Calculus

Differential calculus $\frac{dy}{dx}$

For an **introduction to differentiation**:

- A brief refresher on basic differentiation, critical points and their nature, and with applications to economics.
  Introduction to calculus (pdf, 78KB)
- A more in-depth treatment to differentiation: rates of change, tangents and derivatives, the product, quotient and chain rule, stationary points and optimisation problems.
  Introduction to differential calculus (pdf, 2.1MB)

For specific help on calculating derivatives using the **rules of differentiation**:

- Differentiating constants $y = k$, polynomial functions $y = x^n$, constant multiples $y = cf(x)$, addition and subtraction of functions $y = f(x) \pm g(x)$, the product rule for $y = uv$, and the quotient rule for $y = \frac{u}{v}$.
  The rules of calculus (pdf, 89KB)
- The chain rule for composite functions $y = h(g(x))$, and its two formulations:
  $$y' = h'\big(g(x)\big)g'(x) \qquad \text{and} \qquad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}.$$
  Composite function rule (the chain rule) (pdf, 88KB)
- For derivatives of functions with exponentials e^x and logarithms $\ln x$. Some of the examples assumes knowledge of the chain rule.
  Derivatives of exponential and logarithmic functions (pdf, 81KB)
- For derivatives of functions with $\sin x$, $\cos x$, $\tan x$. Some of the examples assumes knowledge of the product, quotient and chain rules.
  Differentiation of trigonometry (pdf, 78KB)

For **what derivatives can tell us** about the shape of a graph:

- The first derivative $\frac{dy}{dx}$: increasing and decreasing functions, stationary points and their nature, relative maximum and minimum.
  The first derivative and stationary points (pdf, 99KB)
- The second derivative $\frac{d^2y}{dx^2}$: concave up, concave down and points of inflection.
  Second derivative and points of inflection (pdf, 95KB)

Integral calculus $\int f(x)dx$

For an introduction to the indefinite integral $\int f(x)dx$: anti-derivatives, calculating some elementary anti-derivatives and reversing the Chain Rule.
Introduction to integration part 1: the anti-derivative (pdf, 191KB)

For an introduction to the definite integral $\int_a^b f(x)dx$: limiting sums, the Fundamental Theorem of Calculus, and finding areas under and between curves.
Introduction to integration part 2: the definite integral (pdf, 281KB)

Applications of calculus

For demonstrations of how to use the concepts and tools of differential calculus to sketch graphs and curves of functions through several worked examples.
Curve sketching using calculus (pdf, 119KB)

For demonstrations of how to apply differential calculus in optimisation problems such as maximising or minimising functions over an interval.
Optimisation using calculus (pdf, 115KB)

For various applications of exponentials and logarithms: linearisation using logarithms (log transforms of $y = ax^b$ and $y = ae^{kx}$), logarithmic differentiation, exponential growth and decay, half-life, and the differential equation $\frac{dP}{dt} = kP$.
Exponentials and logarithms: applications and calculus (pdf, 845KB)

To develop your understanding of how calculus can be used to model real world phenomena: an introduction to differential equations, general and particular solutions, separation of variables. A useful resource to be used alongside the modelling and calculus recordings.
Modelling and calculus (pdf, 544KB)

Audiovisual recordings
Introduction to Calculus
For some `just in time' videos related to introductory calculus concepts. Also a useful resource for the *Introduction to Calculus* MOOC.
Entire playlist: **Introduction to calculus tutorial recordings (YouTube video)**

Individual videos:

- **Module 1**: The Sieve of Eratosthenes: finding all the primes less than 100.
- **Module 2**: Solving the inequality equation $\frac{4}{2-x} \leq 1$.
- **Module 3**: Factorising the quadratic expression $x^2 + 5x + 6$.
- **Module 4**: Solving the quadratic equation $x^2 + 5x + 6 = 0$ using factorisation.
- **Module 5**: Revision of the sine, cosine and tangent ratios in a right angled triangle.
- **Module 6**: How to complete the square along with applications.
- **Module 7**: Sketching the graph of $f(x) = x^2 - 2x + 3$ using shifting transformations.
- **Module 8**: Inverse functions: motivations, domain restrictions, finding formula of f^{-1}, and properties.
- **Module 9**: Solving the cubic polynomial equation $x^3 + 3x^2 + 4x + 2 = 0$ using polynomial long division.
- **Module 10**: Revision of logarithms and comparing logs with different bases such as $\log_4 17$ and $\log_5 24$.
- **Module 11**: Finding the equation of a straight line through two points.
- **Module 12**: Revision of derivatives: differentiating $y = \sqrt{x}$ from first principles and using the power rule.
- **Module 13**: Solving the absolute value equation $|x - 2| < |x + 4|$.
- **Module 14**: Revision of limits: extending the domain of the discontinuous function $f(x) = \frac{x^2-1}{x-1}$.
- **Module 15**: Revision of composite functions: using the chain rule to differentiate $y = \sin(x^2 + 1)$.
- **Module 16**: Using the Quotient Rule to differentiate $y = \tan x$.
- **Module 17**: Using derivatives to sketch the cubic polynomial $f(x) = 2x^3 - 9x^2 + 12x - 3$.
- **Module 18**: Curve sketching tips: the roles of f, f' and f'' and when to solve for $f = 0, f' = 0$ and $f'' = 0$.
- **Module 19**: Revision of even and odd functions and some of their properties.
- **Module 20**: Evaluating the indefinite integral of a simple polynomial.
- **Module 21**: Integration by substitution.

- **Module 22**: Using Riemann sums to approximate a definite integral: showing that $\frac{\pi}{6} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \, dx \leq \frac{\pi}{3}$.

Mathematical Modelling

For developing your understanding of some first and second order differential equations used in mathematical modelling. A useful accompaniment to the `modelling and calculus' pdf booklet.

Entire playlist: **Mathematical modelling recordings (YouTube video)**

Individual videos:
Some first order equations.

- **Module 1**: Solving the constant **absolute growth rate** (AGR) differential equation $\frac{dP}{dt} = a$.
- **Module 2**: Solving the constant **relative growth rate** (RGR) differential equation $\frac{dP}{dt} = kP$.
- **Module 3**: Solving the **general linear** differential equation $\frac{dP}{dt} = kP + d$.
- **Module 4**: Solving the **Logistic** differential equation $\frac{dP}{dt} = rP(1 - P)$ or $\frac{dp}{dt} = rp\left(1 - \frac{p}{K}\right)$.
- **Module 5**: An introduction to **Recurrence Relations (difference equations):** solving $X_{n+1} = X_n + nd$.

Second order constant coefficient differential equations $a\frac{d^2P}{dt^2} + b\frac{dP}{dt} + cP = 0$.

- **Module 6**: Part 1: The real exponential form
- **Module 7**: Part 2: The repeated roots form
- **Module 8**: Part 3: The trigonometric form