and Rasmussen (2009b).

![Base plate uplift](image)

Figure 44: Base plate uplift

6.2 Experimental test set-up

To determine the base plate uplift stiffness, a 500 mm long upright fitted with a base plate is bolted to the lower platen of a 300 kN capacity MTS Sintech machine, as shown in Figure 45 (a). A vertical tension force is applied along the centroidal axis of the upright and vertical displacements are recorded at the interface between the base plate assembly and the upright, at the front of the upright, using two LVDTs symmetrically positioned about the upright centroidal axis. Due to the nature of the deformed shape, see Figure 45 (b), the displacements recorded by the transducers at the front of the upright correspond to the vertical displacement of the upright centroidal axis. The load is gradually increased until significant uplift displacements develop. Two sets of upright are tested at crosshead speed of 4 mm/min.
The uplift stiffness \( k_{\text{uplift}} \) is defined as,

\[
k_{\text{uplift}} = \frac{F}{\delta_1 + \delta_2}
\]

where \( F \) is the uplift force and \( \delta_1 \) and \( \delta_2 \) are the recorded displacements of LVDT 1 and 2 respectively.

**6.3 Experimental test results and multi-linear approximation**

Figure 46 plots the experimental uplift force against the uplift displacement and a representative multi-linear approximation for use in Finite Element analyses. It was observed in the tests that a gap of approximately 1 mm existed between the base plate and the floor when no axial load was applied to the upright. This gap is taken into account in the multi-linear approximation by allowing a maximum downwards displacement of 1 mm associated with the initial stiffness, as shown in Figure 46.
Figure 46: Base plate uplift test results experimental test result and multi-linear approximation

The multi-linear stiffness value are reported in Table 12.

<table>
<thead>
<tr>
<th>Multi-linear approximation</th>
<th>Point 1 (Axial force , Uplift) (kN, mm)</th>
<th>Point 2 (Axial force , Uplift) (kN, mm)</th>
<th>Stiffness $k_{\text{uplift}}$ (kN/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st straight line</td>
<td>($F&lt;-2.2 , d_{\text{uplift}}&lt;-1.0$)</td>
<td>(-2.2 , -1.0)</td>
<td>$\infty$</td>
</tr>
<tr>
<td>2nd straight line</td>
<td>(-2.2 , -1.0)</td>
<td>(6.78 , 3.04)</td>
<td>2.230</td>
</tr>
<tr>
<td>3rd straight line</td>
<td>(6.78 , 3.04)</td>
<td>(10.0 , 6.0)</td>
<td>1.088</td>
</tr>
<tr>
<td>4th straight line</td>
<td>(10.0 , 6.0)</td>
<td>($F&gt;10.0 , d_{\text{uplift}}&gt;6.0$)</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Table 12: Base plate multi-linear curves

6.4 Summary

This section has presented an experimental test set-up to determine the base plate uplift stiffness of steel storage racks. The base plate uplift stiffness is found to significantly influence the overall behaviour of braced storage racks.

7 SHEAR STIFFNESS OF UPRIGHT FRAME TESTS

7.1 General

In steel storage racks, uprights are connected in the cross aisle direction by bracing members forming an upright frame as shown in Figure 47. The bracing members are typically bolted to the upright flanges and made from cold-formed lipped C-sections. There may be one or two bracing members connected to the upright at the same bracing point, as shown in Figure 47.
The transverse shear stiffness of the upright frame is required to accurately determine the cross-aisle displacement of the rack as well as the cross-aisle elastic buckling load \( P_{cr} \) of the upright frame. The RMI (2008) and AS 4084 (1993) calculate the elastic buckling load \( P_{cr} \) for upright frames braced with diagonals as,

\[
P_{cr} = \frac{\pi^2 EI}{k^2 L^2} \left(1 + \frac{\pi^2 I}{k^2 L^2 A_d \sin \phi \cos^2 \phi} \right)
\]

Equation 48 is based on Timoshenko and Gere (1961)’s shear formulae in which the upright frame shear deformation is assumed to be solely due to the axial deformation of the bracing members. In Equation 48 \( A_d \) is the cross-sectional area of the diagonal braces, \( E \) is the Young’s modulus of steel, \( I \) is the minimum net moment of inertia of the uprights perpendicular to the plane of the upright frame, \( k \) is a factor which depends on the position of the loads, \( L \) is the total height of the upright frame and \( \Phi \) is the angle between horizontal and diagonal braces, as shown in Figure 47.

A different approach is adopted by the EN 15512 (2009) which requires testing to calculate the transverse shear stiffness per unit length of an upright frame. The approach is to experimentally determine the upright frame longitudinal shear stiffness and to evaluate the transverse shear stiffness using Timoshenko and Gere (1961)’s shear formulae. In the EN 15512 (2009) proposed test set-up shown in Figure 48, an upright frame with at least two panels longitudinally is restrained in the transverse direction at each corner. One upright is pinned in the longitudinal direction at one end and a longitudinal load \( F \) is applied at one end of the other upright through its centroidal axis. This test set-up varies slightly from the previous FEM (1998) in which the frame was restrained in the transverse direction at each connection point between the upright and bracing members. This change in test set-up follows recommendations by Sajja et al. (2006).

The longitudinal displacement at the point of application of the load is recorded. The stiffness \( k_{lt} \) at the point of application of the load, taken as the average slope of the load-displacement experimental curve, is used to calculate the upright frame transverse shear stiffness \( S_{lt} \) as,
where \( d \) and \( h \) are the distances between the centroidal axes of the uprights and the length of the frame respectively.

\[
S_n = \frac{k_i d^2}{h} \tag{49}
\]

As mentioned previously Equation 49 is based on Timoshenko and Gere (1961)’s shear formulae and only considers the axial deformation of the bracing members as shown in Appendix 4 for triangular frames.

### 7.2 Literature review

Very few investigations have been reported on the shear stiffness of steel storage rack upright frames consisting of cold-formed steel profiles and bolted connections. Godley and Beale (2008) experimentally investigated the influence of the looseness of bracing member connectors on the upright frame shear stiffness and on the ultimate load carrying capacity of pallet rack frames. Cyclic tests were performed according to the FEM (1998) test set-up and the results showed that the upright frame shear stiffness is low when sliding occurs between the uprights and the bracing members at the bolts, and that the frame stiffens considerably when the bolts are in bearing at the connection between the uprights and bracings. This behaviour is very similar to that observed in the portal frame test described in Section 4. When determining the upright frame elastic buckling load, a significant difference is found when using the shear stiffness associated with the looseness of the bolts rather than the shear stiffness of the frame associated with the bolts in bearing. Godley and Beale (2008) recommended that the effect of looseness should be included as a cross-aisle initial out-of-plumb and that the higher unloading shear stiffness of the frame should be used when determining the upright frame elastic buckling load.

Rao et al. (2004) and Sajja et al. (2006, 2008) investigated experimentally and numerically the shear stiffness of rack upright frames at Oxford Brookes University. In total, 80 tests were performed according to the FEM (1998) test set-up using different numbers of panels, aspect ratios of the panels, upright sizes, restraints and bracing configurations. Rao et al. (2004) showed that the Timoshenko and Gere (1961)’s theory overestimates the shear stiffness by a factor up to 20. Finite Element models were not able to accurately reproduce the experimental test results and produced stiffness values 2 to 5 times greater than the test results. Sajja et al. (2006) attribute the difference between the FE results and experimental test results to the torsional distortion of the
upright, caused by the eccentricity between the uprights and the bracing members at the joints, which was not considered in the FE analysis.

More importantly, by considering one by one the effects of the axial and flexural stiffness values of the uprights, the eccentricities between bracing members, uprights and bolts, the bending of bolts and the bolt rotational release in their FEA, Rao et al. (2004) and Sajja et al. (2006, 2008) proved the importance of considering all these effects in determining the upright frame shear stiffness. However, it is not clear whether the effects would manifest themselves to a different extend if the shear deformations were induced in the transverse, rather than the longitudinal direction, and consequently, it is not clear whether the shear stiffness obtained using the test set-up stipulated in the EN 15512 (2009) will produce the correct stiffness for determining the stability of upright frames and serviceability deformations in the cross-aisle direction, or whether a different test set-up is needed which directly measures the stiffness when the upright is subjected to shearing deformations in the transverse direction. Section 7.3 proposes such a test.

7.3 Alternative test set-up n°1

An alternative to the test set-up specified in the EN 15512 (2009) is proposed in this section. The purpose of the test set-up is to determine the overall sway stiffness of the upright frame in the cross-aisle direction. To achieve this objective a frame was tested in combined bending and shear with the base plates bolted to the concrete floor. The top of the frame was restrained from down-aisle displacements and a load F was applied in the cross-aisle direction at the elevation of the top horizontal bracing member using a 250 kN stroke-controlled hydraulic jack. The test set-up is shown in Figure 49. The cross-aisle displacement at the point of application of the load was recorded as was the vertical displacement of the base plates using two LVDTs per base plate located at the interface between the base plate assembly and the upright.

![Figure 49: Alternative test set-up n°1](image)

The total horizontal displacement $\Delta$ at the point of application of the load is the sum of a rigid body deformation $\Delta_1$ of the frame due to uplift of the base plates (Figure 50 (b)), pure bending deformation $\Delta_2$ of the frame (Figure 50 (c)) and a pure shear deformation $\Delta_3$ of the frame (Figure 50 (d)).
The rigid body deformation $\Delta_1$ is calculated as,

$$
\Delta_1 = \left( \frac{\delta_2 + \delta_3}{2} - \frac{\delta_4 + \delta_5}{2} \right) \frac{H_F}{D_{ext}} \tag{50}
$$

where $\delta_2$ to $\delta_5$ are the displacements of LVDTs 2 to 5, $H_F$ is the vertical distance from the floor to the point of application of the load and $D_{ext}$ is the external width of the upright frame corresponding to distance between the points where the LVDTs are recording the vertical displacements. The combined bending and shear deformation can be experimentally found as,

$$
\Delta_2 + \Delta_3 = \delta_1 - \Delta_1 \tag{51}
$$

where $\delta_1$ is the displacement of LVDT 1. The combined bending and shear transverse stiffness $k_{\Delta_2+\Delta_3}$ of the frame in the cross-aisle direction is then defined as,

$$
k_{\Delta_2+\Delta_3} = \frac{F}{\Delta_2 + \Delta_3} \tag{52}
$$

### 7.4 Alternative test set-up n°2

For simplicity, in the test set-up shown in Figure 49, the upright frame was fitted with base plates bolted to the concrete floor. However, this set-up can be substantially improved by pin-connecting the upright ends to a fixed frame, thus avoiding undesirable base plate uplift displacements. Figure 51 shows the test set-up alternative to Figure 49.
7.5 Test outcomes

In general, the cross-aisle sway deformation of storage racks has little effect on their static design. However, determining the cross-aisle sway stiffness is essential in the design of high-bay racks supporting the building enclosure as the outer rack frames must withstand cross-aisle deflections due to wind loadings. For static design, the sway stiffness value mainly affects the serviceability limit state rather than the ultimate limit state.

Analysis of braced and unbraced racks in the cross-aisle direction is specified in Section 10.3 of the EN 15512 (2009), in which the stability of upright frames in the cross-aisle direction “shall be demonstrated by a rational analysis” which takes into account the “shear flexibility of the bracing system”. As the shear flexibility of the upright frame is measured in the test set-up introduced in Section 7.4, the experimental results can then be used to check the frame stability in the cross-aisle direction. From a practical viewpoint, this is usually achieved using FEA by reducing either the Young’s modulus or the cross-section area of the bracing members, i.e. reducing the axial stiffness of the bracing members. The axial stiffness of the bracing members is chosen so that the FEA sway stiffness value matches the experimental sway stiffness value. The calibrated FE model is used for serviceability checks or to calculate the elastic buckling load in the cross-aisle direction as per the EN 15512 (2009).

7.6 Experimental and FE results

The approach introduced in Section 7.5 is used in this report by building an FE model of the upright frame using Abaqus (2005) and calibrating it against experimental shear stiffness results. The uprights are modelled using open section quadratic beam elements (B32OS) which consider torsional warping and the bracing members are modelled using cubic beam elements (B33) which do not consider warping torsion. The two uprights are considered to be pinned at their base about the down-aisle direction as the compressive axial force is found not to lock the base plate flat on the floor as shown in Appendix 5. Shear centre eccentricities, bracing centroidal axis eccentricities and bolt location eccentricities are modelled, and a second order analysis is carrying out. For more details about the FE model refer to Gilbert and Rasmussen (2009a). The dimensions of the cross-sections and upright frame are given in Appendix 6. The FE model is presented in Figure 52.
In the shear tests conducted as part of this investigation, the bracing members were connected to the uprights as per connection type 2 in Figure 47. During installation the technician had to force the bracing members into place between the flanges of the uprights, resulting in the uprights naturally clamping the bracing members. This effect, combined with the standard 20 N.m torque applied to tension the bolts, generated a significant amount of friction between the two members. While testing the full-scale drive-in rack it was not expected that the cross-aisle displacement of the upright frames would induce sufficient moment in the connection to overcome the frictional moment and consequently, the experimental horizontal force \( F \) applied to the upright frame was kept below the critical value inducing rotation between the bracing members and the uprights. Accordingly, a “welded” type connection is then considered between the bracing members and the upright in the FE model.

![Figure 52: Upright frame FE model for (a) hydraulic jack pulling on frame and (b) hydraulic jack pushing on frame](image)

The experimental bending and shear displacement \( \Delta_2 + \Delta_3 \) of the upright frame is plotted against the applied force \( F \) in Figure 53. The experimental load-displacement stiffness \( k_{\Delta_2+\Delta_3} \) is found by performing a linear regression analysis of the experimental results and shown in Figure 53. The FE stiffness \( k_{\text{FE,nom}} \) using the nominal cross-section area of the bracing of 283.7 mm\(^2\) (non-calibrated model) is also plotted in Figure 53. The FE stiffness \( k_{\text{FE,calibrated}} \) (calibrated model) is found to be equal to the experimental stiffness value \( k_{\Delta_2+\Delta_3} \) when the cross-section area of the bracing members is reduced by a factor of 2.7 to 104.5 mm\(^2\). All stiffness values are reported in Table 13.
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Figure 53: Upright frame bending and shear deformation vs jack load

<table>
<thead>
<tr>
<th>Experimental stiffness $k_{2+3}$ (kN/mm)</th>
<th>FE stiffness $k_{FE}$ (kN/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal cross section = 283.7 mm$^2$ (non calibrated model)</td>
</tr>
<tr>
<td></td>
<td>When pulling</td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>0.95</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 13: Combined bending an shear stiffness of the upright frame

It is observed from Table 13 that without reducing the cross section, the average FE combined bending and shear deformation stiffness is 1.6 times higher than the experimental stiffness.

7.7 Summary

This section has reviewed current practice for determining the shear stiffness of upright frames of steel storage racks. A literature review shows that the test set-up proposed in the EN 15512 (2009) may lead to inaccurate results when determining the cross-aisle upright frame shear stiffness. An alternative test set-up to that specified in EN 15512 (2009) is proposed and investigated by testing an upright frame in combined bending and shear. The test set-up allows the overall sway deformation of the upright frame in the cross-aisle direction to be determined. The experimental results can be used to calibrate FE models following industry current practice, and the stability of the rack in the cross-aisle direction can then be checked according to Section 10.3 of the EN 15512 (2009) using the FE models.

8 FOUR POINTS BENDING TEST ON UPRIGHTS

8.1 General

The presence of perforations in the upright section has an influence on its flexural rigidity and four points bending tests were carrying out, as per the FEM (1998) set up, to determine the flexural rigidity $E_{Ix}$ and $E_{Iy}$ of the uprights about their major and minor axes of bending respectively. The FEM (1998) test set-up, shown in Figure 54 (a), induces a constant bending moment in the middle section of the uprights. The span $L$ between supports of the upright shall be such as,

$$30 \leq \frac{L}{D} \leq 40$$  \hspace{1cm} (53)
where \( D \) is the depth of the upright. Storage rack uprights are usually singly symmetric open sections and to avoid flexural-torsional buckling of the section when testing about the major axis of bending, the FEM (1998) recommends that two upright sections be fixed back to back at intervals along their length (Figure 54 (c)), to create a section not prone to flexural-torsional buckling. However, one upright should be used when testing about the minor axis of bending as shown in Figure 54 (d) and Figure 54 (e).

The load \( F \) was applied in the elastic range and vertical displacements at mid-span were recorded.

![Figure 54: Four points bending test set-up from the FEM (1998) (a) test set-up, (b) upright frame configuration, (c) major axis configuration and (d), (e) minor axis configuration](image)

### 8.2 Test set-up

#### 8.2.1 Major axis - bending test set-up

To determine the flexural rigidity \( EI_x \) of the upright section about its major axis of bending, two 4.9 metres long uprights were bolted together web to web at regular intervals of 200 mm as detailed in Figure 55 (a). The tests were performed using a 300 kN capacity MTS Sintech machine with a distance \( L \) between supports of 4.4 metres, satisfying Equation 53 with a depth \( D \) of 125 mm.

The force \( F \) was applied through a loading frame mounted on half rounds to allow the force to be equally distributed into the two loading points as shown in Figure 55 (b). Similarly, the uprights were supported on two half-rounds, and TEFLON pads were fitted below each half round to avoid axial forces from developing in the upright sections. Two LVDTs symmetrically located on each side of the uprights recorded the vertical displacement at mid-span as shown in Figure 55 (b).

Two sets of uprights were tested and for each set, two loading cycles were performed at a crosshead speed of 1 mm/min.
The flexural rigidity $EI_x$ of one upright is given using beam theory as,

$$EI_x = \frac{11F L_x^3}{1536 \left( \frac{\delta_1 + \delta_2}{2} \right)}$$

(54)

where $\delta_1$ and $\delta_2$ are the vertical displacements of LVDTs 1 and 2 respectively, $E$ is the Young’s modulus of the upright and $I_x$ is the second moment of area of the upright about its major axis of bending.

### 8.2.2 Minor axis - bending test set-up

The flexural rigidity $EI_y$ of the upright section about the minor axis is determined using a similar set-up to that described in Section 8.2.1. A 4.9 metres long upright was positioned into the 300 kN capacity MTS Sintech machine with the lips facing downwards. Tests were performed with a distance $L$ between supports of 3.6 metres, satisfying Equation 53 with a depth $D$ of 98 mm. The test set-up is shown in Figure 56.

Three uprights were tested and for each upright two loading cycles were performed at a Sintech crosshead speed of 1 mm/min.

The flexural rigidity $EI_y$ of the upright is given using beam theory as,
\[ EI_y = \frac{11FL_{support}^3}{768 \left( \frac{\delta_1 + \delta_2}{2} \right)} \]  

(55)

where \( I_y \) is the second moment of area of the upright about its minor axis of bending.

8.3 Experimental results

8.3.1 Results for major axis

Figure 57 shows the experimental load-deflection results in the linear range for all major axis bending tests. Excellent agreement is found between all tests. The experimental flexural rigidity values are detailed in Table 14 using a linear interpolation for each loading cycle.

![Graph showing experimental results for bending about major axis](image)

**Figure 57: Experimental results for bending about major axis**

<table>
<thead>
<tr>
<th>Test n°</th>
<th>Upright n°</th>
<th>Loading cycle</th>
<th>( EI_x ) (N.mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 and 3</td>
<td>1st cycle – loading</td>
<td>2.875E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st cycle – unloading</td>
<td>2.940E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – loading</td>
<td>2.904E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – unloading</td>
<td>2.894E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Average</strong></td>
<td>2.902E+11</td>
</tr>
<tr>
<td>2</td>
<td>4 and 5</td>
<td>1st cycle – loading</td>
<td>2.927E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st cycle – unloading</td>
<td>2.886E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – loading</td>
<td>2.901E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – unloading</td>
<td>2.870E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Average</strong></td>
<td>2.896E+11</td>
</tr>
</tbody>
</table>

**Table 14: Experimental flexural rigidity \( EI_x \) for bending about major axis**

Uprights n°2, 4 and 5 used in the experimental tests, as shown in Table 14, were cold-formed from steel coil n°2 (see Section 2) while upright n°3 was cold-formed from steel coil n°1. The average flexural rigidity \( EI_{x,\text{average}} \) for the nominal upright thickness \( t_{\text{nominal}} \) of 1.9 mm can be calculated using the proportional relationship between the upright thickness and the second moment of area i.e.,
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\[ EI_{x,\text{average}} = \frac{1}{2} \left( EI_{x,\text{test1}} \frac{t_{\text{nominal}}}{t_{\text{coil1}}} + EI_{x,\text{test2}} \frac{t_{\text{nominal}}}{t_{\text{coil2}}} \right) \]  

(56)

where \(t_{\text{coil1}}\) and \(t_{\text{coil2}}\) are the thicknesses of steel coils n°1 and n°2 respectively, and \(EI_{x,\text{test1}}\) and \(EI_{x,\text{test2}}\) are the average flexural rigidity values from tests n°1 and 2 in Table 14 respectively. \(EI_{x,\text{average}}\) is found to be equal to \(2.899E+11\) N.mm². The measured flexural rigidity is 5.4% lower than the nominal value of \(3.064E+11\) N.mm², based on a Young’s modulus of 200,000 MPa or 13.3% lower than the nominal value of \(3.344E+11\) N.mm², based on the actual average Young’s modulus of 218,325 MPa in Section 2.2.

8.3.2 Results for minor axis

Figure 58 shows the experimental load-deflection results for all minor axis bending tests. As in Section 8.3.1, excellent agreement is found between all tests. The experimental flexural rigidity values are detailed in Table 15 using a linear interpolation for each loading cycle.

![Experimental results for bending about minor axis](image)

**Figure 58: Experimental results for bending about minor axis**

<table>
<thead>
<tr>
<th>Test n°</th>
<th>Upright n°</th>
<th>Loading cycle</th>
<th>( EI_x ) (N.mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1st cycle – loading</td>
<td>1.512E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st cycle – unloading</td>
<td>1.490E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – loading</td>
<td>1.518E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – unloading</td>
<td>1.504E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>1.506E+11</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1st cycle – loading</td>
<td>1.499E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st cycle – unloading</td>
<td>1.494E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – loading</td>
<td>1.536E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – unloading</td>
<td>1.498E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>1.507E+11</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1st cycle – loading</td>
<td>1.502E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st cycle – unloading</td>
<td>1.483E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – loading</td>
<td>1.520E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd cycle – unloading</td>
<td>1.483E+11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>1.497E+11</td>
</tr>
</tbody>
</table>

**Table 15: Experimental flexural rigidity \( EI \) for bending about major axis**
Similar to Section 8.3.1, the average flexural rigidity $E_{I_{y,\text{average}}}$ for the nominal upright thickness $t_{\text{nominal}}$ of 1.9 mm can be calculated using,

$$E_{I_{y,\text{average}}} = \frac{1}{3} \left( \frac{E_{I_{y,\text{test1}}}}{t_{\text{coil1}}} + \frac{E_{I_{y,\text{test2}}}}{t_{\text{coil2}}} + \frac{E_{I_{y,\text{test3}}}}{t_{\text{coil3}}} \right)$$

(57)

where $E_{I_{y,\text{test1}}}$, $E_{I_{y,\text{test2}}}$ and $E_{I_{y,\text{test3}}}$ are the average flexural rigidity values from tests n°1, 2 and 3 in Table 15 respectively. $E_{I_{y,\text{average}}}$ is found to be equal to 1.496E+11 N.mm². The measured flexural rigidity is 8.4% lower than the nominal value of 1.634E+11 N.mm², based on a Young’s modulus of 200,000 MPa or 16.1% lower than the nominal value of 1.784E+11 N.mm², based on the actual average Young’s modulus of 218,325 MPa in Section 2.2.

9 CONCLUSIONS

This report presents experimental results on individual components of steel storage racks. Due to the nature of the bolted connection, the portal beam to column connector experiences three distinct phases associated with three distinct stiffness values; the stiffness values being related to friction forces between the upright and the portal beam and to the possible bearing of the bolts on the upright and the portal beam.

This report also clarifies and explains the test set-up proposed in the EN 15512 (2009) specification for measuring the stiffness of steel storage rack base plates. An appropriate location of the transducers is proposed for best capturing the base plate behaviour. An alternative test set-up is also proposed, which allows both base plates to fail without triggering catastrophic failure. While international design specifications do not consider the effect of base plate uplift, this phenomenon is found to influence the global behaviour of braced storage racks and an experimental test set-up is proposed in this report to determine the base plate uplift stiffness.

An alternative testing method to that stipulated in the EN 15512 (2009) is proposed in this report. The method allows the global sway displacement of upright frames in the cross-aisle direction to be accurately determined. The obtained stiffness value is then used to check the cross-aisle stability of racks as specified in the EN 15512 (2009).

10 ACKNOWLEDGEMENTS

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APPENDIX 1

Portal beam to upright connection – Portal frame test

Upright and portal beam cross-section dimensions
Measured cross-section dimensions of upright RF12519

Measured front and side dimensions of upright RF12519
Nominal cross-section dimensions of portal beam SB15019

Nominal side and top dimensions of portal beam SB15019
APPENDIX 2

Portal frame test, calculation of the second moment of area $I_b$ of the portal beam
The nominal second moment of area $I_{b,\text{full}}$ of the portal beam without perforations is extracted from ColdSteel and given in Table 1.

The nominal portal beam shown in Figure 1 is approximated to the portal beam shown in Figure 2 in which the circular holes are substituted by rectangular holes. The circular holes have the same area as the circular holes and the same height as the circular hole diameters.

![Figure 1: Portal beam with circular holes (nominal dimensions)](image1)

The second moment of area $I_{b,A-A'}$ corresponding to the full section without the hole in the middle of the web and shown in cross section “section A-A’” in Figure 2, is calculated from the nominal section of area $I_{b,\text{full}}$ and reported in Table 1. Similarly, the second moment of area $I_{b,B-B'}$ of the cross section shown in “section B-B’” in Figure 2, corresponding to the full section with all perforations is reported in Table 1.

![Figure 2: Approximate portal beam with rectangular holes](image2)
The approximate second moment of area $I_b$ taking into account the presence of the perforations is calculated as the contribution of all second moments of area $I_{b,A-A'}$, $I_{b,B-B'}$ and $I_{b,full}$ over the length of the beam as,

$$I_b = \frac{\left(d_{full} + d_{A-A'} + d_{B-B'}\right) I_{b,full} I_{b,A-A'} I_{b,B-B'}}{d_{full} I_{b,A-A'} I_{b,B-B'} + d_{A-A'} I_{b,full} I_{b,B-B'} + d_{B-B'} I_{b,full} I_{b,A-A'}} \tag{1}$$

where $d_{full}$, $d_{A-A'}$ and $d_{B-B'}$ are the lengths over the beam associated with the second moments of area $I_{b,full}$, $I_{b,A-A'}$ and $I_{b,B-B'}$ respectively. $d_{full}$, $d_{A-A'}$, $d_{B-B'}$ as well as the resulting value of $I_b$ are given in Table 1.

<table>
<thead>
<tr>
<th>$I_{b,full}$ (mm$^4$)</th>
<th>$I_{b, A-A'}$ (mm$^4$)</th>
<th>$I_{b, B-B'}$ (mm$^4$)</th>
<th>$I_b$ (mm$^4$)</th>
<th>$d_{full}$ (mm)</th>
<th>$d_{A-A'}$ (mm)</th>
<th>$d_{B-B'}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.7460 \times 10^6$</td>
<td>$1.7448 \times 10^6$</td>
<td>$1.3427 \times 10^6$</td>
<td>$1.6527 \times 10^6$</td>
<td>34.3</td>
<td>5.5</td>
<td>10.2</td>
</tr>
</tbody>
</table>

Table 1: Values of the second moments of area and distances

*ColdSteel/4600, version 2.03, January 2006, in-house software by Dematic Pty Ltd*
APPENDIX 3

Portal frame test comments
### Portal frame test comments

<table>
<thead>
<tr>
<th>Test</th>
<th>Description</th>
<th>Date</th>
<th>Jack speed</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 cycles performed up to 2 kN</td>
<td>22/12/06</td>
<td>6 mm/min</td>
<td>A jack force equal to 2 kN is taken as a criterion to reverse the cycle. Because this force is set too low, phase 3 has not been captured when unloading. After testing, a jack manipulation error yielded the upright at bolt locations.</td>
</tr>
<tr>
<td>2</td>
<td>2 cycles performed up to 2 kN</td>
<td>04/01/07</td>
<td>6 mm/min</td>
<td>As for test 1, the jack force is set too low and equal to 2 kN when reversing the cycle. Phase 3 has not been captured when unloading.</td>
</tr>
<tr>
<td></td>
<td>Loading to 3 kN</td>
<td>04/01/07</td>
<td>6 mm/min</td>
<td>Portal frame is loaded up to 3 kN to capture phase 3.</td>
</tr>
<tr>
<td></td>
<td>1 cycle performed up to 3 kN</td>
<td>04/01/07</td>
<td>6 mm/min</td>
<td>One cycle with jack load up to 3 kN is performed to completely capture phase 3.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>No local deformation at bolt location after test has been noticed</td>
</tr>
<tr>
<td>3</td>
<td>2 cycles performed up to 3 kN</td>
<td>04/01/07</td>
<td>6 mm/min</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Loading to failure</td>
<td>04/01/07</td>
<td>6 mm/min</td>
<td>Test stopped at jack load equal to 5.1 kN due to local deformation of the loading device.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Local deformation of upright at bolt location has been noticed</td>
</tr>
</tbody>
</table>
APPENDIX 4

Upright frame shear stiffness test: Shear frame test, Timoshenko shear formula for upright frame in the longitudinal and transverse directions
The transverse and longitudinal shear stiffness is calculated in this Appendix using Timoshenko shear formulas for a typical upright frame with horizontal and diagonal bracings.

1 SHEAR STIFFNESS OF ONE MESH

Figure 1 shows a typical mesh of a truss with one diagonal bracing member. A unit load deforms the mesh in shear as shown in Figure 1.

Figure 1: Shear deformation of one mesh

1.1 Change of length beams 1 and 2

The change in length $\Delta L_{1,2}$ of horizontal beams 1 and 2 in Figure 1 is given as,

$$L^2 + \Delta s^2 = (L + \Delta L_{1,2})^2$$

(1)

$$\Delta s^2 = 2L \Delta L_{1,2} + \Delta L_{1,2}^2$$

(2)

$$\Delta L_{1,2} = \frac{\Delta s^2}{2L} - \frac{\Delta L_{1,2}^2}{2L}$$

(3)

where $\Delta s$ is the shear deformation of the mesh and $L$ is the width of the mesh. For small displacements,

$$\frac{\Delta s}{2L} \ll 1 \Rightarrow \frac{\Delta s^2}{2L} \ll \Delta s$$

(4)

$$\frac{\Delta L_{1,2}}{2L} \ll 1 \Rightarrow \frac{\Delta L_{1,2}^2}{2L} \ll \Delta L_{1,2}$$

(5)

Equation 3 combined with equations 4 and 5 gives,

$$\Delta L_{1,2} \ll \Delta s \Rightarrow \Delta L_{1,2} \approx 0$$

(6)

The change of length of the horizontal members is then negligible.
1.2 Change of length beam 3

The change in length $\Delta L_3$ of the diagonal beam 3 is given as,

$$L^2 + (L \tan \theta - \Delta s)^2 = \left( \frac{L}{\cos \theta} + \Delta L_3 \right)^2$$

(7)

$$L^2 + L^2 \tan^2 \theta - 2L \tan \theta \Delta s + \Delta s^2 = \frac{L^2}{\cos^2 \theta} + 2\frac{L \Delta L_3}{\cos \theta} + \Delta L_3^2$$

(8)

where $\theta$ is the angle of the diagonal member with the horizontal member. For small displacements,

$$\left( \frac{\Delta s}{L} \right)^2 \ll \frac{\Delta s}{L}$$

(9)

$$\left( \frac{\Delta L_3}{L} \right)^2 \ll \frac{\Delta L_3}{L}$$

(10)

Equation 8 combined with equations 9 and 10 gives,

$$\Delta L_3 = -\Delta s \sin \theta$$

(11)

using,

$$1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$$

(12)

1.3 Shear stiffness of one mesh in the x direction

The virtual forces theorem for the mesh gives,

$$1 \times \Delta s = \sum_i f_i \Delta L_i = \frac{E A_3}{L/\cos \theta} \Delta L_3^2$$

(13)

where $f_i$ is the axial force in member i, E is the Young modulus and $A_3$ is the section area of the diagonal beam 3. Equations 11 and 13 give,

$$\Delta s = \frac{E A_3}{L/\cos \theta} \sin^2 \theta \Delta s^2$$

(14)

$$\Delta s = \frac{L}{E A_3 \cos \theta \sin^2 \theta}$$

(15)

The shear stiffness $(G\Omega)_x$ for the mesh in the x direction shown in Figure 1 is then equal to,

$$(G\Omega)_x = \frac{L}{\Delta s} = \frac{E A_3 \cos \theta \sin^2 \theta}{\Delta s}$$

(16)
1.4 Shear stiffness of one mesh in the y direction

The shear stiffness \((G\Omega)_y\) for the mesh in the y direction shown in Figure 1 is found from equation 16 as,

\[
(G\Omega)_y = EA_3\cos\frac{\pi}{2}\theta\sin^2\frac{\pi}{2}\theta
\]

\[
(G\Omega)_y = EA_3\sin\theta\cos^2\theta
\]  

(17)  

(18)

1.5 Relationship between shear stiffness of one mesh in the x and y directions

Equation 18 can be written as,

\[
(G\Omega)_y = EA_3\sin^2\theta\cos\theta\frac{1}{\tan\theta}
\]

using the relationship between the height \(H\) of the mesh and the width \(L\), previous equation gives,

\[
(G\Omega)_y = EA_3\sin^2\theta\cos\theta\frac{L}{H} = \frac{L}{H}(G\Omega)_x
\]

(19)  

(20)

2 SHEAR STIFFNESS OF A TRUSS IN THE TRANSVERSE DIRECTION

Figure 2 shows the shear deformation of a truss composed of \(n\) meshes in the transverse direction subjected to a unit load.

\[
1\times n\Delta s = \sum_i f_i\Delta L_i = \sum_i \frac{EA_i}{L_i} \Delta L_i^2 = n \frac{EA_3}{L/cos\theta} \sin^2\theta\Delta x^2
\]

(21)
\[ \Delta s = \frac{L}{E A_3 \sin^2 \theta \cos \theta} \]  

(22)

The transverse shear stiffness \((G\Omega)_{\text{transverse}}\) of the truss is then determined as,

\[ (G\Omega)_{\text{transverse}} = \frac{nL}{n\Delta s} = E A_3 \cos \theta \sin^2 \theta \]  

(23)

3 SHEAR STIFFNESS OF A TRUSS IN THE LONGITUDINAL DIRECTION

Figure 3 shows the shear deformation of a truss composed of \(n\) meshes in the longitudinal direction subjected to a unit load.

\[ \text{Figure 3: Shear deformation of a truss in the longitudinal direction} \]

Using the change of length equations found for one mesh, the virtual forces theorem applied to the truss gives,

\[ 1 \times \Delta s = \sum_i f_i \Delta L_i = \sum_i \frac{E A_i}{L_i} \Delta L_i^2 = n \frac{E A_3}{L} \cos^2 (\theta) \Delta s^2 \]  

(24)

\[ 1 \times \Delta x = n \frac{E A_3}{L} \cos^2 (\theta) \Delta s^2 = n \frac{E A_3}{H} \cos^3 \theta \sin \theta \Delta x^2 \]  

(25)

The transverse shear stiffness \((G\Omega)_{\text{longitudinal}}\) of the truss is then determined as,

\[ (G\Omega)_{\text{longitudinal}} = \frac{H}{\Delta s} = n E A_3 \cos^2 \theta \sin \theta \]  

(26)

\[ (G\Omega)_{\text{longitudinal}} = \frac{nL}{H} E A_3 \cos \theta \sin^2 \theta \]  

(27)

4 RELATIONSHIP BETWEEN LONGITUDINAL AND TRANSVERSE SHEAR STIFFNESS OF A TRUSS

Equations 23 and 27 give the following relationship between the longitudinal and transverse shear stiffness for a truss,
\[
(G\Omega)_{\text{longitudinal}} = \frac{nL}{H} (G\Omega)_{\text{transverse}}
\]  

Equation 28 is equivalent to the FEM (1998) equation for determining the transverse upright frame shear stiffness from the upright frame longitudinal shear stiffness.
APPENDIX 5

Upright frame shear stiffness test: Boundary conditions for FE analysis
In this appendix, while testing the frame following alternative test set-up n°1 it is checked if the axial force in the upright in compression is high enough to lock the base plate flat on the floor. Two states can occur:

a) If the axial force in the upright in compression locks the base plate flat on the floor, then the base plate can be ideally considered as fixed and the overturning bending moment is not able to overcome the stabilising moment due to the axial load.

b) If the axial force in the upright in compression is not able to lock the base plate flat on the floor, then the base plate can be ideally considered as pinned and is free to rotate.

The FE model of the upright frame introduced in Section 7.6 of the present report is used to perform this investigation with the base of the upright in compression considered as fixed and the base of the upright in tension considered as pinned. The vertical non-linear uplift stiffness found in Section 7.6 of the present is also considered in the FE model as shown in Figure 1 to simulate the real test condition. The area of the bracing members is reduced according to Section 7.6 results and second-order analysis is carrying out.

![Figure 1: Upright frame FE model for hydraulic jack pulling on frame](image)

The upright can be considered fixed if the stabilising moment is greater than the overturning moment as,

$$ R_1 d_c \geq M_1 $$

where $R_1$ and $M_1$ are the reaction force and moment at the base of the upright in compression respectively and $d_c$ is the distance from the web of the upright to the upright centroidal axis as shown in Figure 2.

Figure 3 plots the FE results for the stabilising and overturning moments in equation 1 for an applied load $F$ varying from 0 kN to 3 kN. Figure 3 shows that the overturning moment is always greater than the stabilising moment contradicting the assumption that the axial force locks the base plate flat on the floor. The upright in compression is then pinned at its base as considered in Section 7.6 of the present report.
Figure 2: Upright stabilising moment and overturning moment

Figure 3: FE results for stabilising and overturning moments
APPENDIX 6

Upright frame shear test, upright and bracing section characteristics. Tested upright frame dimensions
Upright section properties – RF12519 – 450 MPa
Upright frame dimensions