Adaptive Local Ratio

Julían Mestre

MPII
Local Ratio

* Technique for algorithm design
* Related to Primal-Dual scheme
* Powerful, yet simple
Local Ratio

i) Decompose \( w = w_1 + w_2 + \ldots + w_n \)

ii) Construct a solution \( S \)

iii) Argue \( w_k(S) \leq \rho \cdot w_k(A) \quad \forall A \)

\[ \Rightarrow S \text{ is } \rho-\text{approximate} \]
Local Ratio

- Large number of papers
- And many more PD papers!
- Weight decomposing using "simple" models

Optimizing the LR can pay off!
Talk Outline

☑ Background
☆ Data Migration Problem
☆ Adaptive Local Ratio
Data Migration

\[
\text{Transfer graph } G = (V, E) \quad \left\{ \begin{array}{l}
V: \text{disks} \\
E: \text{transfers}
\end{array} \right.
\]

Want to schedule the transfers \( \equiv \) Partition \( E \) into matchings \( M_1, \ldots, M_k \)

\[
\text{minimize } \sum_{v} w_v C_v
\]
\[ \sum C_v = 2 \times 4 + 3 \times 3 = 17 \]
Previous Results

- NP-hard
- 3-approx (LP rounding) [K03]
- shown to be tight [GHKS]
- 3-approx (primal-dual) [GMJ]
Talk Outline

✓ Background

✓ Data Migration Problem

* Adaptive Local Ratio
Local Ratio

i) Decompose $w = w_1 + w_2 + \ldots + w_n$

ii) Construct a solution $S$

iii) Argue $w_k(S) \leq \rho w_k(A) \quad \forall A$

$\Rightarrow S$ is $\rho$-approximate
Algorithmic Framework

\[ \omega = \omega - \alpha \hat{\omega} \]

Lemma: \( C_i \leq \Delta + d_i - 1 \)

Find \( \hat{\omega} \) so that \( \hat{\omega}(S) \leq \rho \hat{\omega}(A) \forall A \)
Bounding the Local Ratio

\[ \mu \xleftarrow{\nu} \hat{\nu}_1 \quad \vdots \quad \hat{\nu}_i \]

Lemma: \( C_i \leq \Delta + d_i - 1 \)

\[ \hat{\nu}_\Delta \quad \hat{\nu}_\Delta \]

\[ \hat{\nu}(S) \leq \Delta^2 + \sum d_i \]

\[ \hat{\nu}_1 = 1 \quad \Rightarrow \hat{\nu}(A) \geq \sum d_i \]

\[ \Rightarrow \hat{\nu}(A) \geq \frac{1}{2} \Delta (\Delta + 1) \]
Using two models

\[ \hat{w}_i = 1 \qquad \text{or} \qquad \hat{w}_i = \begin{cases} 1 & d_i \geq d_j \forall j \end{cases} \]

Lemma: \( \hat{w}(S) \leq 2.82 \hat{w}(A) \quad \forall A \)
Using the best model

Given \( d = (d_1, d_2, \ldots, d_\Delta) \)

find \( \hat{\omega} = (\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_\Delta) \)

minimizing \( f(d) = \frac{UB(d, \hat{\omega})}{LB(d, \hat{\omega})} \)
\[ UB(\hat{\omega}) = \sum_{i} \hat{\omega}_i \left( d_i + \Delta - 1 \right) \]

\[ \mu \xrightarrow{2} v_1 \quad \sigma(1) = 2 \]
\[ \mu \xrightarrow{1} v_2 \quad \sigma(2) = 1 \]
\[ \mu \xrightarrow{3} v_3 \quad \sigma(3) = 3 \]

\[ LB(\hat{\omega}) = \min_{\sigma \in \mathcal{P}_A} \sum_{i} \hat{\omega}_i \max(d_i, \sigma(i)) \]
Towards an LP formulation

\[ \rho(d) = \min \frac{UB(\hat{w})}{LB(\hat{w})} = \min \frac{UB(\hat{w})}{LB(\hat{w})} \]

\[ \text{s.t.} \quad LB(\hat{w}) \geq 1 \quad \hat{w}_i \geq 0 \]

\[ UB(\hat{w}) = \sum_i \hat{w}_i \left( d_i + \Delta - 1 \right) \]

\[ LB(\hat{w}) = \min \sum_{\delta \in \mathcal{P}_A} \sum_i \hat{w}_i \max(\delta d_i, \sigma(i)) \]

\[ \hat{w}_i \geq 0 \]
Towards an LP formulation

$$p(d) = \min \sum \hat{\omega}_i \left( d_i + \Delta - 1 \right)$$

$$\sum \hat{\omega}_i \max (d_i, \delta(i)) \geq 1 \quad \forall k \in \mathcal{P}_A$$

$$\hat{\omega}_i \geq 0$$

Elipsoid method is not practical
LP formulation

\[ p(d) = \min \sum \hat{\omega}_i (d_i + D - 1) \]

s.t.

\[ \sum (y_i - z_i) \geq 1 \]

\[ y_i - 2j \leq \hat{\omega}_i \max (d_i, j) \]

\[ \hat{\omega}_i, y_i, z_i \geq 0 \]
Bounding the Local Ratio

Def: \( p = \sup_d \rho(d) \)

Thm: \( S \) is \( \rho \)-approximate and this is tight

Now we only need to bound \( \rho \)
Experimental Evaluation

Exhaustive search for small values of $\Delta$

\[ p_\Delta = \max_{d: \|d\| = \Delta} p(d) \]
\[ P = \sup_d P(d) \]

\[ P_\Delta = \max \alpha \]

\[ \sum_i x_{ij} \geq \alpha \quad \forall j \]

\[ \sum_j x_{ij} \leq \alpha \quad \forall i \]

\[ \sum_i \sum_j x_{ij} \max(d_{ij}, j) \leq d_i + \Delta - 1 \quad \forall i \]

\[ d_{ij}, x_{ij}, \alpha \geq 0 \]
Thm. \( p = 1 + \phi \approx 2.61 \)

Intuition:
- weights exploit irregularities of \( d \)
- if \( d \) is "flat", only two models are useful
Talk Outline

☑ Background
☑ Summary of Results
☑ Adaptive Local Ratio
Concluding Remarks

- Generalizations
- First LP-guided local-ratio Alg
- Optimizing the local ratio can make a difference
Thanks for your attention!
Do you really need \( l(\cdot) \)?

Forget about \( l(\cdot) \) and just go with greedy

Sort \((u,v) \in E\) by

\[
\begin{cases} 
\min(d_{u}, d_{v}) \\
\max(d_{u}, d_{v})
\end{cases}
\]
Thanks for your attention!