Trigonometric Identities

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Contents

1 Introduction

1.1 How to use this book

You will not gain much by just reading this booklet. Have pencil and paper ready to work through the examples before reading their solutions. Do *all* the exercises. It is important that you try hard to complete the exercises on your own, rather than refer to the solutions as soon as you are stuck.

1.2 Introduction

This unit is designed to help you learn, or revise, trigonometric identities.

You need to know these identities, and be able to use them confidently. They are used in many different branches of mathematics, including integration, complex numbers and mechanics.

The best way to learn these identities is to have lots of practice in using them. So we remind you of what they are, then ask you to work through examples and exercises. We've tried to select exercises that might be useful to you later, in your calculus unit of study.

1.3 Objectives

By the time you have worked through this workbook you should

• be familiar with the trigonometric functions sin, cos, tan, sec, csc and cot, and with the relationships between them,

• know the identities associated with $\sin^2\theta + \cos^2\theta = 1$,

• know the expressions for sin, cos, tan of sums and differences of angles,

• be able to simplify expressions and verify identities involving the trigonometric functions,

• know how to differentiate all the trigonometric functions,

• know expressions for $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ and use them in simplifying trigonometric functions,

• know how to reduce expressions involving powers and products of trigonometric functions to simple forms which can be integrated.

1.4 Pretest

We shall assume that you are familiar with radian measure for angles, and with the definitions and properties of the trigonometric functions sin, cos, tan. This test is included to help you check how well you remember these.

1. Express in radians angles of

i. $60°$ ii. $135°$ iii. $270°$

2. Express in degrees angles of

i. $\frac{\pi}{4}$ ii. $-\frac{3\pi}{2}$ iii. 2π

3. What are the values of

i. $\sin\frac{\pi}{2}$ ii. $\cos\frac{3\pi}{2}$ iii. $\tan\frac{3\pi}{4}$

iv. $\sin\frac{7\pi}{6}$ V. $\cos\frac{5\pi}{3}$ vi. $\tan 2\pi$

4. Sketch the graph of $y = \cos x$.

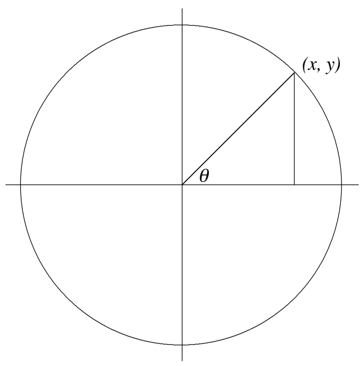
2 Relations between the trigonometric functions

Recall the definitions of the trigonometric functions by means of the unit circle, $x^2 + y^2 = 1$.

$$\sin\theta = y$$

$$\cos\theta = x$$

$$\tan\theta = \frac{y}{x}$$



Three more functions are defined in terms of these, secant (sec), cosecant (cosec or csc) and cotangent (cot).

$$\sec\theta = \frac{1}{\cos\theta} \qquad (1)$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad (2)$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad (3)$$

The functions cos and sin are the basic ones. Each of the others can be expressed in terms of these. In particular

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad (4)$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \qquad (5)$$

These relationships are identities, not equations. An equation is a relation between functions that is true only for some particular values of the variable.

For example, the relation $\sin\theta = \cos\theta$ is an equation, since it is satisfied when $\theta = \frac{\pi}{4}$, but not for other values of θ between 0 and π.

On the other hand, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ is true for all values of θ, so this is an identity.

The relationships (1) to (5) above are true for all values of θ, and so are identities. They can be used to simplify trigonometric expressions, and to prove other identities. Usually the best way to begin is to express everything in terms of sin and cos.

Examples

1. Simplify the function $\cos x \tan x$.

$$\begin{aligned}\cos x\tan x &= \cos x \times \frac{\sin x}{\cos x}\\ &= \sin x\end{aligned}$$

2. Show that $\dfrac{\sin\theta + \tan\theta}{\csc\theta + \cot\theta} = \sin\theta\tan\theta$.

To show that an identity is true, we have to prove that the left hand side and the right hand side are different ways of writing the same function. We usually do this by starting with one side and using the identities we know to transform it until we obtain the expression on the other side.

$$\begin{aligned}\frac{\sin\theta + \tan\theta}{\csc\theta + \cot\theta} &= \frac{\sin\theta + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}\\ &= \frac{(\sin\theta\cos\theta + \sin\theta)}{1+\cos\theta} \times \frac{\sin\theta}{\cos\theta}\\ &= \frac{\sin^2\theta(1+\cos\theta)}{\cos\theta(1+\cos\theta)}\end{aligned}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$= \sin\theta\tan\theta$$

Exercises 1

1. Simplify

a. $\sin x \cot x$

b. $\dfrac{\csc\theta}{\sec\theta}$

c. $\dfrac{\sin x + \tan x}{1 + \sec x}$

2. Show that

a. $\dfrac{\cot\theta + 1}{\cot\theta - 1} = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

b. $\dfrac{\cot x + 1}{\sin x + \cos x} = \csc x$

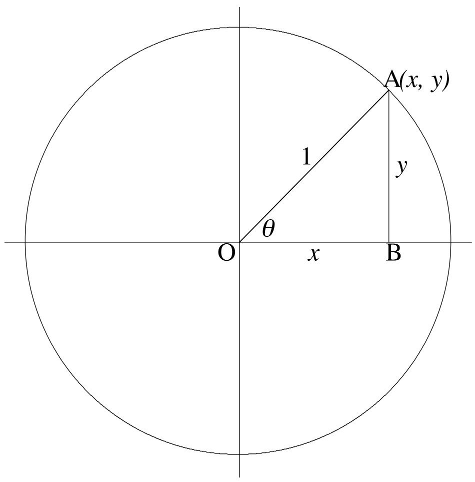
c. $(1 + \tan x)\dfrac{\sin x}{\sin x + \cos x} = \tan x.$

3 The Pythagorean identities

Remember that Pythagoras' theorem states that in any right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

In the right angled triangle OAB, $x = \cos\theta$ and $y = \sin\theta$, so

$$\cos^2\theta + \sin^2\theta = 1 \qquad (6).$$



Remember that $\cos^2\theta$ means $(\cos\theta)^2 = \cos\theta\cos\theta$.

Two other important identities can be derived from this one.
Dividing both sides of (6) by $\cos^2\theta$ we obtain

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

ie $1 + \tan^2\theta = \sec^2\theta$.
If we divide both sides of (6) by $\sin^2\theta$ we get

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

ie $\cot^2\theta + 1 = \csc^2\theta$.
Summarising,

$$\begin{aligned} \cos^2\theta + \sin^2\theta &= 1 && (6)\\ 1 + \tan^2\theta &= \sec^2\theta && (7)\\ \cot^2\theta + 1 &= \csc^2\theta && (8) \end{aligned}$$

Examples

1. Simplify the expression $\dfrac{\sec^2\theta}{\sec^2\theta - 1}$.

$$\begin{aligned} \frac{\sec^2\theta}{\sec^2\theta - 1} &= \frac{\sec^2\theta}{\tan^2\theta}\\ &= \frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\\ &= \frac{1}{\sin^2\theta}\\ &= \csc^2\theta. \end{aligned}$$

2. Show that

$$\frac{1 - 2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \cot\theta.$$

$$\begin{aligned} \tan\theta - \cot\theta &= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}\\ &= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}\\ &= \frac{1 - 2\cos^2\theta}{\sin\theta\cos\theta}. \end{aligned}$$

Exercises 2

1. Simplify

a. $\dfrac{1}{\tan x + \cot x}$

b. $(1 - \sin^2 t)(1 + \tan^2 t)$

c. $\dfrac{1 + \cos\theta}{\sec\theta - \tan\theta} + \dfrac{\cos\theta - 1}{\sec\theta + \tan\theta}$.

2. Show that

a. $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

b. $\tan x \csc x = \tan x \sin x + \cos x$

c. $\dfrac{1 + \sec\theta}{\tan\theta} = \dfrac{\tan\theta}{\sec\theta - 1}$.

Remember that you used these identities in finding the derivatives of tan, sec, csc and cot.

Recall that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.

Then

$$
\begin{aligned}
\frac{d}{dx}(\tan x) &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) \\
&= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} \\
&= \sec^2 x.
\end{aligned}
$$

Exercises 3

Find

1. $\dfrac{d}{dx}(\cot x)$,   2. $\dfrac{d}{dx}(\sec x)$,   3. $\dfrac{d}{dx}(\csc x)$.

4 Sums and differences of angles

A number of useful identities depend on the expressions for $\sin(\alpha + \beta)$ and $\cos(\alpha - \beta)$. We shall state these expressions, then show how they can be derived.

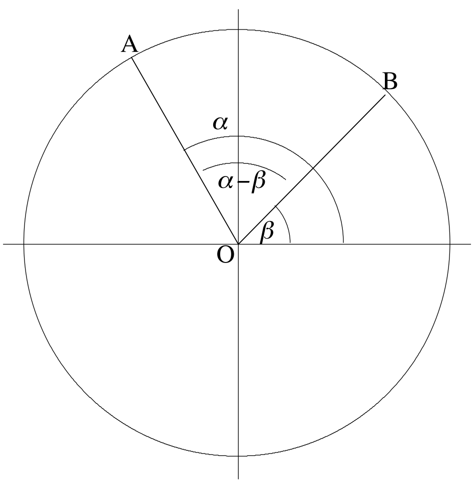
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \quad (9)$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \quad (10)$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \quad (11)$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \quad (12)$$

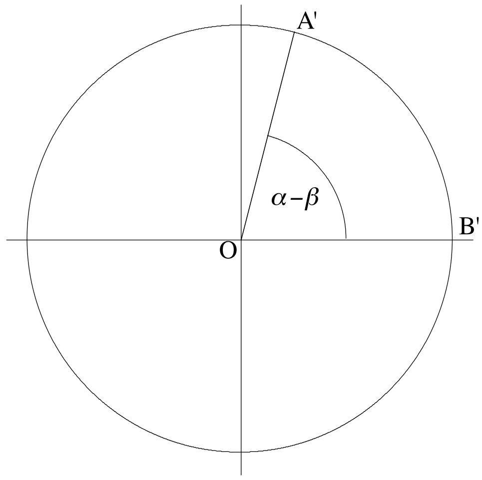
The expressions for $\sin(\alpha + \beta)$, $\sin(\alpha - \beta)$ and $\cos(\alpha + \beta)$ can all be derived from the expression for $\cos(\alpha - \beta)$. We derive that expression first.

Look at the two diagrams below containing the angle $(\alpha - \beta)$. We assume α is greater than β.

We draw α and β in standard position (ie from the positive x-axis), and let A and B be the points where the terminal sides of α and β cut the unit circle.

We draw the angle $\alpha - \beta$ in standard position and let A′ be the point where its terminal side cuts the unit circle.





A is the point $(\cos\alpha, \sin\alpha)$.
B is the point $(\cos\beta, \sin\beta)$.

A′ is the point $(\cos(\alpha - \beta), \sin(\alpha - \beta))$.
B′ is the point $(1, 0)$.

The triangles OAB and OA′B′ are congruent, since triangle OA′B′ is obtained by rotating OAB until OB lies along the x-axis. Therefore AB and A′B′ are equal in length.

Recall that the distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula

$$(PQ)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

So the distance AB is given by

$$\begin{aligned}(AB)^2 &= (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2 \\ &= \cos^2\beta - 2\cos\alpha\cos\beta + \cos^2\alpha + \sin^2\beta - 2\sin\alpha\sin\beta + \sin^2\alpha \\ &= 2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta.\end{aligned}$$

The distance A′B′ is given by

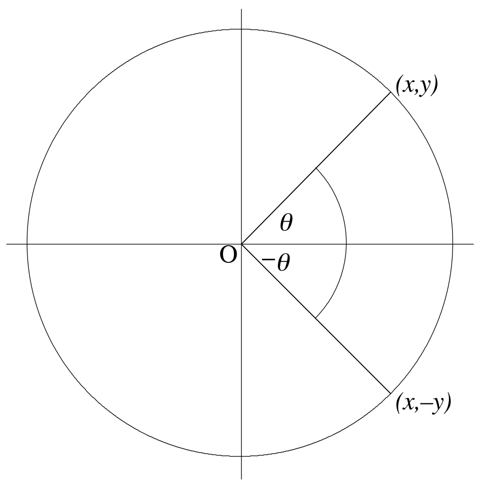
$$\begin{aligned}(\mathrm{A'B'})^2 &= (\cos(\alpha-\beta)-1)^2+(\sin(\alpha-\beta))^2\\ &= \cos^2(\alpha-\beta)-2\cos(\alpha-\beta)+1+\sin^2(\alpha-\beta)\\ &= 2-2\cos(\alpha-\beta).\end{aligned}$$

These distances are equal so

$$\begin{aligned}2-2\cos(\alpha-\beta) &= 2-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta\\ \cos(\alpha-\beta) &= \cos\alpha\cos\beta+\sin\alpha\sin\beta.\end{aligned}$$

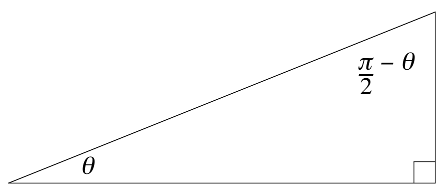
From this we can derive expressions for $\cos(\alpha+\beta)$, $\sin(\alpha+\beta)$ and $\sin(\alpha-\beta)$. In order to do this we need to know the following results:

$$\begin{aligned}\sin(-\theta) &= -\sin\theta\\ \cos(-\theta) &= \cos\theta\end{aligned}$$



and

$$\begin{aligned}\sin(\theta) &= \cos(\frac{\pi}{2}-\theta)\\ \cos(\theta) &= \sin(\frac{\pi}{2}-\theta).\end{aligned}$$



Now

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha-(-\beta))\\ &= \cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)\\ &= \cos\alpha\cos\beta-\sin\alpha\sin\beta\end{aligned}$$

$$\begin{aligned}
\sin(\alpha+\beta) &= \cos[\frac{\pi}{2}-(\alpha+\beta)] \\
&= \cos[(\frac{\pi}{2}-\alpha)-\beta] \\
&= \cos(\frac{\pi}{2}-\alpha)\cos\beta+\sin(\frac{\pi}{2}-\alpha)\sin\beta \\
&= \sin\alpha\cos\beta+\cos\alpha\sin\beta
\end{aligned}$$

$$\begin{aligned}
\sin(\alpha-\beta) &= \sin(\alpha+(-\beta)) \\
&= \sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta) \\
&= \sin\alpha\cos\beta-\cos\alpha\sin\beta.
\end{aligned}$$

These formulae can be used in many different ways.

Examples

1. Simplify $\sin(a+b)+\sin(a-b)$.

$$\begin{aligned}
\sin(a+b)+\sin(a-b) &= \sin a\cos b+\cos a\sin b+\sin a\cos b-\cos a\sin b \\
&= 2\sin a\cos b.
\end{aligned}$$

2. Prove $\sin(\frac{\pi}{2}+\theta)=\cos\theta$ using the addition formulae.

$$\begin{aligned}
\sin(\frac{\pi}{2}+\theta) &= \sin\frac{\pi}{2}\cos\theta+\cos\frac{\pi}{2}\sin\theta \\
&= 1\times\cos\theta+0\times\sin\theta. \\
&= \cos\theta.
\end{aligned}$$

Exercises 4

1. Simplify

a.

$$\frac{\sin(A+B)-\sin(A-B)}{\sin A\sin B}$$

b.

$$\frac{\cos(A+B)+\cos(A-B)}{\cos A\cos B}$$

c.

$$\frac{\cos(A+B)-\cos(A-B)}{\cos A\sin B}.$$

2. Prove

a. $\sin(\pi - \theta) = \sin\theta$

b. $\cos(\pi - \theta) = -\cos\theta$

c. $\cos(\frac{\pi}{2} - \theta) = \sin\theta$

d. $\cos(\frac{\pi}{2} + \theta) = -\sin\theta$.

Expressions for $\tan(A+B)$ and $\tan(A-B)$ follow in a straightforward way. Try to derive them for yourself first.

$$\begin{aligned}\tan(A+B) &= \frac{\sin(A+B)}{\cos(A+B)}\\ &= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}\\ &= \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}\\ &= \frac{\tan A + \tan B}{1 - \tan A\tan B}.\end{aligned}$$

$$\begin{aligned}\tan(A-B) &= \frac{\sin(A-B)}{\cos(A-B)}\\ &= \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B + \sin A\sin B}\\ &= \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}}\\ &= \frac{\tan A - \tan B}{1 + \tan A\tan B}.\end{aligned}$$

Summary

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad (13)$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B} \qquad (14)$$

Exercises 5

1. Show that

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}.$$

2. Setting $\alpha = \frac{2\pi}{3}$ and $\beta = \frac{\pi}{3}$, write down values of $\tan\alpha$, $\tan\beta$ and verify the expressions for $\tan(\alpha+\beta)$ and $\tan(\alpha-\beta)$.

5 Double angle formulae

Expressions for the trigonometric functions of 2θ follow very easily from the preceding formulae.

We shall summarise them and ask you to derive them as an exercise.

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{15}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \tag{16}$$

$$\cos 2\theta = 2\cos^2\theta - 1 \tag{17}$$

$$\cos 2\theta = 1 - 2\sin^2\theta \tag{18}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \tag{19}$$

Example

Show $\cos 2\theta = 2\cos^2\theta - 1$.

$$\begin{aligned}\cos 2\theta &= \cos(\theta+\theta)\\ &= \cos\theta\cos\theta - \sin\theta\sin\theta\\ &= \cos^2\theta - \sin^2\theta\\ &= \cos^2\theta - (1-\cos^2\theta)\\ &= 2\cos^2\theta - 1.\end{aligned}$$

Exercise Derive the rest of the expressions above.

Example

Simplify $\dfrac{\sin 2\theta}{1-\cos 2\theta}$.

$$\begin{aligned}\frac{\sin 2\theta}{1-\cos 2\theta} &= \frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)}\\ &= \frac{2\sin\theta\cos\theta}{2\sin^2\theta}\\ &= \cot\theta.\end{aligned}$$

Exercises 6

1. Simplify $\dfrac{1+\sin(\frac{\pi}{2}-2x)}{1-\sin(\frac{\pi}{2}-2x)}$.

2. Simplify $\dfrac{1+\cos 2\theta}{\sin 2\theta}$.

3. Simplify $\dfrac{1+\sin A-\cos 2A}{\cos A+\sin 2A}$.

6 Applications of the sum, difference, and double angle formulae

A number of relations which are very useful in integration follow from the identities in sections 4 and 5.

From (17) $\cos 2\theta = 2\cos^2\theta - 1$ it follows that

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \qquad (20)$$

and from (15) $\cos 2\theta = 1 - 2\sin^2\theta$ it follows that

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad (21)$$

These identities are very useful in integration. For example

$$\begin{aligned}\int \cos^2\theta d\theta &= \int \frac{1}{2}(1+\cos 2\theta)d\theta \\ &= \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + \mathrm{C}\end{aligned}$$

so you need to be expert in using them to simplify expressions.

Example

Show that $\sin^2 x\cos^2 x = \frac{1}{8}(1-\cos 4x)$.

$$\begin{aligned}\sin^2 x\cos^2 x &= \frac{1}{2}(1-\cos 2x)\times\frac{1}{2}(1+\cos 2x) \\ &= \frac{1}{4}(1-\cos^2 2x) \\ &= \frac{1}{4}(1-\frac{1}{2}(1+\cos 4x)) \\ &= \frac{1}{4}(\frac{1}{2}-\frac{1}{2}\cos 4x) \\ &= \frac{1}{8}(1-\cos 4x).\end{aligned}$$

Exercises 7

Simplify

1. $\cos^4 3\theta$

2. $\sin^4\theta$.

We showed earlier that $\sin(A+B)+\sin(A-B)=2\sin A\cos B$, so
$\sin A\cos B=\frac{1}{2}(\sin(A+B)+\sin(A-B))$.
Obtain similar expressions for $\sin A\sin B$, and $\cos A\cos B$ by using the expressions for $\cos(A+B)$ and $\cos(A-B)$. These relationships are also useful in integration.

Summary

$$\begin{aligned}\sin A\cos B &= \frac{1}{2}(\sin(A+B)+\sin(A-B)) \qquad (22)\\ \cos A\sin B &= \frac{1}{2}(\sin(A+B)-\sin(A-B)) \qquad (23)\\ \cos A\cos B &= \frac{1}{2}(\cos(A+B)+\cos(A-B)) \qquad (24)\\ \sin A\sin B &= \frac{1}{2}(\cos(A-B)-\cos(A+B)) \qquad (25)\end{aligned}$$

Example

Find $\int \sin 6x\cos 2x dx$.

$$\begin{aligned}\int \sin 6x\cos 2x &= \frac{1}{2}\int(\sin 8x+\sin 4x)dx\\ &= -\frac{1}{16}\cos 8x-\frac{1}{8}\cos 4x+\mathrm{C}\end{aligned}$$

Exercises 8

Express as sums or differences the following products:

1. $\sin 7x\cos 3x$

2. $\cos 8x\cos 2x$

3. $\cos 6x\sin 5x$

4. $\sin 4x\sin 2x$.

7 Self assessment

1. Simplify $\dfrac{\sin\theta\csc\theta}{\sin^2\theta+\cos^2\theta}$.

2. Simplify $\dfrac{\sin\theta+\sin\theta\tan^2\theta}{\tan\theta}$.

3. Simplify $\sin(\frac{3\pi}{2}+\theta)$.

4. Verify $\cos^4\theta-\sin^4\theta=\cos 2\theta$.

5. Verify $\dfrac{\sin(A+B)+\sin(A-B)}{\sin(A+B)-\sin(A-B)}=\tan A\cot B$.

8 Solutions to exercises

Pretest

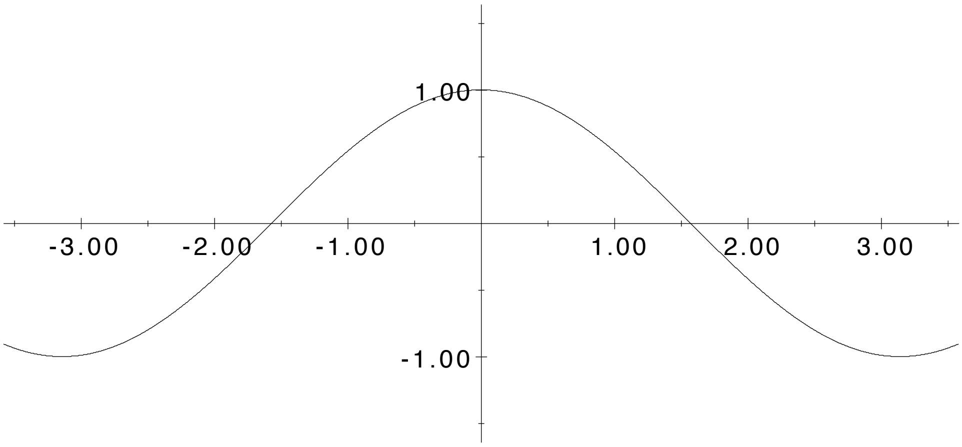
1. a. $\dfrac{\pi}{3}$ b. $\dfrac{3\pi}{4}$ c. $\dfrac{3\pi}{2}$

2. a. $45°$ b. $-270°$ c. $360°$

3. a. 1 b. 0 c. -1

   d. $-\frac{1}{2}$ e. $\frac{1}{2}$ f. 0

4. A graph of the function $y = \cos x$.



Exercises 1

1. a. $\cos x$ b. $\cot\theta$ c. $\sin x$

Exercises 2

1. a. $\sin x\cos x$ b. 1 c. $2 + 2\tan\theta$

Exercises 3

1. $\dfrac{d}{dx}\cot x = -\csc^2 x$

2. $\dfrac{d}{dx}\sec x = \sec x\tan x$

3. $\dfrac{d}{dx}\csc x = -\csc x\cot x$

Exercises 4

1. a. $2\cot A$ b. 2 c. $-2\tan A$

Exercises 5

2. $\tan\alpha = -\sqrt{3}$ and $\tan\beta = \sqrt{3}$

Exercises 6

1. a. $\cot^2 x$ b. $\cot\theta$ c. $\tan A$

Exercises 7

1. $\frac{1}{8}(3 + 4\cos 6\theta + \cos 12\theta)$

2. $\frac{1}{8}(3 - 4\cos 2\theta + \cos 4\theta)$

Exercises 8

1. $\frac{1}{2}(\sin 10x + \sin 4x)$

2. $\frac{1}{2}(\cos 10x + \cos 6x)$

3. $\frac{1}{2}(\sin 11x - \sin x)$

4. $\frac{1}{2}(\cos 2x - \cos 6x)$

Self assessment

1. $\dfrac{\sin\theta\csc\theta}{\sin^2\theta + \cos^2\theta} = 1$

Use $\csc\theta = \dfrac{1}{\sin\theta}$ and $\sin^2\theta + \cos^2\theta = 1$.

2. $\dfrac{\sin\theta + \sin\theta\tan^2\theta}{\tan\theta} = \sec\theta$

Use $1 + \tan^2\theta = \sec^2\theta$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$.

3. $\sin(\dfrac{3\pi}{2} + \theta) = -\cos\theta$

Use $\sin\dfrac{3\pi}{2} = -1$ and $\cos\dfrac{3\pi}{2} = 0$.

4.

$$\begin{aligned} cos^4\theta - \sin^4\theta &= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) \\ &= \cos 2\theta \times 1 \\ &= \cos 2\theta. \end{aligned}$$

5.

$$\begin{aligned} \frac{\sin(A+B) + \sin(A-B)}{\sin(A+B) - \sin(A-B)} &= \frac{2\sin A\cos B}{2\cos A\sin B} \\ &= \tan A\cot B. \end{aligned}$$

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