APPLIED MATHEMATICS 4

2018

THE UNIVERSITY OF SYDNEY

SCHOOL OF MATHEMATICS AND STATISTICS

Contents

1 Entry Requirements

Entry to the Honours Programme in Applied Mathematics is usually based on satisfying the following (and subject to approval by the Head of the School of Mathematics & Statistics):

1. Faculty requirement: The candidate must have qualified for the Pass Degree with a SciWam of at least 65 %.
2. Mathematics requirement: The candidate must have completed 24 credit points of senior (third-year) mathematics with a Pass Degree average of at least 65 % for advanced level units and a Pass Degree average of at least 75 % for normal level units.
3. Essay / Project supervision: The candidate is expected to find a prospective supervisor from among the Applied Mathematics staff, who is agreeable to supervise the candidate's essay or project in the candidate's chosen topic.

Students from institutions other than the University of Sydney must possess qualifications which are deemed equivalent to the above. There is some flexibility in these conditions; students not quite meeting them, but keen to pursue Honours in Applied Mathematics are invited to contact the Course Coordinator for advice. Students are expected to meet with prospective supervisors to discuss the potential for essays or projects before submitting the honours application.

Applications must be submitted to the Faculty of Science.

Application and enrolment information should also be obtained from the Faculty of Science, either in person of from their website. The Faculty will also provide AAM computations once the final third-year results are in.

Graduate Diploma in Science and MSc(Qualifying) applicants should see the Director of Postgraduate Studies before enrolling.

2 Course Administration

The Course Coordinator for Applied Mathematics 4 in 2018 is:

Dr. Robert Marangell
Room 720, Carslaw Building
Phone: (02) 9351 5795
Email: `robert.marangell@sydney.edu.au`

3 Structure of Applied Mathematics 4

Full-time students normally attend three lecture courses each Semester, for a total of six courses. All six courses will count towards the student's final assessment. If a students takes more than six courses in total then the top six results will count towards the student's final assessment.

In addition to the courses, each student is also required to write an essay or project on an Applied Mathematics topic, under the supervision of a member of staff of the School of Mathematics and Statistics. This is considered to be the major aspect of the Honours Programme, and is detailed in Section 6.

The primary choice for the six courses come from the Applied Mathematics courses we offer each year. Each such course runs for the first 12 weeks of each semester, at 2 lectures per week. There are usually no formal tutorials, but lecturers are happy to help students with their questions by arrangement.

The following Applied Mathematics 4 courses are expected to be offered this year:

First Semester

Asymptotic Methods and Perturbation Thoery	A/Prof S. Stephen
Computational Projects in Applied Mathematics	Prof G. Gottwald
Mathematical Biology	Prof M. Myerscough
Random Graphs vs. Complex Networks	A/Prof E. Altmann

Second Semester

Integrable Systems	Prof N. Joshi & Dr M. Radnovic
Introduction to Optimal Control	Prof B. Goldys
Spectral and Dynamical Stability of Nonlinear Waves	Dr R. Marangell

Students are also welcome to choose any number of courses from Pure Mathematics 4 or Mathematical Statistics 4 subject to approval by the student's supervisor. The choice of course work has to ensure that the student covers the proper material required for the student's project or essay. (Details of the available courses, and any entry requirements, should be obtained from the Fourth-Year Course Coordinators in Pure Mathematics and Statistics.)

Students also have the option of choosing a few courses from the following, *for which approval from the Essay/Project Supervisor and the Course Coordinator needs to be obtained prior to enrolling*:

- Fourth-Year Courses from related disciplines (e.g., Physics) or other universities (e.g., UNSW).
- Access Grid Room (AGR) Courses offered from other Australian Universities; see `http://www.maths.usyd.edu.au/u/UG/accessgrid.html` for further information;
- Third-Year Courses at the Advanced level offered by the School of Mathematics & Statistics;

4 Course Summaries for 2018 Courses

Asymptotic Analysis and Perturbation Theory *(Sem-1)* *A/Prof S. Stephen*
Assessment: 60% Exam; 40% Assignments

Description

Asymptotic methods are vital techniques to make analytic progress in all areas of applied mathematics. They aid in the determination of the dominant physical mechanisms. Problems involving different lengthscales or timescales are widespread in physical applications. Perturbation methods take advantage of these differing scales or small parameters in the problem to provide rational approximations to the governing differential equations.

The topics to be covered are: asymptotic expansions of integrals; regular and singular perturbation theory; multiple-scale analysis; and WKB theory.

Prerequisite knowledge

Theory of, and solutions of, ordinary differential equations, including nonlinear equations, second order and non-constant coefficients; complex variable theory, including contour integration.

Course objectives and outcomes

At the end of this course students will be able to: determine the asymptotic expansions of functions defined by integrals; and find approximate solutions to regular and singular perturbation problems.

Recommended text books

- Introduction to Perturbation Techniques, A. H. Nayfey, Wiley, 1981
- Advanced Mathematical Methods for Scientists and Engineers, C. M. Bender and S. A. Orszag, McGraw-Hill, 1978
- Perturbation Methods, E. J. Hinch, Cambridge University Press, 1991
- Nonlinear Ordinary Differential Equations (second ed.), D. W. Jordan and P. Smith, Oxford University Press, 1987

Computational Projects in Applied Mathematics *(Sem-1)* *Prof G. Gottwald*
Assessment: Three assignments (project reports) worth 33.3% each, no final exam

The computational approach to study scientific problems serves a twofold purpose in mathematics. Numerical formulations and analysis help scientists in exploring complex systems, often giving rise to hitherto unknown phenomena which deserve further analytical treatment and understanding, or they can be used to verify existing mathematical theories. On the other hand, the need for numerical simulations, for example in studying complex systems such as the ocean-atmosphere systems, requires new mathematics to allow for an efficient simulation. This course will highlight examples of this fruitful interplay of analytical mathematics and numerical computations.

The course will present a variety of numerical methods together with some of the underlying theory covering the numerical integration of partial differential equations, geometric integration, stochastic differential equations as well as some basic machine learning. When solving continuous time systems such as ordinary, partial or stochastic differential equations, we need to discretize time and space. How this affects the accuracy of our computations will be discussed, for example. The lectures will prepare the theory and background.

MATH2x63 and/or MATH3x76 or similar courses (such as COSC) provide a useful background for this course, but are not required.

Mathematical Biology *(Sem-1)* *Prof M. Meyerscough*

Mathematics has a plethora of applications to biological systems and biological models throw up many examples of interesting mathematics. This subject covers some of the models and techniques of classical mathematical biology, including population biology, epidemiology, oscillating systems and pattern formation, and associated mathematical techniques.

The course will include some or all of the following topics. Exactly which topics are covered will depend on the interests of the class.

- Basic techniques of nonlinear ODEs: phase planes and linear analysis of steady states. Example: predator-prey models and models for two competing species.
- Limit cycles, the Hopf bifurcation theorem, the Poincare-Bendixson theorem, limit cycle stability. Example: Schnackenberg kinetics, the Brusselator model.
- Epidemiological models. SIR models and extensions. Endemic disease and R0. Examples: SIR, SIS diseases. Sexually transmitted diseases.
- Stationary bifurcations. Classifying bifurcations using singularity theory. Examples: Spruce budworm population dynamics, the cubic autocatalator.
- Pattern formation in reaction-diffusion systems. Turing instabilities.
- Travelling wave analysis. Examples: spruce budworm (again); rabies in foxes.
- Travelling waves in excitable media. Examples: travelling pulses in the FitzHugh-Nagumo system.
- Discrete age- or stage- structured models. Leslie matrices. Coates graphs. The Perron-Frobenious Theorem. Examples: noxious weeds, Teasel and *Spartina.*

Random Graphs vs. Complex Networks *(Sem-1)* *A/Prof E. Altmann*
Assessment: 50% Exam 50% Assignments/Projects

The representation of the relationship (links) between objects (nodes) in form of a network is increasingly popular in social, economical, biological, and technological sciences. Real-world examples of networks coming from these various fields show striking differences to the random-graph models proposed by mathematicians (in the mid XX century). For instance, real networks show (i) a large number of triangles, short loops, and other small subgraphs which are absent in simple random-graph models; and (ii) the distribution of the number of links that nodes receive deviates from a Binomial and shows large skewness (fat-tailed distribution). The goals of this course are to characterize these disagreements between complex networks and random graphs, to show why they matter, and to present more advanced mathematical methods to model complex networks.

The course will start with a general introduction to the field of complex networks, discussing different examples of networks and how to characterize them (e.g., clustering and centrality measures). We will then introduce and apply computational methods to empirically-measured networks. The main part of the course will be the definition and characterization of different random-graph ensembles. Some significant deviations between random graphs and real networks will be explored, with focus on the small-world effect, the fat-tailed degree distribution, and the consequences to network robustness. In the final part of the course we will discuss how more advanced random-graph ensembles (e.g., exponential random-graph models) can be used to characterize real-world networks. This will be interpreted in terms of – and serve as an introduction to – more general ideas and methods, such as the Principle of Maximum Entropy and Importance Sampling Monte Carlo methods (e.g., the Metropolis Algorithm).

This course involves computer simulations and data analysis. Familiarity with a programming language (e.g., Python or Matlab) and basic statistical concepts are desired.

Reference: - M. Newman, Networks: An Introduction, Oxford Univ. Press 2010.

Introduction to Optimal Control *(Sem-2)* *Prof B. Goldys*

The assessment will be based on two assignments ($2 \times 20\%$) *and a final mini-project* (60%)*.*

This course is devoted to the theory of optimal control of systems described by ordinary differential equations. This theory has a wide variety of applications, ranging from classical mechanics, physics and biology to engineering, economics and finance. Specific examples include steering of missiles and space mission design [4], designing cancer therapies [8], dynamic portfolio optimisation and production planning [9]. The control theory also provides tools to study certain problems in pure mathematics, such as finding the geodesic lines on Riemannian manifolds and finding optimal constants in Sobolev inequalities [1, 3, 6].

The control theory draws on many fields of mathematics but the prerequisites for this course are modest. It is expected that students are familiar with linear algebra, ordinary differential equations and basics of analysis. Students who completed the course on Hamiltonian dynamics, will find many concepts familiar.

The course content:

1. Examples and formulation of control problems
2. Review of ODEs
3. Controllability of linear systems
4. Local controllability of nonlinear systems

5. Existence of optimal controls, convexity
6. Necessary conditions of optimality, duality
7. Linear-quadratic regulator
8. Optimal control of constrained systems, time optimal control
9. Dynamic Programming Principle
10. Sufficient conditions of optimality: Hamilton-Jacobi-Bellman equation
11. Infinite time horizon control problems and stationary Hamilton-Jacobi-Bellman equations
12. Visocsity solutions of Hamilton-Jacobi-Bellman equations

Complete lecture notes with exercises will be provided. The following books and lecture notes may be useful.

References

[1] Bressan A. and Piccoli B.: Introduction to the mathematical theory of control. Springer, 2015

[2] Berkovitz LD. and Medhin NG: Nonlinear optimal control theory. Chapman&Hall, 2013

[3] Evans LC: An Introduction to Mathematical Optimal Control Theory, https://math.berkeley.edu/ evans/control.course.pdf

[4] Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D. Dynamical Systems, the Three-Body Problem and Space Mission Design. Marsden Books, 2011

[5] Lewis FL, Vrabie DL and Syrmos VL: Optimal control. Third edition. John Wiley&Sons, Inc., 2012

[6] Liberzon D: Calculus of variations and optimal control theory. A concise introduction. Princeton University Press, 2012

[7] Schättler H and Ledzewicz U: Geometric optimal control. Theory, methods and examples. Springer, 2012

[8] Schättler H. and Ledzewicz U.: Optimal control for mathematical models of cancer therapies. An application of geometric methods. Springer, 2015

[9] Weber TA: Optimal control theory with applications in economics. MIT Press, 2011

Integrable Systems *(Sem-2)* *Prof N. Joshi, & Dr M. Radnović*

Assessment: Assignments & Take-home exam (60/40%).

Outline: The mathematical theory of integrable systems has been described as one of the most profound advances of twentieth century mathematics. They lie at the boundary of mathematics and physics and were discovered through a famous paradox that arises in a model devised to describe the thermal properties of metals (called the Fermi-Pasta-Ulam paradox).

In attempting to resolve this paradox, Kruskal and Zabusky discovered exceptional properties in the solutions of a non-linear PDE, called the Korteweg-de Vries equation (KdV). These properties showed that although the solutions are waves, they interact with each other as though they were particles, i.e., without losing their shape or speed, until then thought to be impossible for solutions of non-linear PDEs. Kruskal invented the name *solitons* for these solutions.

Solitons are known to arise in other non-linear PDEs and also in partial difference equations. These systems and their symmetry reductions are now called "integrable systems". These systems occur as universal limiting models in many physical situations.

This course introduces the mathematical properties of such systems. In particular, we will study their solutions, symmetry reductions called the Painlevé equations and their discrete versions. It focuses on mathematical methods created to describe the solutions of such equations and their interrelationships.

Course Objectives and Outcomes:

- Understand the *inverse scattering transform method*: how to use it to solve integrable systems and find solitons; how to prove that it works for certain initial conditions.
- Understand the transformation theory that relates integrable systems to each other and the reductions from PDEs to ODEs.
- Understand how to use transformations to find special solutions, recurrence relations and related discrete integrable systems.
- Describe other properties of solutions of integrable systems, in particular behaviours that occur in limits.

References and Supporting Material: Useful texts include

- P.G. Drazin and R.S. Johnson *Solitons : an introduction* Cambridge University Press, Cambridge, UK, 1989.
- M. J. Ablowitz and H. Segur, *Solitons and the inverse scattering transform* SIAM, Philadelphia, USA, 1981.
- M. J. Ablowitz and P.A. Clarkson, *Solitons, nonlinear evolution equations and inverse scattering* Cambridge University Press, Cambridge, UK, 1991.
- M. Noumi, *Painlevé equations through symmetry*, American Mathematical Society, Providence, R.I., USA, 2004.

Interesting Links: There are many interesting links on solitons, provided on the course webpage. Have a look at

- An account of John Scott Russell's discovery of "that singular and beautiful phenomenon, which I have called the wave of translation."
- A modern attempt by mathematicians to recreate Scott Russell's wave in the Union Canal near Edinburgh.
- The wikipedia page on Solitons.
- The wikipedia page on the Korteweg-de Vries equation.

Spectral and Dynamical Stability of Nonlinear Waves *(Sem-2)* *Dr R. Marangell*
Assessment: TBD

This course will focus on the stability theory of nonlinear waves that has been developed over the last few decades, and which lies at the interface of dynamical systems, partial differential equations (PDEs), topology, and functional analysis.

I will begin with a review of Sturm-Liouville theory, and an extension of it to functions on the line with some classical results regarding the stability of pulses and fronts. This will also give me a chance to cover some 'background' material from functional analysis, in particular the relevant theory for continuous, point and absolute spectra of the operators arising in the stability analysis of nonlinear waves.

From here there are a few places we can go, mostly driven by examples and the tastes of the class. These include but are not limited to:

- orbital stability of waves in Hamiltonian (and integrable) systems,
- the role of the Krein signature of a Hamiltonian operator in stability analysis,
- the role that a topological object called the Maslov index, plays in stability,
- the Evans function and its role in stability for non self adjoint problems,
- the Evans function as a characteristic class of a sphere bundle,
- matrix Riccati equations and the splitting of the Evans bundles,
- edge bifurcations and the onset of instability,
- the dynamics of instabilities, convective, point and absolute instabilities,
- resonance poles and 'ghost' instabilities.

In all of these topics, the emphasis will be on examples and using these ideas to identify stable and unstable waves, as well as to compute the spectrum of a travelling wave. The Functional Analysis course offered by the School in first semester will provide a useful background for this course, but is not required.

Reference: This course will primarily follow the book below, but I will also provide supporting materials as necessary for the topics that are covered.

- T. Kapitula and K. Promislow *Spectral and Dynamical Stability of Nonlinear Waves.* Springer, New York, USA 2013.

5 Assessment Procedures

The Honours mark for each student is computed based on the following:

- 40 % for the Project/Essay assessment;
- 60 % for 6 courses (10 % for each).

Students are required to attend at least 6 courses during the academic year. Only the best 6 results will be included in the overall assessment. The assessment procedure for the Project/Essay is outlined in Section 6.

The marking scale for Honours is significantly different from the undergraduate marking scale at the University of Sydney. The Essay/Project, in addition to all the fourth-year courses, will be marked with this scale in mind. This scale appears below.

Grade of Honours	Faculty-Scale
First Class, with Medal	95–100
First Class (possibly with Medal)	90–94
First Class	80–89
Second Class, First Division	75–79
Second Class, Second Division	70–74
Third Class	65–69
Fail	00–64

Note: All assessable student work (such as assignments, Honours essays and projects) should be completed and submitted by the advertised date. If this is not possible, approval for an extension should be sought *in advance* from the lecturer concerned or (in the case of Honours essays and projects) from the Course Coordinator. Unless there are compelling circumstances, and approval for an extension has been obtained in advance, late submissions will attract penalties as determined by the Board of Examiners (taking into account any applications for special consideration).

Appeals against the assessment of any component of the course, or against the class of Honours awarded, should be directed to the Head of School.

6 The Essay/Project

A significant part of the Honours year is the completion of an Honours Essay or Project by each student. There is a distinction between an essay and a project. A project involves intensive research, analysis or computation and normally requires a greater level of supervision than an essay. An essay may cover a classical problem of acknowledged importance and mathematical depth with the student providing his/her own critical evaluation.

Each student must choose an Essay/Project supervisor who is willing to supervise the student's chosen topic for the Essay or Project. The supervisor must be a member of the Applied Mathematics staff of the School of Mathematics and Statistics. A list of available topics appears in Section 6.4. However, student are welcome to choose different topics, provided that they are able to obtain a supervisor for that topic from within the School. Essay/project topics and supervisors should be finalised by the beginning of the First Semester, so that students can commence work immediately on their Essays/Projects.

The following list shows the main applied mathematics resarch areas:

- Dynamical Systems
- Financial Mathematics and Mathematical Economics
- Geophysical and Astrophysical Fluid Dynamics
- Industrial and Biomedical Modelling
- Integrable Systems
- Mathematical Biology

For detailed information about these areas and the corresponding staff, please have a look at the webpage
`http://www.maths.usyd.edu.au/res/AppMaths.html`

6.1 Assessment

The Essay or Project will be marked according to the following.

- 90 % for the final written report
  This will be marked by 3 different markers, one of whom is the supervisor, and each marking will therefore constitute 30 % of the final Essay/Project mark. Note that the assessment also includes a one page report submitted at the end of the first semester (see section 6.3).
- 10 % for a seminar presentation on the Essay/Project

Three typed and bound copies of the final Essay/Project should be submitted to the Applied Mathematics Honours Course Coordinator, who will then distribute these copies to three markers (one of which is the supervisor) for marking. Due dates for submission appear in Section 6.2.

The seminar is an opportunity for each student to present the material of his or her Essay or Project to a mathematically literate audience. The seminar talk will usually be of 25 minutes duration, with an additional 5 minutes set aside for questions. The Course Coordinator will provide additional information and help Honours students in their preparation for the seminar. The presenter of the best AM4 seminar will be awarded the Chris Cannon Prize.

6.2 Important Dates

The following are important dates for all students intending to complete their essay/projects by the end of the first or the second semester of 2018 .

Semester 1:

- **Seminar: Friday 16th March, 2018 (week 2)**
- **Essay Submission: Tuesday, 24th April 2018 (week 7)**
  An electronic file (pdf format) and three typed and bound copies of the Essay/Project are to be handed in to the Applied Mathematics Honours Course Coordinator by this date and time. Note the Electronic pdf file must be submitted to the Honours Coordinator and uploaded via the Turnitin system to the University's LMS website by this deadline.

Semester 2:

- **Seminar: Week of 17-21st September, 2018 (week 8)**
- **Essay Submission: 5:00pm Monday, 29th October 2018 (week 13)**
  An electronic file (pdf format) and three typed and bound copies of the Essay/Project are to be handed in to the Applied Mathematics Honours Course Coordinator by this date and time. Note the Electronic pdf file must be submitted to the Honours Coordinator and uploaded via the Turnitin system to the University's LMS website by this deadline.

6.3 Essay/Project Guidelines

- The student should consult the supervisor on a regular basis, preferably at least once a week. This is the student's responsibility.
- A realistic schedule for work on the essay or project should be drawn up at an early stage, and adhered to as closely as possible. If it proves necessary to modify the original plans, a revised schedule should be drawn up after discussion with the supervisor.
- At the end of Semester 1, a one page report has to be submitted to the Honours coordinator. This report includes a half page description about the students aim/scope of the project/essay and a half page description about what the student has achieved in semester 1 and what the student wants to achieve in semester 2. This report has to be approved by the supervisor before submission.
- The essay/project should be both a discursive and a critical account of the selected topic. It should be written at a level that an expert Applied Mathematician can be expected to understand, though he/she need not be an expert in the field covered. The work must contain substantial mathematical content.
- The essay/project should be based on some four to six original primary source articles, which themselves represent a substantial contribution to the topic. Secondary sources, such as books, review papers, etc., should also be consulted and cited.
- Original research is not essential.
- The length of the essay/project should be between 40 to 60 typed A4 pages. Only in exceptional circumstances, and after consultation with the supervisor, should the essay exceed 60 pages. This number includes all figures, contents pages, tables, appendices, etc. Computer programs essential to the work should be included (with adequate commentary) as additional material.
- Students should be careful to provide full and correct referencing to all material used in the preparation of essays and projects. Be explicit in stating what is your contribution and what is someone else's contribution. Avoid quoting verbatim unless reinforcing an important point.
- Three examiners will be appointed to assess each essay/project. One of these examiners will be the student's supervisor. Although marking schemes may differ, marks will generally awarded for:

  (i) selection and synthesis of source material;
  (ii) evidence of understanding;
  (iii) evidence of critical ability;
  (iv) clarity, style and presentation;
  (v) mathematical and/or modelling expertise.

- Students are advised to read the pamphlet entitled "*Guide to Essay Writing for Science Students*" available from the Science Faculty Office.

- The preferred method of typesetting mathematical documents these days is using LaTeX. This is available from the computers at the School. Students are recommended to use LaTeX in typesetting their Essays/Projects. Additional information on LaTeX is available from the Course Coordinator.
- Students who have worked on their essays or project topics as Vacation Scholars are required to make a declaration to that effect in the preface of their essay/project.

6.4 Suggested Topics for the Essay/Project

The following is a list of possible essay/project topics for Applied Mathematics 4 students in 2018 . Prospective students interested in any of these topics are encouraged to discuss them with the named supervisors as early as possible.

However, this list is not exhaustive. Students may wish to suggest their own topics for essays or projects. Before commencing work, however, each student must find a member of staff who will agree to supervise the essay/project. For topics other than those listed below, the student and supervisor must submit a brief written outline of the proposed project or essay for approval by the Course Coordinator.

Statistical laws, information theory, and natural language (Project)
Supervisor: A/Prof. E. Altmann (Carslaw room 615; phone 9351-2448)

How dissimilar are texts from different authors, epochs, or literary genres? Information theory provides a quantitative and mathematically well-defined answer to these questions. For instance, suppose a single (random) word is shown to you and your task is to guess from which of two texts this word was extracted from. The dissimilarity between the two texts is then quantified as the (expected) amount of (Shannon) information you obtain from the word to solve your task. While these and other natural-language questions have long been a main motivation for information theory, the recent availability of large amounts of texts (e.g., on the Internet and due to the digitization of information) opens new possibilities of investigations and applications. In this project we will explore the consequences of statistical regularities observed in large datasets of written language (e.g., Zipf's law) to the computation of similarity measures. Example of questions we will address from an information-theoretic perspective are: how similar are spoken and written English? Is Australian English more similar to British or to American English? Given a series of observations of two sources, how can we quantify the extent into which one source is affected by the other? This project requires programming skills and the interest in applications of statistical methods.

Fractals in transiently chaotic systems (Project)
Supervisor: A/Prof. E. Altmann (Carslaw room 615; phone 9351-2448)

The most interesting dynamics of dynamical systems often happens during the transient time it needs to reach an asymptotic state which is often trivial (e.g., the rest state of autonomous systems with friction). This project will investigate how chaos and fractality are defined and quantified during such transients. After a revision of the main concepts of transient chaos theory, it will focus on one of two areas of current research: (i) the appearance of fractal basin boundaries in undriven systems showing "double transient chaos"; or (ii) optical billiards in which the intensity of light is partially reflected and partially transmitted. The challenge in both cases is to to generalize and apply measures of fractal dimension used in traditionally transiently-chaotic systems (in particular, the uncertainty dimension). The project involves a combination of analytical calculations and numerical simulations.

Clustering and Communities in Complex Networks (Project)
Supervisor: A/Prof. E. Altmann (Carslaw room 615; phone 9351-2448)

A variety of complex systems is described in form of networks, from social communities (e.g., online social networks) to chemical reactions (e.g., of the metabolism) and ecological environments (e.g., food webs). A universal feature of these networks is the tendency of nodes to cluster in groups of strongly connected subgraphs. From a local (egocentric) perspective, this effect is well understood: "friends of my friends tend to be my friend as well". What is less understood is what is the effect of these local clustering tendencies in global properties of the network, such as the appearance of (meso-scale) communities and the tendency of high-degree nodes to preferentially connect with each other (assortativity). This project will investigate the effect of clustering and short loops on the onset of communities and assortativity in networks through a combination of analytical calculations, numerical simulations, and data analysis.

Analysis and Simulation of Simpler Mechanical Systems (Project)
Supervisor: Prof H. Dullin (Carslaw room 714; phone 9351 4083)

This project is about the dynamics of mechanical system with few degrees of freedom (for example the double or triple pendulum, the Foucault pendulum, a spinning top, the Levitron, ... I am open to your suggestions, please come talk to me if you have further ideas/suggestions). The goal of the project is to get a good understanding of the dynamics of the chosen systems. Depending on the example different techniques will be applied, ranging from analytical computations to the the numerical integration of the problem.

Simulating a Closed Chain of Planar Rigid Bodies (Project)
Supervisor: Prof H. Dullin (Carslaw room 714; phone 9351 4083)

A chain of planar rigid bodies is a simple mechanical system with n segments connected by joints that allow free rotation. Connecting the first segment to the last by another joint gives a closed chain. Since the distance between the joints is fixed the closed chain has n degrees of freedom. Reduction by translations and rotations leaves $n-3$ degrees of freedom specifying the shape. For certain parameters the dynamics of this system is chaotic in the sense of Anosov. The goal of the project is the numerical solution of the equations of motion for the case of the five-gon, with the ultimate goal of finding many periodic orbits.

Rotation number near the 1:3 resonance (Project)
Supervisor: Prof H. Dullin (Carslaw room 714; phone 9351 4083)

Consider a Hamiltonian system with two degrees of freedom near an equilibrium point with frequencies (of the harmonic oscillator approximation) in 1:3 resonance. Every such system can be approximated by the so called resonant Birkhoff normal form. This resonant Birkhoff normal form is an integrable system, and the goal of the project is to compute the rotation number (i.e. the ratio of frequencies) of the motion for general initial condition near the equilibrium point. These are given by complete elliptic integrals, which need to be analysed.

Symplectic integration of the regularised planar ciruclar restricted 3 body problem (Project)
Supervisor: Prof H. Dullin (Carslaw room 714; phone 9351 4083)

The restricted three body problem describes the motion of a test particle in the field of two heavy masses rotating around each other in circular orbits. The problem has a singularity when the test particle collides with either of the other masses. The collision can be regularised, such that the solutions are defined for all times. The goal of the project is to construct and implement a symplectic integration method for the regularised problem. This can then be used to study periodic orbits in this chaotic system, in particular including collision orbits.

Designing optimal contract (Project)
Supervisor: Prof B. Goldys (Carslaw room 709; phone 9351-2976)

Contract theory is part of economics focused on designing optimal contracts between principals and agents. Only recently, mathematical methods have become available in this area. The breakthrough has been made by Sannikov in his seminal paper: A continuous-time version of the principal-agent problem. Rev. Econom. Stud. 75 (2008) 957-984. We will study the Sannikov paper and will try to extend it to more realistic situations. Familiarity with stochastic analysis is necessary for this project.

Stochastic PDEs in credit modelling (Project)
Supervisor: Prof B. Goldys (Carslaw room 709; phone 9351-2976)

Pricing and hedging of credit derivatives is one of the most challenging problems of Mathematical Finance. A standard tool to build mathematical models in this area is the theory of copula functions. A major deficiency of this method is that it is difficult to incorporate dynamics in the model. This problem has been recently addressed by using the mean field method well known in Physics, see Bush, N.; Hambly, B. M.; Haworth, H.; Jin, .; Reisinger, C. Stochastic evolution equations in portfolio credit modelling. SIAM J. Financial Math. 2 (2011), no. 1, 627-664. Using this approach we can derive a stochastic partial differential equation that describes time evolution of the fraction f(t,x) of agents that did not default before a given time and are in a distance x to the default. The aim of this project is to study this approach and develop more realistic models.

Stochastic differential equations with memory and applications to stochastic volatility models (Project)
Supervisor: Prof B. Goldys (Carslaw room 709; phone 9351-2976)

IA standard assumption of the Black-Scholes model for pricing of options is that the volatility of the asset, if random, is driven by Wiener process. However, recent careful analysis of market data show that the volatility is a much rougher process than Wiener process, see for example Gatheral, Jaisson, Rosenbaum. Volatility is rough, https://arxiv.org/abs/1410.3394 , In this project we will investigate a simple model of stock prices in the case when volatility is modelled as a solution to a stochastic differential equation with memory. The main question will be how to derive (approximate) option prices and hedges.

Reaction-diffusion equation on a half-line driven by the boundary noise (Project)
Supervisor: Prof B. Goldys (Carslaw room 709; phone 9351-2976)

Partial differential equations with random boundary conditions arise in many problems of Science and Engineering. In this project we will study an important nonlinear PDE with random boundary conditions, for details see from Brze?niak, Zdzis?aw; Goldys, Ben; Peszat, Szymon; Russo, Francesco Second order PDEs with Dirichlet white noise boundary conditions. J. Evol. Equ. 15 (2015), no. 1, 1?26. We will focus on the existence and uniqueness of stationary states and the rate of convergence to equilibrium. The project will extend some results

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Linear processes driven by space-time homogeneous noise (Project)
Supervisor: Prof B. Goldys (Carslaw room 709; phone 9351-2976)

Processes evolving randomly in space and time are often described using partial differential equations driven by external (independent of the solution) noise. Such PDEs are nontrivial even in the linear case and they are frequently applied to model random phenomena in physics, fluid dynamics, biology and engineering, see for example Sturm, Anja On convergence of population processes in random environments to the stochastic heat equation with colored noise. Electron. J. Probab. 8 (2003), no. 6.

In this project we will focus on linear PDEs driven by the so-called space-time homogeneous noise. Such an assumption leads to a rich class of examples useful in applications but still rather poorly understood. The project requires knowledge of Fourier analysis and some basic functional analysis.

Data-driven modelling: Finding models for observations in finance and climate (Project)
Supervisor: Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

When given data, which may come from observations of some natural process or data collected form the stock market, it is a formidable challenge to find a model describing those data. If the data were generated by some complex dynamical system one may try and model them as some diffusion process. The challenge is that even if we know that the data can be diffusive, it is by no means clear on what manifold the diffusion takes place. This project aims at applying novel state-of-the-art methods such as diffusion maps and nonlinear Laplacian spectral analysis to determine probabilistic models. You will be using data from ice cores encoding the global climate of the past 800kyrs as well as financial data. In the latter case you might be able o recover the famous Black-Scholes formula (but probably not).

This project requires new creative ideas and good programming skills.

Optimal power grid networks and synchronisation (Project)
Supervisor: Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

Complex networks of coupled oscillators are used to model systems from pacemaker cells to power grids. Given their sheer size we need methods to reduce the complexity while retaining the essential dynamical information. Recent new mathematical methodology was developped to describe the collective behaviour of large networks of oscillators with only a few parameters which we call ?collective coordinates?. This allows for the quantitative description of finite-size networks as well as chaotic dynamics, which are both out of reach for the usually employed model reduction methods.

You will apply this methodology to understand causes of and ways to prevent glitches and failure in the emerging modern decentralised power grids. As modern societies increase the share of renewable energies in power generation the resulting power grid becomes increasingly decentralised. Rather than providing a power supply constant in time, the modern decentralised grid generates fluctuating and intermittent supply. It is of paramount importance for a reliable supply of electric power to understand the dynamic stability of these power grids and how instabilities might emerge. A reliable power-grid consists of well-synchronised power generators. Failing to assure the synchronised state results in large power outages as, for example, in North America in 2003, Europe in 2006, Brazil in 2009 and India in 2012 where initially localised outages cascade through the grid on a nation-wide scale. Such cascading effects are tightly linked to the network topology. Modern power-grids face an intrinsic challenge: on the one hand decentralisation was shown to favour synchronisation in power grids, on the other hand decentralised grids are more susceptible to dynamic perturbations such as intermittent power supply or overload.

The project uses analytical methods as well as computational simulation of models for power grids. You will start with a simple network topology and then, if progress is made, use actual power grid topologies.

Data assimilation in numerical weather forecasting and climate science (Project)
Supervisor: Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

Data assimilation is the procedure in nuemrical weather forecasting whereby the information of noisy observations and of an imperfect model forecast with chaotic dynamics, we cannot trust, is combined to find the optimal estimate of the current state of the atmosphere (and ocean). Data assimilation is arguably the most computationally costly step in producing modern weatehr forecasts and has been topic of intense research in the last decade. There exists several approaches, each of which with their own advantage and disadvantages. Recently a method was introduced to ada[tively ?pick? the best method to perform data assimilation. This method employs a switch which, although it seems to work, has not been linked to any theoretical nor physical properties of the actual flow. This project will be using toy models for the atmosphere to understand the witch with the aim of improving the choice of the switching parameter.

Networks of coupled oscillators (Project)
Supervisor: Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

Many biological systems are structured as a network. Examples range from microscopic systems such as genes and cells, to macroscopic systems such as fireflies or even an applauding audience at a concert.

Of paramount importance is the topography of such a network, ie how the nodes, let's say the fireflies, are connected and how they couple. Can they only see their nearest neighbours, or all of them. Are some fireflies brighter than others, and how would that affect the overall behaviour of a whole swarm of fireflies? For example, the famous 'only 6 degrees of separation'-law for the connectivity of human relationships is important in this context.

In this project we aim to understand the influence of the topography of such a network. Question such as: How should a network be constructed to allow for maximal synchronization will be addressed.

This project requires new creative ideas and good programming skills.

Weather derivatives (Project)
Supervisor: Prof G. Gottwald (Carslaw room 625; phone 9351-5784)

Weather derivatives were introduced to safe guard economies which severely depend on the weather. The question we propose in this project is whether the value of weather derivatives has any information about the actual weather which will occur; in other words: Can they be used to do weather forecasting. The trader probably uses different forecast models from different weather centres across the world to determine the value; can the value of a weather derivative be understood as a multi-model forecast. We will use real data.

This project requires new creative ideas and good programming skills. Some exposure to statistical ideas and Bayesian methodology would be desirable but is not essential.

Discrete soliton equations (Project)
Supervisor: Prof N. Joshi (Carslaw room 629; phone 9351-2172)

Famous PDEs such as the Korteweg-de Vries equation (which have soliton solutions) have discrete versions (which also have soliton solutions). These discrete versions are equations fitted together in a self-consistent way on a square, a 3-cube or an N-dimensional cube. These have simple, beautiful geometric structures that provide information about many properties: solutions, reductions to discrete versions of famous ODEs, and deeper aspects such as Lagrangians. This project would consider generalisations of such structures and/or properties of the solutions, such as finding their zeroes or poles.

Integrable discrete or difference equations (Essay or Project)
Supervisor: Prof N. Joshi (Carslaw room 629; phone 9351-2172)

The field of integrable difference equations is only about 20 years old, but has already caused great interest amongst physicists (in the theory of random matrices, string theory, or quantum gravity) and mathematicians (in the theory of orthogonal polynomials and soliton theory). For each integrable differential equation there are, in principle, an infinite number of discrete versions. An essay in this area would provide a critical survey of the many known difference versions of the classical Painlevé equations, comparisons between them, and analyse differing evidence for their integrability. Project topics would include the derivation of new evidence for integrability. The field is so new that many achievable calculations remain to be done: including derivations of exact solutions and transformations for the discrete Painlevé equations.

Exponential asymptotics (Project)
Supervisor: Prof N. Joshi (Carslaw room 629; phone 9351-2172)

Near an irregular singular point of a differential equation, the solutions usually have divergent series expansions. Although these can be summed in some way to make sense as approximations to the solutions, they do not provide a unique way of identifying a solution. There is a hidden free parameter which has an effect like the butterfly in chaos theory. This problem has been well studied for many classes of nonlinear ODEs but almost nothing is known for PDEs and not much more is known for difference equations. This project would include studies of a model PDE, like the famous Korteweg-de Vries equation near infinity, or a difference equation like the string equation that arises in 2D quantum gravity.

Cellular Automata (Essay or Project)
Supervisor: Prof N. Joshi (Carslaw room 629; phone 9351-2172)

Cellular automata are mathematical models based on very simple rules, which have an ability to reproduce very complicated phenomena. (If you have played the Game of Life on a computer, then you have already seen automata with complicated behaviours.) This project is concerned with the mathematical analysis of their solutions, which lags far behind corresponding developments for differential or difference equations.

In this project, we will consider a family of cellular automata called parity filter rules, for which initial data are given on an infinite set. For example, consider an infinitely long train of boxes, a finite number of which have a ball inside, whilst the remainder are empty. At each time step, there is a simple rule for moving the leftmost ball in a box to the next empty box on the right. Continue until you have finished updating all nonempty boxes in the initial train. (Try this out for yourself with adjacent boxes with three balls, followed by two empty boxes and then two boxes with balls inside. What do you see after one update? Two updates?) It turns out that these box-and-ball systems replicate solitons, observed in solutions of integrable nonlinear PDEs. In this project, we will consider how to derive parity filter rules from nonlinear difference equations, and how to analyse their solutions. One direction for the project is to analyse the solutions as functions of initial data. Another direction is to develop ways to describe long-term behaviours.

Other Possible Research Topics (Essay or Project)
Supervisor: Prof N. Joshi (Carslaw room 629; phone 9351-2172)

Additional projects under the supervision of Prof Joshi include research topics in the areas of dynamical systems and mathematical physics. Please contact Prof Joshi for further details.

Modelling the evolution of human post-menopausal longevity and pari bonding (Project)
Supervisor: A/Prof P. Kim (Carslaw room 621; phone 9351-2970)

A striking contrast between humans and primates is that human lifespans extend well beyond the end of the female reproductive years. Natural selection favours individuals with the greatest number of offspring, so the presence of a long female post-fertile period presents a challenge for understanding human evolution.

One prevailing theory that attempts to explain this paradox proposes that increased longevity resulted from the advent of grandmother care of grandchildren. We have developed preliminary age-structured PDE models and agent-based models to consider the intergenerational care of young proposed by this Grandmother Hypothesis. The project will involve extending the models to consider whether the presence of grandmothering could increase the optimum human longevity while simultaneously maintaining a relatively early end of fertility as seen in humans (and killer whales).

Analytical approaches will involve developing numerical schemes for the PDEs and analytically and numerically studying the steady state age distributions and growth rates of the populations with and without grandmothering and under different life history parameters, e.g. longevity and end of fertility.

We have now also begun to explore mating strategies, especially pair bonding, yet another unique human characteristic among mammals. Speculations about how pair bonding developed from our ancestral roots abound and are open to being quantified, modelled, and analysed. Like the grandmothering models, these investigations will involve PDEs or agent- based models.

Modelling cancer immunotherapy (Project)
Supervisor: A/Prof P. Kim (Carslaw room 621; phone 9351-2970)

A next generation approach to treating cancer focuses on cancer immunology, specifically directing a person's immune system to fight tumours. Recent directions in cancer immunotherapy include

- Oncolytic virotherapy: infecting tumours with genetically-engineered viruses that preferentially destroy tumour cells and induce a local anti-tumour immune response,
- Preventative or therapeutic cancer vaccines: stimulating a person's immune system to attack tumour colonies to prevent or hinder tumour development,
- Cytokine therapy: using immunostimulatory cytokines to recruit immune cells and enhance existing anti-tumour immune responses.

These treatments can be used alone or in combination with each other or with other forms of treatment such as chemotherapy. Since immunotherapy often involves immune responses against small tumours, often close to inception, they are highly spatially dependent and often probabilistic.

The goal of the will be to develop differential equation and possibly probabilistic agent-based models to understand the tumour-virus-immune dynamics around a small, developing tumour and determine conditions that could lead to effective tumour reduction or complete elimination. The project will involve developing the models and schemes for numerically simulating the ODE and PDE systems, and if possible, performing a stability analysis of the ODE system.

The Schrödinger Equation on Quantum Graphs (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795)

A metric graph is a (usually planar) set of vertices and edges where the edges have a defined length. A "quantum graph" is a metric graph equipped with a differential operator (called a *Hamiltonian*) and some boundary conditions on the vertices of the graph.

The study of quantum graphs involves aspects from a wide variety of mathematical areas, including graph theory, combinatorics, spectral theory, mathematical physics, PDEs and spectral theory.

This is essay or project will focus on the study of the Schrödinger operator

$$H = -\partial_{xx} + V(x)$$

on Quantum Graphs. We will be interested in the spectrum (eigenvalues and eigenfunctions) of the Schrödinger operator for various potentials V(x). This will depend heavily on the shape of the graph - and in general the spectrum will be very different for trees than it will be for graphs with cycles. It will also be different depending on whether the graph is bounded or not. This project will study as many of these cases as possible.

The spectrum of the Schrödinger equation on a quantum graph will share spectral properties of operators in both one and two dimensions, and as such some pretty interesting things can happen. In particular compactly supported eigenfunctions of the Schrödinger operator can exist on a quantum graph which is not a thing that can happen for (a lot of) ODEs (whenever there is uniqueness of solutions).

PDEs on Quantum Graphs (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795)

See the previous entry for the definition of a Quantum Graph. This is actually a series of projects. For this type of project we'll pick one of the following (depending on your tastes/random chance):

- The Heat Equation (this will be similar to the Schrödinger equation project above)
- The Wave Equation
- Reaction-diffusion equations (with or without advection - this is a huge number of PDEs)
- The Korteweg de Vries equation
- The Nonlinear Schrödinger Equation
- The sine-Gordon equation
- Any other interesting PDE

The project will focus on the study of such a PDE on graphs. We will investigate application, steady states of such a graph and their stability/instability, other interesting solutions that might arise, how to construct numerical simulations on Graphs, and how the geometry/topology of the graph plays a role in all of this. Much of this project will depend on your taste, so if you are interested in this type of thing, please email me and we can meet to discuss more details.

Periodic wave trains in the nonlinear Klein Gordon Equation (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795)

The 1+1 nonlinear Klein Gordon equation is

$$u_{tt} - u_{xx} + f'(u) = 0$$

Where $f'(u)$ is some (nice enough) function called the potential. For a wide class of potentials, the solutions exhibit periodic travelling wave trains.

Passing to a moving frame and linearising leads to periodic travelling wave equations in which the (temporal) spectral parameter is now part of an advective term.

A type of dynamic instability, called *dynamical Hamiltonian-Hopf instabilities* is known for wavetrains travelling fast enough. This project will search for dynamical Hamiltonian-Hopf instabilities and attempt to numerically compute the spectrum of the periodic travelling wavetrains. Then we will numerically simulate the temporal dynamics of such travelling waves. Because the spectrum of the periodic wavetrain enters the right half plane away from the origin, it is not entirely understood how Hamiltonian-Hopf instabilities will manifest themselves dynamically, and this work will provide useful information for linking dynamic and spectral behaviour.

Symmetry Breaking in the defective Nonlinear Schrödinger equation with a potential (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795) This project will examine how solutions to the Nonlinear Schrödinger (NLS) equation with two defects bifurcate with the addition of a potential. Solutions to the NLS equation with a pair of defects can be constructed via phase portrait techniques, and one can see that a family of symmetric states can be found, some of which can be shown to be stable. Similarly, the linear Schrödinger equation with double-well potential also admits stable symmetric solutions. There are other, unstable solutions in both cases. The purpose of this project is to examine what happens when we use both of these equations to perturb off of each other. If the magnitude of the potential and defects are chosen carefully, then we should be able to observe that each of these cases can be thought of as a bifurcation of the other. Hopefully some new and interesting solutions will emerge.

The vortex filament equation and solitons in firenados (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795)

The vortex filament equation is the relatively simple-to-state PDE for a curve in 3-D

$$\gamma_t = \gamma_x \times \gamma_{xx}.$$

This equation models a thin rotating vortex, like a water sprite, a tornado, or even a firenado. It turns out that this equation is transformable into the Nonlinear Schrödinger (NLS) equation. What this means is that solitons exist (and indeed have even been found) within this equation. Moreover, because the NLS has exact solutions, there is a range of closed solutions to this equation. These closed solutions turn out to be torus knots. This project will explore all of these features, as well as compute, simulate and plot closed form solutions and solitons of thin rotating vortices. Differential Geometry would be a plus for this project as well as familiarity with at least some form of computational software tool, though these are not explicitly necessary.

Chemotaxis in models with zero diffusivity (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795) Chemotaxis is the movement of a cell via advection either towards or away from a chemical source. It has been used in many biological models, from slime-moulds to motile bacteria, to roadway construction by humans. Typically linear diffusivity has been studied, but lately models where the diffusivity is allowed to go to zero have become of interest. This project will examine the existence of travelling wave solutions in such models, as well as some elementary stability properties of such solutions.

Stability in a model of herd grazing and chemotaxis (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795) This project will examine a model of the formation of a herd of grazing animals. The model will focus on two major factors, how the animal seeks food and how the the animals interact with each other. Remarkably, the model shares many properties with another, well studied model, that of so-called bacterial chemotaxis. The aim of this project will be to analyse, both numerically and analytically, such a model, and to understand certain special solutions in the model, called travelling waves, as well as their stability.

Absolute and convective instabilities (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795) Absolute and convective instabilities are instabilities that result from points in the essential spectrum crossing into the real axis. This project will focus on several toy problems which explore what happens as such instabilities are present. The aim of this project is twofold - first to explore the mechanisms that lead to such spectral instabilities, and then secondly to understand the dynamic implications of such instabilities.

Absolute spectrum of St. Venant roll waves (Essay or Project)
Roll waves are a phenomenon that occurs when shallow water flows down an inclined ramp. Mathematically they can be modelled by the St. Venant equations. Typically roll waves occur as periodic solutions, however if they are far enough apart, they can be treated as solitary waves. In this case, the spectrum of the linearised operator governs their dynamics, and in particular, their stability properties. This project will focus on computing the absolute and essential spectrum of these solitary waves. Medium computational skills are required for this project. Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795)

Perron-Frobenius methods in PDEs (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795) The Perron-Frobenius theorem is a cute theorem from linear algebra, which states that a matrix with only positive entries, has a simple largest eigenvalue, and that the eigenvector for this eigenvalue contains only positive entries. Moreover, this is the only eigenvalue whose eigenvector has this property. This theorem can be extended to linear operators, and as such can be used to gain information about the spectral stability of solutions to PDEs. This project will begin by investigating applications of this to simple, scalar PDEs, and will then move up to systems. In particular, investigation of stability in population models which involve an allee effect will be particularly amenable to these methods.

A discrete and continuous dictionary (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795) There are many results in the field of second order linear difference equations, and second order linear ODEs that are the same in 'spirit'. One such example is the Abel/Liouville/Jacobi identity, which states that the determinant of a fundamental set of solutions obeys a straightforward first order linear differential equation. The analogue is that the determinant of a fundamental set of solutions to a second order difference equation obeys a first order difference rule. This project will attempt to highlight which results should be connected, and for theorems with certain hypotheses, how to translate one into the other.

The history of continued fractions (Essay)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795) The origin of continued fractions is not well known, though it is traditionally placed at around the time of Euclid's algorithm. By manipulating the Euclidean algorithm, one can derive a simple continued fraction of the rational number $\frac{p}{q}$. Continued fractions gained much interest after John Wallis published his famous identity in 1655

$$\frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times \cdots},$$

and it was observed that the expression could be transformed into a continued fraction. Continued fractions continued to be explored through the 19th century with incredibly prominent mathematicians of the age such as Hermite, Jacobi, Gauss, Stieltjes and Cauchy making contributions. Continued fractions (in some form) are still of interest today in the fields of orthogonal polynomials as well as special functions, and integrable systems. This essay will explore the use and theory behind continued fractions from their 'inception' nearly 2000 years ago up through the current time.

The history of the integral (Essay)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795)

This essay will follow the history of the integral from the method of exhaustion of the ancient greeks, up through the modern notions of the Lebesgue integral. From here the project can go in a couple of different ways - either into modern notions of stochastic and probablistic integration, into modern notions of integral transforms, or into computer algorithms for symbolic and numeric integration. Perhaps unsurprisingly, there aren't many books available on this subject, so the student will be expected to find most of their own sources for this project, but the Wikipedia articles 'Integral' and 'History of Calculus' and the references therein are a good place to start. If this project is chosen, some care must be taken to ensure that enough mathematics in included in the essay.

Other Possible Research Topics (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw room720; phone 9351-5795)

Other projects under the supervision of Dr Marangell include topics in the areas of nonlinear standing or travelling waves, topics in the application of geometric and topological methods in dynamical systems and PDEs, and other research topics in the history of mathematics and science in general. Examples of nonlinear standing and travelling waves come from models in a wide range of areas which include mathematical biology, chemistry and physics. More specific examples would be standing/travelling waves in population dynamics, combustion models, and quantum computing, but really there are many, many examples, so please contact Dr Marangell for further details.

PDE models for the distribution of ingested lipids in macrophages in atherosclerotic plaques. (Project)

Supervisor: Prof M.R. Myerscough (Carslaw room 626; phone 9351-3724)

Atherosclerotic plaques are accumulations of lipid (fat) loaded cells and necrotic (dead) cellular debris in artery walls. They are caused by LDL (which carries 'bad cholesterol) penetrating the blood vessel wall, becoming chemically modified (usually oxidised) and setting off an immune reaction. In response to this immune reaction, macrophages (a type of white blood cell) enter the artery wall and consume the modified LDL. In this way macrophages accumulate lipids and as more LDL and more cells enter the vessel walls the population of cells also grows. Other processes can affect the growth or regression of the plaque, such as cell death, cells leaving the tissue and lipid export from inside cells to HDL (which carries good cholesterol which is good because its been carried away from the plaque). When atherosclerotic plaques grow very large and rupture they can cause heart attacks and strokes which are one of the two leading causes of death in the developed world. (The other is cancer.)

We have written a partial differential equation model for the accumulation of cells and lipids in plaques. In this model, the number of macrophages in the plaque is a function of both time t and accumulated lipid a. The primary equation is an advection equation with nonlinear source and sink terms, including a term with an integral convolution that models what happens when macrophages phagocytose (=eat) other macrophages that are dead or dying. We have done an analysis of this model at steady state when all the processes (lipid ingestion, macrophages leaving the plaque, the action of HDL) occur at a constant rate. This project will build on this analysis and has the aim of producing numerical solutions to the model when model processes are functions of a, the accumulated lipid inside the cell.

This project is particularly suitable for students who are interested in applications of mathematics to biomedical problems, have completed a third year unit on PDEs and have at least some experience in coding in Matlab, C, Python or similar.

Developing more accurate cellular random walk models (Project)

Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049) & Mark Read (Charles Perkins Centre)

Many critical processes within the body, such as the development of an immune response, arise from inter-cellular interactions, which are in turn dictated by the motility dynamics of cells. Recent advances in imaging technology allow us to track individual cells as they move through tissue. Accurately modelling the motility of these cells is a fundamental requirement for simulating and exploring how complex emergent behaviours arise from cellular interactions. Research has shown that immune cells, as with many other larger biological creatures, perform a Levy flight random walk. Levy flights entail choosing a random direction, and moving in a straight line along that path for a duration/speed drawn from a long tailed distribution. It is thought to be evolutionarily optimal. However, closer examination shows that Levy flights fail to capture much of the finer-level nuances in the paths that cells actually follow. This project seeks to develop a more accurate and intricate random walk, which captures the essence of Levy flight on a broader scale, but encompasses the finer details that cells are known to exhibit.

Developing a fuzzy-valued multi-objective optimisation framework for biological simulation parameterization (Project)
Supervisor: A/Prof L. Poladian (Room 713; phone 9351 2049) & Mark Read (Charles Perkins Centre)

Agent-based simulation is transforming the way biological investigation is carried out, and is increasingly used as a complement to wet-lab techniques. A key challenge in this field is model selection and parameterization; being abstractions, many model parameter values cannot be derived experimentally or from literature, and there often exist several alternative ways in which some biology might be modelled. Multi-objective optimisation (MOO) offers a solution to both problems. The dynamics of the biological system are captured as potentially conflicting objectives, and MOO identifies the best parameter values it can for a given model. The best solutions obtained can be contrasted to inform model selection.

The technology works, yet agent-based simulations present a highly challenging domain for MOO to operate in. Being stochastic in nature, many simulation executions are required to obtain representative behaviours, and the computational expense is significant. This project will investigate how fuzzy values can be incorporated into the MOO framework to increase efficiency, and tackle biological problems of greater complexity.The project has access to a number of biological simulations in which to develop and test the framework: a simulation of immune cell motility in the skin and lymph nodes, a simulation of how the gut bacterial community responds to different diets, and a simulation of EAE (a mouse model of multiple sclerosis).

Poncelet Porisms (Essay)
Supervisor: Dr Milena Radnovic (Room 633; phone 9351-4543)

Suppose that two conics are given in the plane, together with a closed polygonal line inscribed in one of them and circumscribed about the other one. The Poncelet porism states that then infinitely many such closed polygonal lines exist and all of them with the same number of sides. That statement is one of most beautiful and deepest contributions of the XIX-th century geometry and has many generalisations and interpretations in various branches of mathematics. In this essay, the student will present rich history and current developments of the Poncelet porism.

Elliptical billiards and their periodic trajectories (Essay or Project)
Supervisor: Dr Milena Radnovic (Room 633; phone 9351-4543)

We consider billiards in a domain bounded by arcs of several conics belonging to a confocal family. When the boundary of such a billiard does not contain reflex angles, the system turns out to be integrable. Geometrically, the integrability has the following manifestation - for each billiard trajectory, there is a curve, called caustic, which is touching each segment of the trajectory. For elliptical billiards, the caustics are conics from the same confocal family.

Integrability implies that the trajectories sharing the same caustic are either all periodic with the same period or all non-periodic.

On the other hand, if there is at least one reflex angle on the boundary, the integrability will be broken, although the caustics still exist. Such billiards are thus called pseudo-integrable and there may exist trajectories which are non-periodic and periodic with different periods sharing the same caustic.

An essay on this topic would provide a review of classical and modern results related to the elliptical billiards. In a project, the student would explore examples of billiard desks.

Other Possible Research Topics (Essay or Project)
Supervisor: Dr Milena Radnovic (Room 633; phone 9351-4543)

Additional projects under the supervision of Dr Radnovic include research topics in the areas of dynamical systems and mathematical physics. Please contact Dr Radnovic for further details.

Hedging of counterparty credit risk (Essay or Project)
Supervisor: Prof M. Rutkowski (Carslaw room 814; phone 9351-1923)

The risk that a counterparty cannot meet its contractual obligations has become the hot subject since the last financial crisis. Intertwined studies were recently conducted in different directions: the systemic risk (the risk of a domino effect following the bankruptcy of a major financial institution, the systemic impact of centralized clearing, the effects of an asymmetric information in regard of securitised products and exposure of banks to these products), the liquidity effects (the risk of fire sales, the impact of collateral policies of central banks) and, last but not least, the counterparty risk due to the possibility of default of a counterparty (in particular, the computations of the credit value adjustment). The goal of the project is to examine hedging strategies for unilateral and bilateral counterparty risks within the set-up of intensity-based models of defaults.

Modeling of risky interest curves (Essay or Project)
Supervisor: Prof M. Rutkowski (Carslaw room 814; phone 9351-1923)

The recent credit-crunch has changed the markets perceptions and banks are nowadays more conservative about the possibility of default of other banks and their own future funding costs, that is, the rate at which they can borrow. These risk are being priced into money market lending rates (e.g. LIBOR) as well as derivatives written on these rates. The project aims at understanding apparent anomalies that appeared recently in the interest rate curves. Mathematical goal is to develop a model reflecting the fact that the LIBORs embed options on the creditworthiness of the counterparty and to show how this model can be used to explain the basis swaps patterns during the financial crisis by taking as inputs the level of counterparty risk and credit volatility.

Robustness of credit risk models (Essay or Project)
Supervisor: Prof M. Rutkowski (Carslaw room 814; phone 9351-1923)

The concept of robustness of a financial model hinges on the postulate that the knowledge of a 'perfect' model for the real-world dynamics is not available to traders, who instead need to use some 'imperfect' method in order to to value financial assets and hedge the risk exposure. The issue of robustness was examined in detail for the case of the classic Black-Scholes model for equities/currencies and some of its extensions. In the context of credit risk, it is natural to compare a fully dynamical model of defaults to the industry-standard static one-factor Gaussian copula with periodic recalibration to the market data. The goal of the project is to examine the efficiency of hedging strategies derived using practitioner's method under the assumption that the market prices are computed using the perfect model.

Control of boundary-layer flows (Essay or Project)
Supervisor: A/Prof S. Stephen (Carslaw room 525; phone 9351-3048)

This project is in the field of hydrodynamic stability of boundary-layer flows where viscous effects are important. The aim is towards understanding more fully the transition process from a laminar flow to a turbulent one.

We will consider rotating flows which are relevant to the flow over a swept wing and to rotor-stator systems in a turbine engine. Experiments show that the boundary layer becomes unstable to stationary or travelling spiral vortices.

The project will investigate the effect of different surface boundary conditions on boundary-layer flows over rotating bodies. Effects such as suction, partial-slip, compliance and wall shape can be modelled. Suction, for example, is used to achieve laminar flow control on swept wings.

The resulting system of governing ordinary differential equations will be solved numerically for the basic flow, determining important values such as the wall shear. The linear stability of these flows to crossflow instabilities will be investigated. These take the form of co-rotating vortices, observed in experiments, and only occur in three-dimensional boundary layers.

The flow for large Reynolds number, corresponding to large values of rotation, will be considered. In this case the boundary layer thickness will be very small so asymptotic methods of solution will be used. Different asymptotic regimes will need to be considered and solutions obtained in each region. Matching the solutions between the regimes and satisfying the boundary conditions will lead to an eigenrelation. Inviscid and viscous instability modes will be considered.

The effect of the surface boundary conditions on the disturbance wave number and wave angle will be determined. This will have applications in possible control of boundary layers as boundaries causing stabilisation of the instabilities could lead to a delay in the transition process from a laminar flow to a turbulent flow.

Inverse Problems for Partial Diffeential Equations - A Geometric Analysis Perspective (Essay or Project)
Supervisor: Dr L. Tzou (Carslaw room 615; phone 9351-1917)

The analysis of mathematical models arising from various imaging techniques has become a prominent research topic due to its potential applications in many fields including oil exploration and early detection of malignant breast tumor. In many cases one asks the question of whether one can deduce interior information about an object by making measurements on its surface. Mathematically this amounts to recovering the model parameters for a partial differential equation from boundary behavior of its solutions.

This project will explore how these mathematical models behave in various geometric settings. We will see how this applied mathematics problem can motivate us to ask interesting questions in both geometry and analysis.

Non-equilibrium statistical mechanics of a chain: the birth of temperature. (Essay or Project)
Supervisor: Dr G. Vasil (Carslaw room 627; phone 9351-4163)

If you release a simple, frictionless pendulum from a given height, it will oscillate forever in periodic motion. If you attach a second pendulum at the bottom of the first, the result becomes chaotic with all the associated rich behaviour. But the system will still more-or-less swing back and forth forever.

If you attach 100, 1,000, or 1,000,000 more frictionless elements to the bottom of the configuration something even more interesting happens. The swinging motion begins to go away. The initial coherent kinetic energy transfers into the small-scale random fluctuations of the individual elements. The transfer of energy is a process called "thermalisation" (or the birth of temperature), and the damping of the motion is the emergence of friction in an otherwise dissipation-free system.

Even more fascinating, the system can generate configurational entropy, and this gives rise to emergent entropic forces. This is a fancy way of saying that the initial inextensible pendulum chain configuration turns into a hot bouncing bungee cord.

How does this happen exactly? How fast does temperature, entropy, friction, and tension emerge? What can we predict about the statistics? What happens if we drive the system externally? What does this say about hydrodynamical turbulence? This project will investigate all any and all of these question and more. The project will start by directly simulating the pendulum system numerically. After analysing the results, we can use theoretical techniques from equilibrium and non-equilibrium statistical mechanics to better understand these kinds of systems.

I also have a number of similar and related problems on non-equilibrium statistical systems. You should come talk to me if this area interests you in general.

Computational projects with Dedalus. (Essay or Project)
Supervisor: Dr G. Vasil (Carslaw room 627; phone 9351-4163)

Dedalus is a multi-purpose solver than can simulate a broad array of nonlinear partial differential equations (PDEs).

You can find out about Dedalus here:

http://dedalus-project.org

I have been leading the development of this project for the past few years. The idea behind Dedalus is that the user can enter their equations in text-based format and experiment with different behaviour much more easily than anything that came before. The code runs on computers ranging from laptops to the world's most powerful supercomputers.

And you can see some examples of past work done here:

http://vimeo.com/dedlaus

In the past, my collaborators and I have solved problems ranging from solar magnetohydrodynamics, the dynamics of boats in Arctic seas, buckling elastic beams, nonlinear optics on topologically complex graphs, turbulent thermal convection, nuclear burning flames in white dwarf stars, cancer modelling, and more.

I have a range of potential projects that students can investigate with Dedalus. Some of these are extensions of past problems, and some are entirely new. I'm also happy to talk to students who have a particular application in mind that they would like to try. The idea is to simulate something interesting and model the behaviour after you know the answer!

Improved numerical algorithms for multidimensional spectral methods. (Essay or Project)
Supervisor: Dr G. Vasil (Carslaw room 627; phone 9351-4163)

Simulating important physical, biological, finical, and mathematical dynamics usually involves solving a large system of linear equations. An important key to many numerical methods is finding a way to solve linear systems in as few operations as possible. The first way to do this is to use a basis that produces matrices with as few non-zero elements as possible. But even with very sparse matrices, then actual time to find the solution can still differ by many orders of magnitude with different methods.

Two general classes of methods exist for solving large, sparse, linear systems. The first class is "direct methods". These are things like Gaussian elimination that you commonly learn in undergraduate courses. The second class are "iterative methods". Iteration involves applying a matrix to a vector over and over again in a clever way that converges to the desired solution. Both of these methods have their pros and cons.

The idea of this project is to study iterative methods in a new context. This project has the potential to accelerate important computational algorithms by many orders of magnitude. The results of this would contribute back to the code base of the Dedalus project and apply to many different areas.

Possible Research Topics (Essay or Project)

Supervisor: Prof M. Wechselberger (Carslaw room 628; phone 9351-3860)

Projects under the supervision of Prof Wechselberger include research topics in the areas of relaxation oscillators, return maps, physiological rhythms, mathematical neuroscience, and more generally, dynamical systems.

These areas of research are concerned with the study of oscillatory patterns of so called 'slow/fast systems'. These systems are ubiquitous in nature and control most of our physiological rhythms. E.g. one cycle of a heart beat consists of a long interval of quasi steady state interspersed by a very fast change of state, the beat itself. This very fast relaxation of energy leads to the notion of a relaxation oscillator and shows the appearance of multiple time-scales (slow/fast) in the system. To study periodic solutions in such systems one can analyse an associated return map. It is well known that already 1D maps can have a rich variety of dynamics, like periodic, quasiperiodic and chaotic solutions, the most famous example is the 1D circle map. Plotting the winding number versus a control parameter in this case yields the famous 'devil's staircase'.

For more information on possible topics, please see the research link on Prof Wechselberger's webpage `http://www.maths.usyd.edu.au/u/wm/`

7 Rights and Responsibilities

Applied Mathematics 4 students will have access to the following.

- Office space and a desk in the Carslaw building.
- A computer account with access to e-mail and the World-Wide Web, as well as TeX and laser printing facilities for the preparation of essays and projects.
- A photocopying account paid by the School for assembling essay/project source material.
- After-hours access key to the Carslaw building. (A deposit is payable.)
- A pigeon-hole in room 728 — please inspect it regularly as lecturers often use it to hand out relevant material.
- Participation in the School's social events.
- Class representative at School meetings.

Applied Mathematics 4 students have the following obligations.

- Regular attendance at the regular weekly seminars in applied mathematics.
- Have regular meetings with project/essay supervisors, and meet all deadlines.
- Utilise all School resources in an ethical manner.
- Contribute towards the academic life in Applied Mathematics at the School of Mathematics and Statistics.

8 Scholarships, Prizes and Awards

The following prizes may be awarded to Applied Mathematics 4 students of sufficient merit. Students do not need to apply for these prizes, which are awarded automatically.

Joye Prize in Mathematics Value: **$5300, with medal and shield**
Awarded to the most outstanding student completing Honours in the School of Mathematics and Statistics.

University Medal
Awarded to Honours students who perform outstandingly. The award is subject to Faculty rules, which require a Faculty mark of 90 or more in Applied Mathematics 4 and a Third Year WAM of 80 or higher. A medal is always awarded when the Faculty mark is 95 or higher. More than one medal may be awarded in any year.

K.E. Bullen Memorial Prize Value: **$1000**
Awarded annually on the recommendation of the Head of the School of Mathematics and Statistics in consultation with the professors of Applied Mathematics to the most proficient student in Applied Mathematics 4, provided that the student's work is of sufficient merit.

Barker Prize Value: **$550**
Awarded at the Fourth (Honours) Year examination for proficiency in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

M.J. and M. Ashby Prize Value: **$360**
Offered annually for the best essay, submitted by a student in the Faculty of Science, that forms part of the requirements of Pure Mathematics 4, Applied Mathematics 4 or Mathematical Statistics 4.

Norbert Quirk Prize No IV Value: **$250**
Awarded annually for the best essay on a given mathematical subject by a student enrolled in a Fourth Year course in mathematics (Pure Mathematics, Applied Mathematics or Mathematical Statistics) provided that the essay is of sufficient merit.

Australian Federation of Graduate Women Prize in Mathematics. Value: **$175**
Awarded annually, on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate for the degree of BA or BSc who graduates with first class Honours in Applied Mathematics, Pure Mathematics or Mathematical Statistics.

Chris Cannon Prize Value: **$100**
For the best adjudged essay/project seminar presentation of an Applied Mathematics 4 student.

Note: some of the prize values may change. A complete list of the prizes offered by the School of Mathematics and Statistics, as well as any changes to current prize values can be found at `http://www.maths.usyd.edu.au/u/About/prizes.html` *on the school's website.*

9 Life After Fourth Year

Postgraduate Studies:
Many students completing the Honours programme have in the past gone on to pursue postgraduate studies at the University of Sydney, at other Australian universities, and at overseas universities. Please see the Director of Postgraduate Studies if interested in enrolling for a MSc or PhD at the School of Mathematics & Statistics. Students who do well in Applied Mathematics 4 may be eligible for postgraduate scholarships, which provide financial support during subsequent study for higher degrees at Australian universities. The honours coordinator is available to discuss options and provide advice to students interested in pursuing studies at other universities.

Careers:
Students seeking assistance with post-grad opportunities and job applications should feel free to ask lecturers most familiar with their work for advice and written references. The Director of the Applied Mathematics Teaching Programme, the Course Coordinator and the course lecturers may also provide advice and personal references for interested students.