Computational & Algorithmic Topology, Sydney

The University of Sydney
27 June – 1 July 2017

Tue, Wed, Thu talks in Carslaw 375
Lunch and afternoon tea in
Staff Tea Room, Carslaw 727

	Tuesday 27/6	Wednesday 28/6		Thursday 29/6
10:00 – 11:00	Hyam Rubinstein *(p.2)*	Hyam Rubinstein *(p.2)*	10:00 – 11:30	Monique Teillaud *(p.2)*
11:15 – 12:15	Serge Gaspers *(p.1)*	Serge Gaspers *(p.1)*	11:45 – 12:15	Nikki Sanderson *(p.6)*
12:15 – 14:00	Lunch	Lunch	12:15 – 14:00	Lunch
14:00 – 15:00	Abby Thompson *(p.2)*	Abby Thompson *(p.2)*	14:00 – 15:30	Vanessa Robins *(p.1)*
15:15 – 15:45	Byunghee An *(p.3)*	Ingrid Irmer *(p.4)*	15:30 – 16:00	Afternoon tea
15:45 – 16:15	Afternoon tea	Afternoon tea		
16:15 – 16:45	Tim Ophelders *(p.5)*	Hwa Jeong Lee *(p.4)*		

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Fri, Sat talks in Abercrombie 1170
Afternoon tea provided

	Friday 30/6	Saturday 1/7
10:00 – 11:00	Joel Hass *(p.3)*	Erin Chambers *(p.3)*
11:15 – 12:15	Yusu Wang *(p.6)*	Murray Elder *(p.3)*
12:15 – 14:00	Lunch	Lunch
14:00 – 15:00	Hubert Wagner *(p.6)*	Dave Letscher *(p.4)*
15:15 – 16:15	Jessica Purcell *(p.5)*	Hyam Rubinstein *(p.5)*
16:15 – 16:45	Afternoon tea	Afternoon tea
16:45 – 17:45	Clément Maria *(p.4)*	
19:00 –	Conference dinner *(p.7)*	

An introduction to parameterized complexity

Serge Gaspers
University of New South Wales

The main motivation for parameterized complexity is that in many applied settings, the difficulty of solving a problem does not merely depend on the size of the instance, but on other parameters of the instance as well. It enables a much more fine-grained complexity analysis than the classical theory around NP-hardness. In this talk, we will see some of the basic algorithmic techniques for taking advantage of small parameters of the input and briefly discuss fixed-parameter intractability. We will mainly focus on concepts that are especially relevant to computational topology, including kernelization, treewidth, integer linear programming, and bidimensionality.

M. Cygan, F. Fomin, L. Kowalik, D. Lokshtanov, D. Marx, M. Pilipczuk, M. Pilipczuk, and S. Saurabh, *Parameterized Algorithms*, Springer, 2015.

R. Downey and M. Fellows, *Fundamentals of Parameterized Complexity*, Springer, 2013.

Discrete Morse theory

Vanessa Robins
Australian National University

Morse theory describes deep connections between smooth manifolds, M, and smooth real-valued functions, f, defined on those manifolds. The initial results are due to Morse in 1934, and the standard reference on the subject is due to Milnor (1962). It is a highly adaptable tool that allows mathematicians to study the topology of spaces via functions. For example, Morse theory was used by Smale (1961) to prove the higher-dimensional Poincare conjecture.

Discrete Morse theory is an analogous theory for cell complexes developed by Robin Forman in the 1990s. It is a fully discrete and combinatorial theory, not a piecewise linear adaption, and it enabled Robin Forman to prove many results similar to the smooth theory. Discrete Morse theory has been applied in mathematical fields from graph theory to combinatorics, and in applied computational topology. This talk will introduce the subject following Robin Forman's papers, and discuss some of its applications in computational topology and even image analysis.

R. Forman, *Morse Theory for Cell Complexes*, Advances in Mathematics, vol. 134, pp. 90–145, 1998.

R. Forman, *A user's guide to discrete Morse theory*, Séminaire Lotharingien de Combinatoire, vol. 48, p. B48c, 2002

V. Robins, P. Wood, and A. Sheppard, *Theory and algorithms for constructing discrete Morse complexes from grayscale digital images*, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 33, no. 8, pp. 1646–1658, 2011.

O. Delgado-Friedrichs, V. Robins, and A. Sheppard, *Skeletonization and Partitioning of Digital Images Using Discrete Morse Theory*, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 37, no. 3, pp. 654–666, 2015.

Sweepouts
Hyam Rubinstein
University of Melbourne

Lecture 1: We will talk about different types of surfaces in 3-manifolds and corresponding sweepouts of interest. Lusternik–Schnirelmann theory of simple closed geodesics on convex surfaces forms a nice 2-dimensional model for sweepouts. Connections to Morse theory, especially the index of critical points of generic functions will be discussed.

Lecture 2: In joint work with Dunfield, Garoufalidis, and Hodgson a new approach to sweepout theory is being developed, inspired by the 3d index coming from mathematical physics. A key relationship is between Haken normalisation of surfaces and sweepouts.

J. Hass, *What is an almost normal surface?*, Contemp Math, 597, pp. 1–13, 2013.

S. Tillmann, *Normal surfaces in topologically finite 3-manifolds*, Enseign Math. (2) 54 no.3–4, pp. 329–380, 2008.

S. Garoufalidis, C. Hodgson, N. Hoffman, H. Rubinstein, *The 3D-index and normal surfaces*, arxiv:1604.02688

J. Milnor, *Morse theory*, Princeton University Press, 1963.

Delaunay triangulations: theory and practice
Monique Teillaud
INRIA Nancy

Delaunay triangulations are one of the classic core data structures in computational geometry. The lecture recalls their definition and their basic properties. It surveys algorithms to compute them, before focusing on issues that arise during their implementation and the solutions used in CGAL [1]. A few applications of Delaunay triangulations are presented, in particular meshing algorithms. Finally, an overview of generalizations and extensions is provided. This lecture does not assume any prior knowledge on Delaunay triangulations.

M. de Berg, O. Cheong, M. van Kreveld, M. Overmars, *Computational Geometry: Algorithms and Applications*, Springer, 2008.

J. Boissonnat, M. Yvinec, *Algorithmic Geometry*, Cambridge University Press, 1998.

[1] CGAL, *The Computational Geometry Algorithms Library*, www.cgal.org

Thin position
Abby Thompson
University of California, Davis

Talk 1: Thin position for knots in the 3-sphere. I'll define thin position for knots in the 3-sphere, and describe results and applications. I'll also discuss some generalizations to higher index width.

Talk 2: Thin position for 3-manifolds. The ideas behind thin position for knots can be generalized to describe "efficient" handle decompositions of 3-manifolds, with some analogous applications. I'll describe these, as well as some generalizations to other varieties of Morse functions. Most of this work is joint with M. Scharlemann.

D. Gabai, *Foliations and the topology of 3-manifolds III*, J. Diff Geom, 26, no. 3, p 479–536, 1987.

A. Thompson, *Thin position and bridge number for knots in the 3-sphere*, Topology, 36, no. 2, pp. 505–507, 1997.

M. Scharlemann, A. Thompson, *Thin position and Heegaard splittings of the 3-sphere*, J. Diff Geom, 39, no. 2, p 343–357, 1994.

Configuration spaces of graphs

Byunghee An
Centre of Geometry and Physics, Korea

We consider the total configuration space $B\Gamma$ of a graph Γ equipped with an action of edges $E(\Gamma)$ and introduce a minimal model $S_*(\Gamma)$ for its singular chain complexes $C_*(B\Gamma)$, which is a DG-module over $\mathbb{Z}[E]$. By using this model, we compute homology groups for several interesting examples, and also discuss about the formality. This is a joint work with Gabriel C. Drummond-Cole and Ben Knudsen.

Burning the medial axis

Erin Chambers
Saint Louis University

The medial axis plays a fundamental role in many shape matching and analysis, but is widely known to be unstable to even small boundary perturbations. Methods for pruning the medial axis are usually guided by some measure of significance, with considerable work done for both 2 and 3 dimensional shapes. However, the majority of significance measures over the medial axis are locally defined, and hence are unable to recognize more global topological features, or are difficult to compute and sensitive to perturbations on the boundary. In this talk, I will present recent work done in 2d and 3d to compute a new significance measure on the medial axis, which we call the burn time function. Using this function, we are able to generalize the classical notion of erosion thickness measure over the medial axes of 2D shapes. We demonstrate the utility of these shape significance measures in extracting clean, shape-revealing and topology-preserving skeletons in 2 and 3D which are robust to noise on the boundary. To conclude, I will also discuss future directions and applications of this work.

(This talk is based on prior joint work with Tao Ju, David Letscher, Lu Liu, Kyle Sykes, and Yajie Yan.)

Equations in groups

Murray Elder
University of Newcastle

I will describe work with Volker Diekert and Laura Ciobanu on deciding the solvability of and expressing the set of all solutions to an equation in a free or virtually free group in PSPACE. As motivation I will explain how various algorithmic or decision problems about free, virtually free and hyperbolic groups can be expressed in terms of equations.

A metric on the space of genus zero shapes

Joel Hass
University of California, Davis

I will discuss a method of comparing shapes based on finding an optimal conformal diffeomorphism between two surfaces. This method gives a metric on the space of Riemannian surfaces of genus zero. I will also describe some alternative approaches to comparing genus zero shapes with cone points, based on comparing conformally equivalent hyperbolic orbifolds. This allows specifying landmarks when comparing surfaces and has been used to compare brain cortices.

An algorithm for computing an immersed analogue of scl on surfaces

Ingrid Irmer
University of Melbourne

There are no known algorithms for computing stable commutator length (scl) in general groups, as such computations involve solving infinite dimensional optimisation problems. Calegari has an algorithm for free groups, and it was conjectured that scl in surface groups is also computable. This talk briefly outlines an algorithm for computing an immersed analogue of scl, called rhscl, on surfaces. The algorithm makes use of an even/odd phenomenon to show that rhscl is algebraic on surfaces. This result would seem to have negative consequences for computability of scl in surface and 3-manifold groups.

The number of knot mosaics

Hwa Jeong Lee
Daegu Gyeongbuk Institute of Science and Technology

Lomonaco and Kauffman developed a knot mosaic system to introduce a precise and workable definition of a quantum knot system. This definition is intended to represent an actual physical quantum system. A knot (m, n)-mosaic is an $m \times n$ matrix of mosaic tiles representing a knot or a link by adjoining properly that is called suitably connected. $D^{(m,n)}$ is the total number of all knot (m, n)-mosaics. This value indicates the dimension of the Hilbert space of these quantum knot system. In this presentation, we construct an algorithm producing the precise value of $D^{(m,n)}$ for $m, n \geq 2$ that uses recurrence relations of state matrices that turn out to be remarkably efficient to count knot mosaics. This is a joint work with Seungsang Oh, Kyungpyo Hong, and Ho Lee.

Topological simplification in 3 dimensions

Dave Letscher
Saint Louis University

One of the fundamental problems is topological persistence is how to remove topological noise from a dataset. In 2 dimensions, persistence homology can be used to remove all noise bellow a given threshold. In 3 dimensions this is not possible. In this talk we see how persistent homotopy can be used to remove topological noise. This is possible using techniques from 3-manifold topology. Normal surface theory based algorithms can be used to remove the noise. We will also discuss the practicalities of performing these simplifications on real world data

A polynomial time algorithm to compute quantum invariants of 3-manifolds with bounded first Betti number

Clément Maria
University of Queensland

In this talk, we introduce a fixed parameter tractable algorithm for computing the Turaev-Viro invariants $TV_{4,q}$, using the dimension of the first homology group of the manifold as parameter. This is, to our knowledge, the first parameterised algorithm in computational 3-manifold topology using a topological parameter. The computation of $TV_{4,q}$ is known to be #P-hard in general; using a topological parameter provides an algorithm polynomial in the size of the input triangulation for the extremely large family of 3-manifolds with first homology group of bounded rank.

The algorithm relies on an interpretation of the weights attached to the definition of the Turaev-Viro invariant in terms of the topology of almost normal surfaces assigned to admissible colourings. This reduces the computation, up to a sign, to a set of counting problems, each of which we can solve efficiently. A particular interest of the method is that it gives a more ‘topological’ definition of the invariant $TV_{4,q}$, as opposed to the more ‘combinatorial’ usual definition, that is heavily dependent on the underlying triangulation of the manifold. This is joint work with Jonathan Spreer.

Sweeping surfaces using short curves

Tim Ophelders
Technische Universiteit Eindhoven

For two curves on a surface, one can quantify how similar they are based on how much one has to stretch the curves in order to continuously deform one into the other. Such a deformation will be a homotopy from one curve to the other that minimizes the length of the longest intermediate curve. We give some structural properties that some such homotopy is guaranteed to have in general, as well as in more restricted cases. The first such restriction assumes that the two curves lie on the boundary of the surface.

Secondly, we consider a discrete variant of this problem, where the surface is represented as an edge-weighted combinatorial map. We show that this variant lies in NP by giving a polynomial bound on the complexity of such a homotopy. Furthermore, we show that this discrete variant is dual to an interesting graph-drawing problem, related to a plane variant of cut-width.

Whether there exists a polynomial time algorithm that gives a sublogarithmic approximation factor remains an open question. The presentation combines results of collaborations with Erin Chambers, Gregory Chambers, Arnaud de Mesmay, and Regina Rotman.

Decompositions of 3-manifolds and geometry

Jessica Purcell
Monash University

A highly useful technique for studying a 3-manifold is to decompose it into simpler pieces, such as tetrahedra, and to examine normal surfaces within the pieces. If the pieces admit additional data, e.g. an angle structure, then there are concrete geometric consequences for the manifold and the surfaces it contains. In this talk, we describe a way to generalise these decompositions, and to extend results to broader families of 3-manifolds. For example we allow pieces that are not simply connected, glued along faces that are not disks, and we define normal surfaces in these cases. We give examples of manifolds with these structures (families of knot complements) and show that the existance of such structures ensures geometric consequences. This is joint work with Josh Howie.

Counting isotopy classes of incompressible surfaces

Hyam Rubinstein
University of Melbourne

This is joint work with Dunfield, Garoufalidis, and Hodgson and arises from the 3d index. We show that a cell complex can be built from realisations of a fixed isotopy class of an incompressible surface in a 3-manifold with a given triangulation, very similar to the curve complex. The 1-skeleton of this graph can be used to detect both incompressibility of a given normal surface and whether two normal surfaces are isotopic or not, assuming they are incompressible.

Witness complex for time series analysis

Nikki Sanderson
University of Colorado, Boulder

A scalar time-series can be "unfolded" into $\mathbb{R}^d$ by delay coordinate reconstruction. In the best case, this gives an attractor that is topologically equivalent to that of the underlying dynamical system. We can then compute the persistent homology of the reconstructed data using a variety of complexes, e.g., Čech, Vietoris-Rips, or alpha. To be more computationally efficient we use a witness complex: it can provide a sparser representation and yet be faithful to the homology. Topologically accurate delay reconstruction requires choice of appropriate values for a d and a time delay. In practice, these must be estimated from the data, and the estimation procedures are heuristic and often problematic. Recent work of Garland et al. demonstrates that accurate persistent homology computations are possible from witness complexes built from delay reconstructions with d below that demanded by the theory. Following this, we introduce novel witness relations that incorporate time and explore the robustness of the resulting homology with respect to choice of delay. The new relations seek to inhibit data points from witnessing landmarks traveling in disparate directions and that are on distinct branches of an attractor, as these can suggest a false connection due to the particular reconstruction.

Topological analysis in information spaces

Hubert Wagner
Institute of Science and Technology, Austria

Understanding high dimensional data remains a challenging problem. Topological Data Analysis (TDA) promises to simplify, characterize and compare such data. However, standard TDA focuses on Euclidean spaces, while many types of high-dimensional data naturally live in non-Euclidean ones. Spaces derived from text, speech, image, ... data are best characterized by non-metric dissimilarities, many of which are inspired by information-theoretical concepts. Such spaces will be called information spaces.

I will present the theoretical foundations of topological analysis in information spaces. A family of dissimilarity measures will be defined along with geometric/topological interpretation. I will show how the framework of TDA can be extended to information spaces equipped with these dissimilarity measures. We will then focus on understanding the properties one member of this family, the Kullback-Leibler divergence. Finally, I will explain how existing software packages can be adapted to this new setting, using appropriate approximations.

This is joint work with Herbert Edelsbrunner and Ziga Virk.

Point data sparsification and denoising for topology inference

Yusu Wang
Ohio State University

Recent years have witnessed much progress and development in computational topology, both in theory and in data analysis applications. However, to further broaden the scope of topological (as well as geometric) data analysis methods, we need to address several challenges caused by the "complexity" of modern data. In this talk, I will describe some of our recent work towards addressing (1) the issue of size and (2) the issue of noise associated with modern data. Specifically, first, I will focus on the problem of homology inference from points sampled, without noise, from a smooth manifold sitting in an Euclidean space. I will describe how we sparsify the input point set and build a complex of small size for homology inference on top of the sparsified data, without requiring any user supplied parameter. Next, I will consider the setting where the input points are noisy samples of a hidden compact set where some samples can be far away from the hidden domain. I will present a simple denoising algorithm with only one parameter, and provide its theoretical guarantees (on the output quality). I will further show that this single parameter can be removed if we assume a stronger sampling condition of the input points. This is joint work with several collaborators: M. Buchet, T. Dey, Z. Dong, and J. Wang.

Conference dinner
ThaiRiffic Newtown
109 King St, Newtown NSW 2042

Shared entrées:
Spring rolls (vegetarian)
Curry puffs (vegetarian)
Satay chicken skewers

Shared mains:
Green curry with vegetables and tofu (vegetarian)
Pad thai (vegetarian)
Pumpkin stir fried (vegetarian)
Thai beef Salad
Chicken cashew nut sauce

(BYO)

Participants

Byunghee An	Centre of Geometry and Physics	Korea
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Kamil Bulinski	University of Sydney	Australia
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Sponsors

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